Teaching Precalculus with a Problem Solving Focus

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WHY?

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An Agenda for Action: Recommendations for School Mathematics in the 1980s

"Problem solving should be the focus of school mathematics" (NCTM, 1980, p.1)

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CCSS – Mathematical Practice

The eight Standards for Mathematical Practice are:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

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Mathematics Teaching Practices (NCTM, 2014)

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical expressions.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Dimensions and Core Features of Classrooms that Promote Understanding

DIMENSIONS	CORE FEATURES
Nature of Classroom Tasks	Make mathematics problematic Connect with where students are Leave behind something of mathematical value
Role of the Teacher	Select tasks with goals in mind Share essential information Establish classroom culture
Social Culture of the Classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user Used with purpose--to solve problems Used for recording, communicating, and thinking
Equity and Accessibility	Tasks are accessible to all students Every student is heard Every student contributes

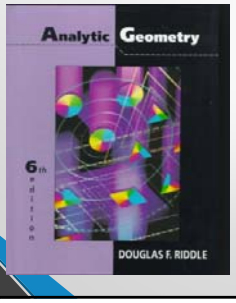
Appropriate Tasks

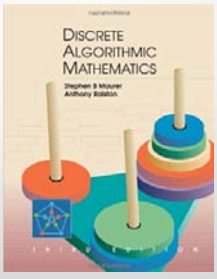
- Make the MATHEMATICS problematic. That is, is it posed as an interesting problem.
- Connect with where the students are. They have to be able to use their current knowledge and skills to complete the task.
- Engage the students in thinking about important mathematics. They take away something of mathematical value.
- Drive the level of questioning.

Research on Tasks

- Not all tasks provide the same opportunities for student thinking and learning (Hiebert et al. 1997; Stein et al. 2009).
- Student learning is **greatest** in classrooms where the tasks **consistently encourage high-level student thinking and reasoning** and **least** in classrooms where the tasks *are routinely procedural in nature* (Boaler and Staples 2009; Hieber and Wearne 1993; Stein and Lane 1996)

Texts:





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Topics Covered…

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Graph Theory

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The Amazon Problem

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Amazon, as you know, is one of the world's largest retailers. In 2017, the company made over 5 billion worldwide Prime shipments alone. An organized and efficient distribution network is needed to manage such a large quantity of shipments. While Amazon does not publicly release exact numbers, one logistics consulting company's research reports that Amazon has 122 fulfillment centers in the United States.

Imagine you work for Amazon and are in charge of planning the locations of these fulfillment centers. Where would you place these 122 centers? What factors would you need to consider.

Work in your group to determine a location for every one of the 122 Amazon fulfillment centers. On the map below you must display the location of each center. You must also include an explanation for how you determined these locations. Identify any resources you used in making your decisions as well as the reasoning behind your decisions.

A Counting Problem

13 books are placed on a shelf. Three of the books are distinct math books. The other ten books are also distinct but are not math books. In how many ways can the books be arranged so **none** of the math books are next to each other?

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Compare and contrast the graphs of the functions within each column below.

COLUMN A	COLUMN B
$y = x^3 + 4x^2$	$y = x^3 - 4x$
$y = x^3 + 4x^2 + 5x$	$y = x^3 + x^2 - 4x$
$y = 2x^3 - 5x^2$	$y = x^3 + x^2 - 4x + 5$
$y = x^3 + 4x^2 - x$	$y = x^3 - 10x$
$y = -x^3 + 3x^2$	$y = 8x^3 - 25x$
$y = x^3 + 4x^2 + 3$	$y = x^4 + 8x^3 - 6x$

What conjecture can your group propose regarding the behaviors you see in these graphs? How can you test it? Why do you think your conjecture holds broadly?

Consider the function:

$$f(x) = 2x^3 + 3x^2 - 36x + 48$$

Alternatively,

$$2x^3 + 3x^2 - 36x - y + 48 = 0$$

Imagine you translate the *x* and *y* axes so the origin moves to the point (*h*, *k*).

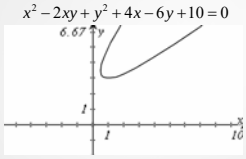
a. Find the equation of the function relative to the new axes. Simplify your equation by grouping like terms.

b. Determine the values of *h* and *k* that will eliminate the *x*-term and the constant term. Explain the significance of the point(s) (*h*, *k*) as they relate to the behavior of the function.

c. Find the value(s) of *h* and *k* that eliminate the second-degree term and the constant. What is the significance of (*h*, *k*) in this case?

- Consider the equation and graph given below:

$$x^2 - 2xy + y^2 + 4x - 6y + 10 = 0$$



Determine the point on the graph that is furthest to the lowest.

Determine the point on the graph that is the furthest to the left.