VOLUME OF A PYRAMID

PART 1: DESIGNING A PYRAMID

1. Build three faces of a square pyramid using one square Magformers piece and two isosceles right triangle Magformers as shown. Notice that all three faces are perpendicular to each other.

2. Two faces of the square pyramid are missing, but they are not the same shape as any of the three faces we have so far. Explain why the missing faces are two congruent right triangles.

____

____

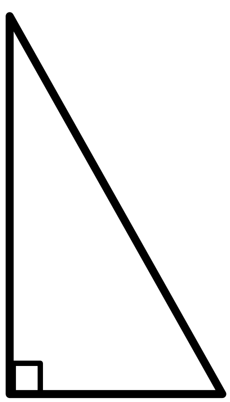
____

____

____

____

3. Find the length of each side of the missing faces by labeling the right triangle below. Show your work. For this activity, we will use 6.1 cm as the length of each side of the square and the length of the legs of the Magformers right isosceles triangles.



4. Cut out the two remaining faces from the cardstock provided.   Use a ruler to be precise. Tape them onto your pyramid.

5. Build a cube using the pyramid pieces you and the other members of your group made. How many pyramids are required to make a cube?

____

6. A cube is a type of prism. Recall that the volume of a prism is given by the equation $V = Bh$, where B represents the area of the base of the prism and h represents the height of the prism. Use this information to determine an equation for the volume of this pyramid in terms of the area of its base, B, and height, h. Justify your reasoning.

____

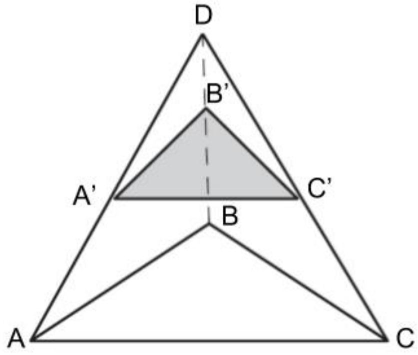
____

____

____

PART 2: CROSS SECTIONS OF A PYRAMID

7. Consider the following triangular pyramid with the cross section of $\Delta A'B'C'$ taken parallel to the base ΔABC. Can you describe a dilation that would take ΔABC to $\Delta A'B'C'$?

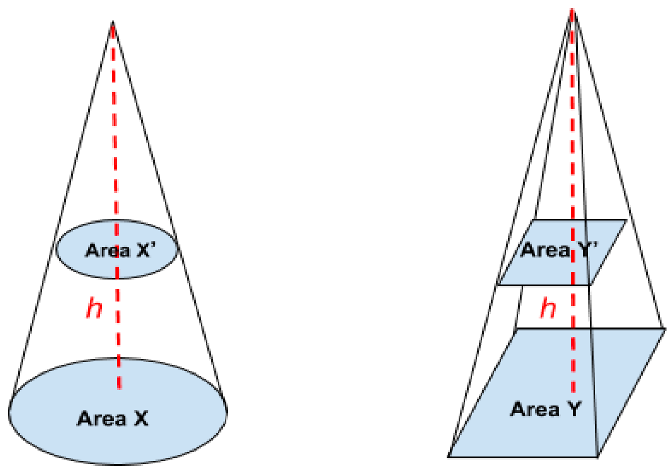


8. What can you conclude about the relationship between ΔABC and $\Delta A'B'C'$?

9. Let k be the constant of proportionality between the length of the sides in ΔABC and $\Delta A'B'C'$. What can you conclude about the area of $\Delta A'B'C'$?

10. What generalizations can you make about pyramids with any polygon base and cross sections taken parallel to the base? Explain.

11. Consider the cone and the pyramid below. Both figures have the same base area and the same height, h. Explain why the cross sections for the figures halfway between the base and the vertex have the same area.



____

____

____

____

____

PART 3: USING CAVALIERI'S PRINCIPLE TO DESCRIBE VOLUMES

Cavalieri's Principle

An Italian mathematician named Bonaventura Cavalieri (1598-1647) proposed an interesting idea about volumes. He looked at cross sections of solids and discovered that if you are given two solids of the same height and the resulting horizontal cross sections at every height have the same area, then the solids have the same volume.
We call his idea Cavalieri's Principle.

12. Revisit your equation in question 6. Use Cavalieri's principle to make an argument about the volume equation of a pyramid with any polygon base in terms of its base area, B, and height, h?

____

____

____

____

____

____

13. Now consider a cone with a height h, and a circle for its base. Make an argument for the volume equation of a cone in terms of its base area, B, and height, h?

____

____

____

____

____

____

14. Each stack of pennies contains 14 pennies. What can you conclude about the volume of each stack? Explain.

____

____

____

____

____

15. Consider a pyramid constructed with one square and 4 equilateral triangle faces and a pyramid constructed with one super square and 4 super equilateral triangle faces. How do the volumes of each pyramid compare to each other? Explain your reasoning.

____

____

____

____

____