Lesson Title:	Dilation	**Grade Level:**	High School

Common Core Standards and Standards for Math Practice:

G.SRT.1b Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

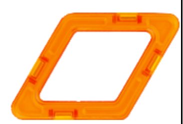
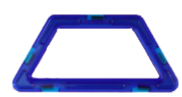
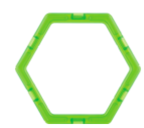
CA Only Standard:
G.GMD.5 Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k, k^2, and k^3, respectively; determine length, area and volume measures using scale factors.

SMP 1: Make sense of problems and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 4: Model with mathematics.
SMP 5: Use appropriate tools strategically.
SMP 7: Look for and make use of structure.

Teacher Preparation:

- Prepare sets of Magformers for each group of students.
  - 16 Equilateral triangles
  - 18 Squares
  - 4-9 Rhombuses
  - 4-9 Trapezoids
  - 3-4 Hexagons

Magformers Used: *(not to scale)*

			
Equilateral triangle	Rhombus	Trapezoid	Hexagon

Brief Overview of the Lesson:

Students build dilations of polygons out of Magformers and conjecture about the effect of dilations on area, volume, and surface area. They prove their conjectures for some shapes.

Teacher Notes:

Students may not be able to extend their understanding of dilations to 3-D objects. Make sure they understand that, as in the plane, all lengths are multiplied by the scale factor. Thus, a 3-D shape that has been dilated by a scale factor greater than one will be longer, wider, and taller. The surface area section is placed last in this activity to emphasize that volumes increase by a factor of k^3 not just because we are dealing with a 3-D shape. The surface area of 3-D shapes does NOT increase by a factor of k^3.

REVIEW: DILATION AND LENGTH

A dilation is a a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

1. What makes dilation different from the other transformations: translation, rotation and reflection?

   Sample student response: Rotations, reflections, and translations do not alter the size of a figure. They always produce a congruent figure. A dilation can produce a smaller or larger figure. A dilated figure is not always congruent to the original figure.

2. If a line segment is dilated by a factor of k, how will the length of the new line segment compare to that of the original? Assume k>0.

   Sample student response: The length of the new segment will be |k| times the length of the original segment.

3. If a given parallelogram $ABCD$ is dilated by a factor of 3, what will the effect be on the:

   a. length of side AB? It will be 3 times longer than the original length.

   b. length of side BC? It will be 3 times longer than the original length.

   c. height of the parallelogram? It will be 3 times the original height

   d. length of diagonal AC? It will be 3 times longer than the original length.

   e. perimeter? It will be 3 times longer than the original perimeter.

4. We are going to be working with area in this activity. We can measure area using many different units including square inches, hectares, square meters, and acres. For this activity, we are going to define 1 trimag to be equal to the area of an equilateral triangle Magformers piece. What is the area of the hexagon? 6 Trimags.

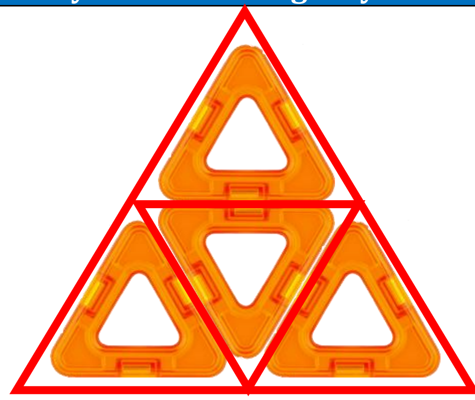
PART 1: DILATION AND AREA

6. Take a wild guess: A given shape has an area of 10 trimags. If this shape is dilated by a factor of 3, what will the area of the new shape be?

   Answers will vary. Some students may guess that the new area will be 30 trimags.

7. Equilateral Triangle: Place one equilateral triangle Magformers piece on your desk. Use Magformers to create a triangle that could represent a dilation of the original triangle by a scale factor of 2. Sketch your creation.

Sketch of My Dilated Triangle by Scale Factor of 2



8. Record the area of your dilated triangle in the table below. Complete the remainder of the table. Remember that an equilateral triangle has an area of 1 trimag.

Area in Trimags after Dilation				
	Original Area	**Area after dilation with scale factor of 2**	**Area after dilation with a scale factor of 3**	**Area after dilation with a scale factor of 4**
Equilateral Triangle	1	4	9	16

9. Make a conjecture by completing the following sentence.

   When an equilateral triangle is dilated by a factor of k, with $k>0$, the area of the new triangle… can be found by multiplying by k^2.

10. Does your conjecture hold true with other polygons? Experiment with your Magformers set to complete the table below with your group. Remember that an equilateral triangle represents an area of 1 trimag.

Area in Trimags after Dilation			
Polygon	**Original Area**	**Area after a dilation with a scale factor of 2**	**Area after a dilation with a scale factor of 3**
Rhombus	2	8	18
Trapezoid	3	12	27
Hexagon	6	24	

11. Write a conjecture about the effect of dilation with on the area of any given polygon.

Sample student response: When a polygon is dilated by a scale factor of k, the area of the dilated polygon is k^2 times the area of the original polygon.

12. Write a formal proof to show that your conjecture is true for all trapezoids. *(Hint: What is the area of a trapezoid with height h and bases b_1 and b_2?)*

The area, A, of a trapezoid with height h and bases b_1 and b_2 is given by the formula $A = \frac{(b_1+b_2)h}{2}$. If a trapezoid is dilated by a factor of k with $k>0$, the distance between any two points on the original trapezoid will be k times as long on the dilated figure. This means the area of the dilated trapezoid will be $A = \frac{(kb_1+kb_2)kh}{2} = \frac{k^2(b_1+b_2)h}{2} = k^2 \cdot \left(\frac{(b_1+b_2)h}{2}\right)$. This means the area of the dilated trapezoid is k^2 times the area of the original trapezoid.

13. Prove that your conjecture is also true for one of the following shapes:
    a generic triangle (not necessarily equilateral), a rhombus, or a regular hexagon.

Answers will vary. *Sample student response:* A regular hexagon can be divided into two trapezoids. Since the area of a dilated trapezoid is k^2 times the area of the original trapezoid, the area of a regular hexagon must also be multiplied by k^2 when dilated by a scaled factor of k.

PART 2: DILATION, VOLUME, AND SURFACE AREA

14. Take a wild guess: A given 3-D solid has a volume of 5 cubic units and is dilated by a scale factor of 3. What will the volume of the new solid be?

    Answers will vary.

15. Write a conjecture about the effect of a dilation on the volume of a 3-D solid.

    Answers will vary.

16. Test your conjecture on at least 2 different prisms made from Magformers. Which prisms did you construct? Did your results support your conjecture? If not, write a new conjecture.

    Answers will vary. *Sample student response:* I made an equilateral triangle prism and a trapezoidal prism. When I dilated each prism by a scale factor of 2, I could fit 8 of the original prism inside of the dilated prism. This supports my conjecture that the volume of a solid increases by a factor of k^3 when dilated by a scale factor of k.

17. Prove your conjecture for one of the following right prisms:
    a trapezoidal prism, a triangular prism, or a parallelogram prism.

    Answers will vary. Any prism has a volume equal to the area of the base times the height. A dilation of a figure by a scale factor of k multiplies the area of a 2-D figure by k^2 and any lengths by k. If a prism with height h and base area B is dilated by a scale factor of k, the volume of the new prism would be $V = (k^2B)(kh) = k^3(Bh)$. This proves my conjecture for prisms if my area conjecture is in fact true for all shapes.

18. Write a conjecture about the effect of a dilation on the surface area of a 3-D solid.

    Answers will vary. *Sample student response:* I think the surface area of a 3-D solid will be k^2 times the surface area of the original once the solid is dilated because area increases by a factor of k^2.

19. Test your conjecture on at least 1 prism and at least 1 non-prism. Which solids did you construct? Did your results support your conjecture? If not, write a new conjecture.

    I made a prism with a rhombus base and a square pyramid. When either was dilated by a scale factor of 2, each face had four times the area. The overall surface area was also four times as large. Yes, this supports my conjecture.

20. Summarize what you learned in this activity.

    Answers will vary.