Problem for consideration:

13 books are placed on a shelf. Three of the books are distinct math books. The other ten books are also distinct but are not math books. In how many ways can the books be arranged so **none** of the math books are next to each other?

1. Solution via summation by counting all cases. M = math book

Starting position of math books

1	2	3	4	5	6	7	8	9	10	11	12	13
M_1		M_2		M_3								

Final position of math books

1	2	3	4	5	6	7	8	9	10	11	12	13
								M_1		M_2		M_3

$$\sum_{i=1}^{9}\sum_{j=i+2}^{11}\sum_{k=j+2}^{13} 10!$$

Notice the starting values of the sum are separated by 2

10! is the ways to arrange the other 10 non-math books.

We also need to multiply this summation by 3! which is the ways to arrange the math books.

Total Arrangements: $3!\sum_{i=1}^{9}\sum_{j=i+2}^{11}\sum_{k=j+2}^{13} 10! = 3,592,512,000$

2. **Counting the number of cases with combinations**

- **No math books on the ends**
  - **Think of the 3 math books as dividers. We place 4 non-math books between the dividers to get started to ensure no math book on the end and at least one space between the math textbooks. This leaves 10 – 4 = 6 non-math books that we need to place somewhere between the dividers.** $\binom{6+3}{3}=\binom{9}{3}$
  - **Below is a specific case to illustrate the concept. The | represents the math textbook and the * represents the non-math textbook.**

1	2	3	4	5	6	7	8	9	10	11	12	13
*			*	*			*	*			*	*	*	*	*

- **Math book on the left-end**
  - **We cannot place a non-math text-book to the left of the first math book. We have 10 non-math textbooks to place. We initially place 1 non-math book between the first/second divider, 1 between the second/third divider, and 1 after the 3rd divider. This leaves 10-3 = 7 non-math books to place between the math books.**
    $\binom{7+2}{2}=\binom{9}{2}$
  - **Note the divider in spot 1 is fixed. Specific example below:**

1	2	3	4	5	6	7	8	9	10	11	12	13
		*	*			*	*	*	*			*	*	*	*

- **Math book on the right-end:**
  - **Same as the left end case above:** $\binom{7+2}{2}=\binom{9}{2}$
- **Math books on each end**
  - **The dividers on the end are fixed. We have to place at least one non-math textbook between the first/second divider and at least one non-math textbook between second/third divider. This leaves 10 – 2 = 8 non-math textbooks to place.**
    $\binom{8+1}{1}=\binom{9}{1}$
  - **Example:**

1	2	3	4	5	6	7	8	9	10	11	12	13
		*			*	*	*	*	*	*	*	*	*		

- **Total Cases:**
  - $\binom{9}{3}+2\binom{9}{2}+\binom{9}{1}=165$ **cases.**
  - **Arrange the math books: 3!**
  - **Arrange the non-math books: 10!**
  - **Total Arranges: 165(10!)(3!) = 3,592,512,000**

3. Fundamental Counting Principle

- Consider placing the 10 non-math books onto the shelf… There are **10!** Ways this can be accomplished.
- The math books must be placed into the spaces between the non-math books *or* at the beginning/end. This means there are 11 'spaces' the math books can go: before #1, between 1 & 2, …, between 9 & 10, after #10.
- We select 3 of the 11 spaces and order the distinct books into the positions. We can do this in P(11,3) ways.
- Total ways: 10! P(11,3) = 3,592,512,000