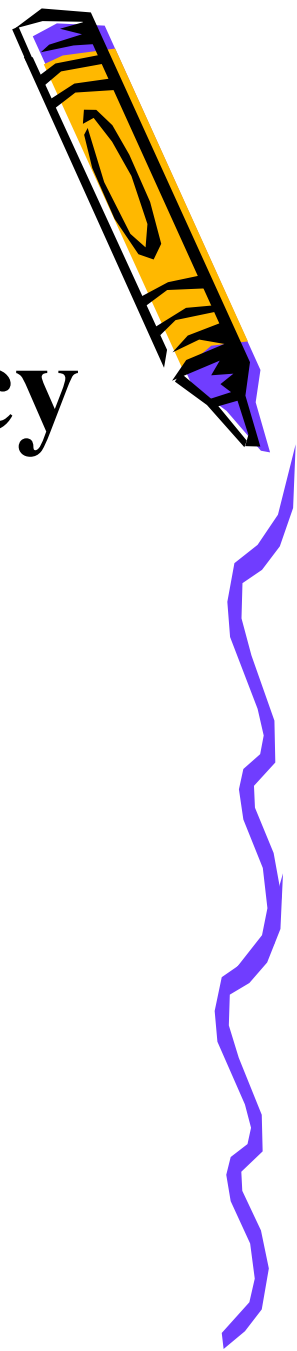# Developing and Assessing Addition Fact *Fluency* in Meaningful Ways

Gina Kling
Jennifer Bay-Williams
NCTM 2019

What do we mean by fluency with basic facts?
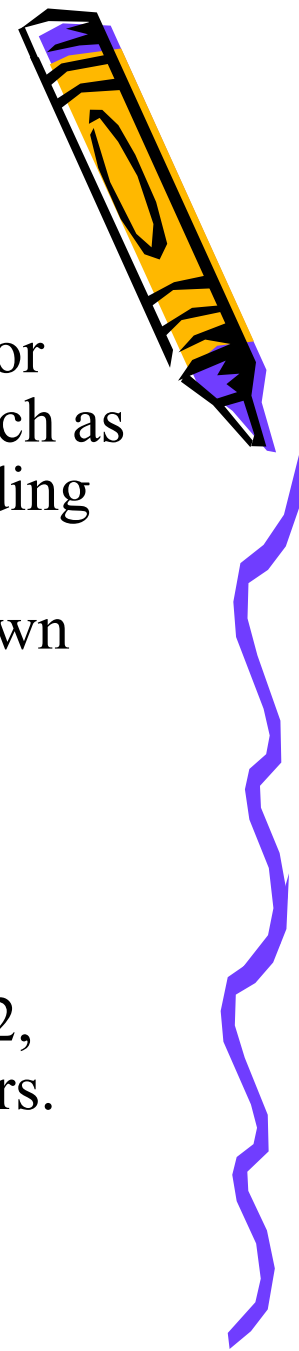
CCSS-M Descriptions

Grade 1 (1.OA.C.6):

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.OA.B.2):

Fluently add and subtract within 20 using mental strategies (reference to **1.OA.C.6**). By end of Grade 2, know from memory all sums of two one-digit numbers.

CCSS-M Descriptions

Grade 1 (1.0A.C.6):

Add and subtract within 20, ***demonstrating fluency*** for addition and subtraction within 10. ***Use strategies*** such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.0A.B.2):

Fluently add and subtract within 20 ***using mental strategies*** (reference to **1.0A.C.6**). By end of Grade 2, ***know from memory*** all sums of two one-digit numbers.

This would suggest that *fluency is different* from automaticity. Research heavily supports this…

So, what does fluency *really* mean?

Procedural <u>fluency</u> is skill in carrying out procedures **flexibly**, **accurately**, **efficiently** and **appropriately**.

(NCTM, 2014; CCSSO, 2010; NRC, 2001)

Procedural Fluency

Knowing from Memory

(an outcome)

$\neq$

Memorization
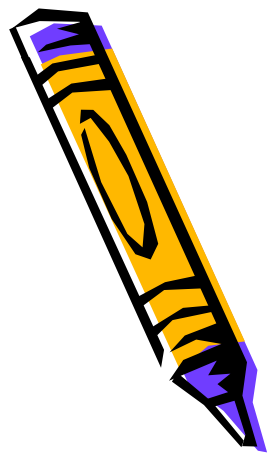
(an instructional strategy)

Developing Addition Fact Fluency

Mastering Basic Facts

Phase 1: Counting
(counts with objects or mentally)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery
**(efficient production of answers)**

Adapted from Baroody, 2006

Mastering Basic Facts

Some programs attempt to push children from Phase 1 directly to Phase 3 through drill and rote memorization.
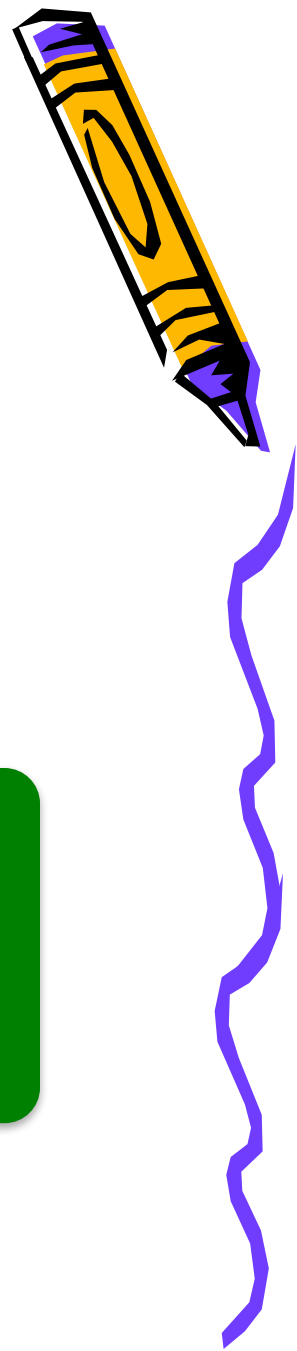
- What aspects of fluency are/are not developed when this happens?

- What are we communicating to children about what it *means to do mathematics* when we do this?

Mastering Basic Facts

NO PASSING ZONE
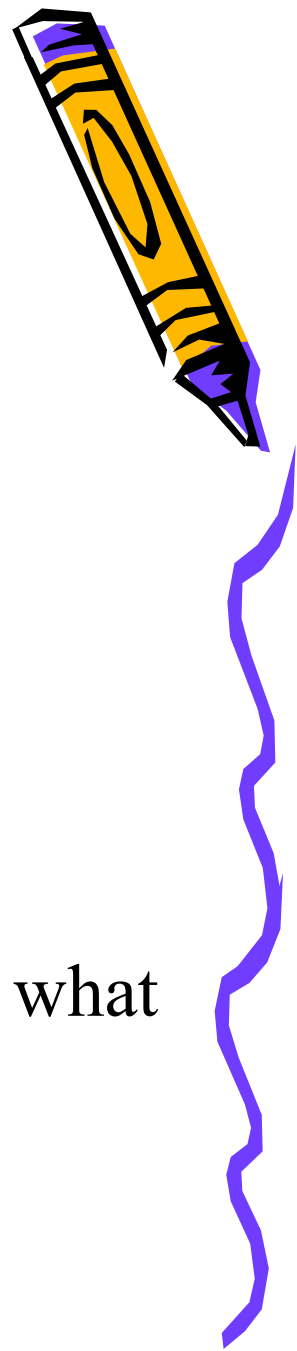
Phase 2: Deriving Fact Strategies
(uses reasoning strategies based on known facts)

Mastering Basic Facts

In contrast, to develop true *fluency*, children need adequate time to make sense of each operation and develop strategies in Phases 1 and 2. Through repeated, meaningful practice, children then naturally progress to Phase 3. This transition occurs with different groups of facts at different times throughout the year.

Addition Facts K-2 Trajectory
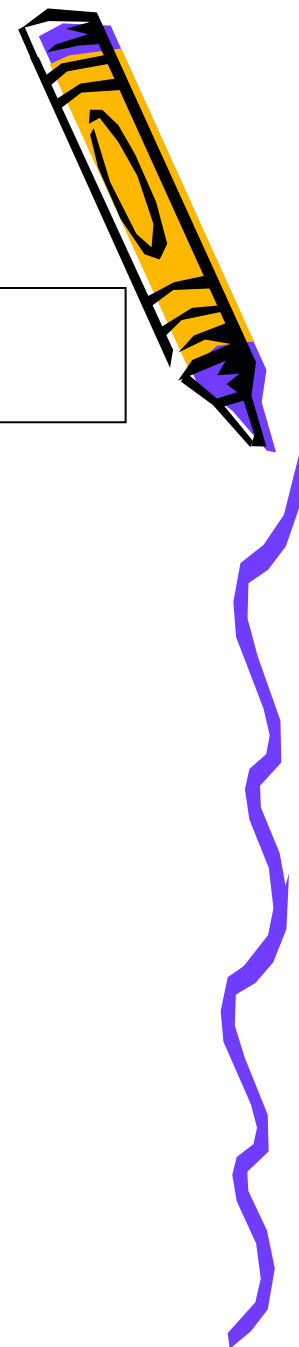
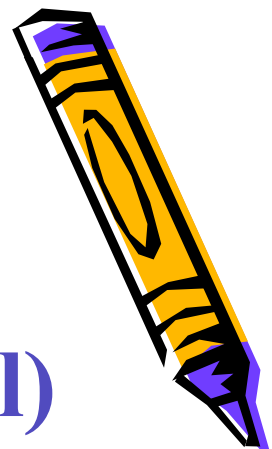| Foundational Fact Strategies	Derived Fact Strategies

Foundational Fact Strategies

- Sums within 5 **K**
- +/– 0, 1, 2 **K**
- Doubles **K** **1**

    (2 + 2, 6 + 6, etc.)

- Combinations of Ten

    (3 + 7, 8 + 2) **K** **1**

- 10 + facts **K**

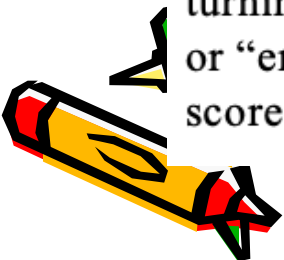# Games for Mastering Combinations of 10 (Foundational)

Go Fish for 10s

This 2–4 player game is played like the card game "Go Fish," only instead of looking for matching cards, children look for combinations of ten. For example, if a child has a 4 in his hand, he would ask another player "Do you have a 6?" Use numeral cards or playing cards with numbers 0–10 (Ace = 1, Queen = 0). Children can continue to draw extra cards as needed, and play continues until all cards are used. Children can then be encouraged to share or record the number sentences for some of the pairs that they found to reinforce the combinations of ten.
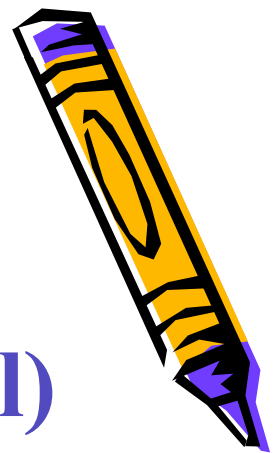
Erase

This 2–4 player game is inspired by the classic game "Sevens." Using a deck of cards with numbers 0–10 (Ace = 1, Queen = 0), calculators (optional), players take turns turning over 7 cards and look for cards that can form a combination of 10 to remove, or "erase" from their hand. The remaining cards are summed to produce the player's score for that round. The player with the lowest score wins.

Games for Mastering Combinations of 10 (Foundational)

Explore the games *Go Fish for 10*s and *Erase* from your handout.

- Compare the games. When might you introduce each to children?

- What other common games could be adapted to provide practice with combinations of 10?

Bay-Williams, J. & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention.* Alexandria, VA: ASCD *and* Reston, VA: NCTM

Meaningful Activities for Learning Basic Addition Facts

- Solve Number Stories
- Use Quick Looks with dot patterns and ten frames
- Discuss/write about strategy use
- Play basic facts games with a focus on reasoning strategies

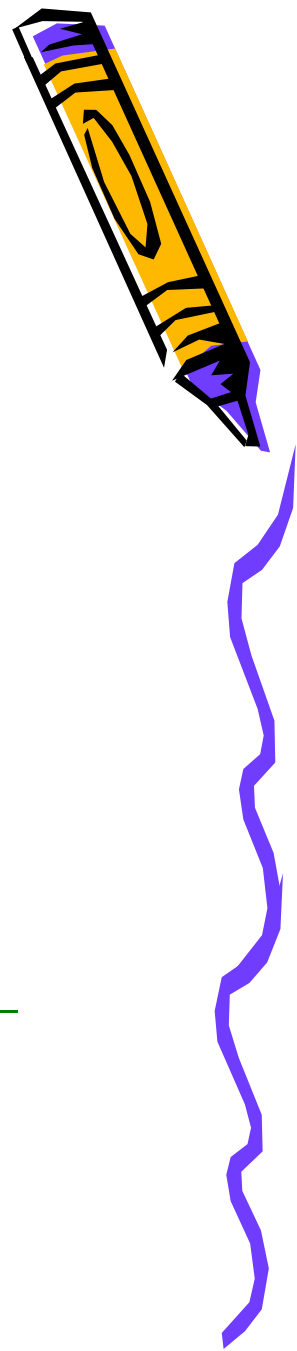# Phase 1 ➡ Phase 2

Quick Looks

PreK:

- ✓ number recognition
- ✓ representation

Kindergarten:

- ✓ decomposing and recomposing numbers
- ✓ subitizing

Grade K- 2:

- ✓ recognizing and developing strategies for basic +/– combinations

Developing Fact Strategies through Quick Looks

- What mathematical concepts and skills can Quick Looks encourage?

- Why is the Quick Look format important?

- How does the arrangement of dots/counters impact the development of mathematical ideas?

The **ABC**s of Quick Looks

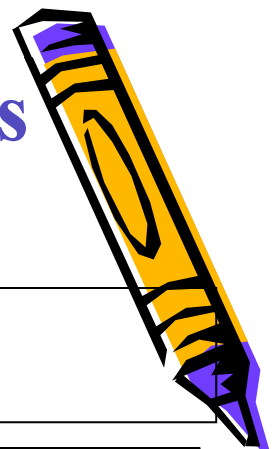**A**lways consider the suggestions for selecting and sequencing cards.

Beyond the quantity…need to elicit *how* they see it (not just how many did they see).

Connections between the visual image and number sentences must be carefully made.

Reasoning Strategies for Addition Facts
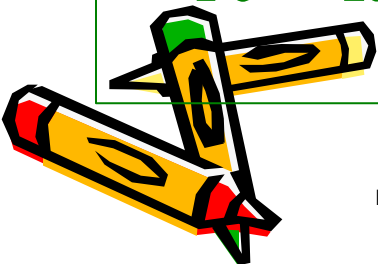## K-2 Trajectory

Foundational Fact Strategies

- Sums within 5   **K**
- +/– 0, 1, 2   **K**
- Doubles   **K 1**
  - (2 + 2, 6 + 6, etc.)
- Combinations of Ten
  - (3 + 7, 8 + 2) **K 1**
- 10 + facts **K**

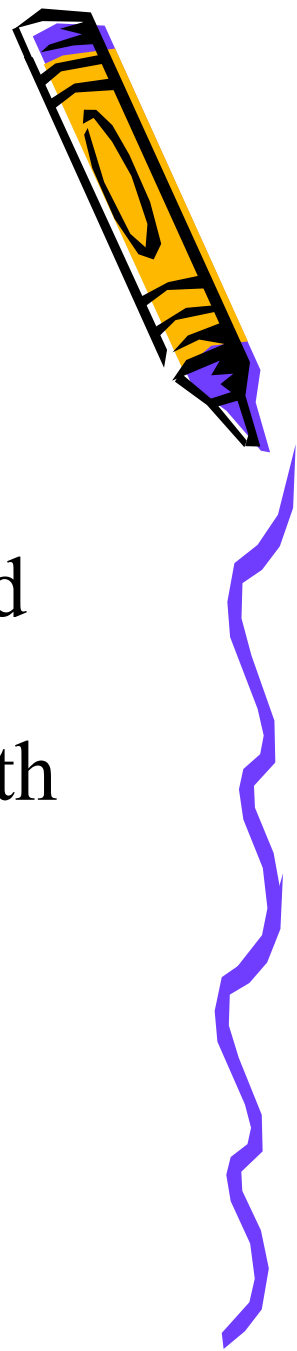| Derived Fact Strategies |
| --- |

- Near Doubles   **1   2**
  - (6 + 7, 8 + 7)
- Making Ten   **1   2**
  - (8 + 3, 9 + 5)
- Pretend-a-Ten   **1   2**
  - (8 + 7, 9 + 5)

Phase 2 ➡ Phase 3
Meaningful Practice

"Practice that follows substantial initial experiences that support understanding and emphasize 'thinking strategies' has been shown to improve student achievement with single-digit calculations." (NRC, 2001).

Games as Meaningful Practice

As you explore *Lucky 13* and *Salute!*, please discuss the following:

- What is the mathematical content of the game?

- What makes this game an example of "meaningful practice"?

- How might you differentiate this game?

- How might you adapt this game to practice different facts or mathematical ideas?

Bay-Williams, J. & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD *and* Reston, VA: NCTM
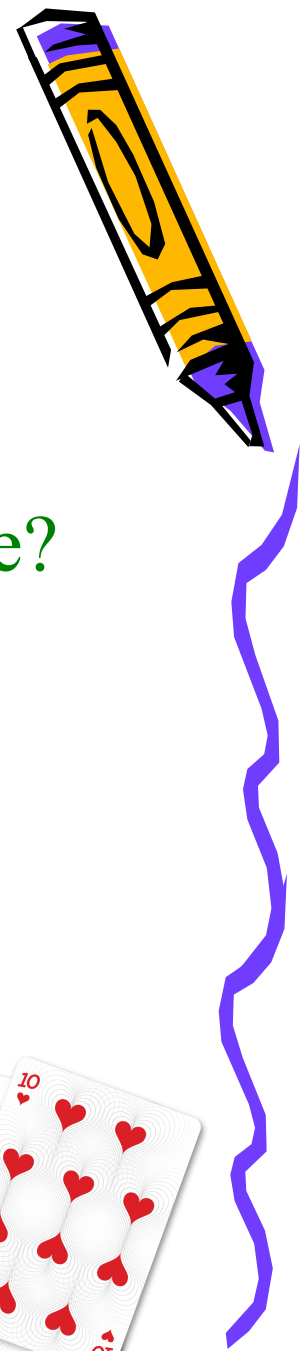
Games as Meaningful Practice

Lucky 13

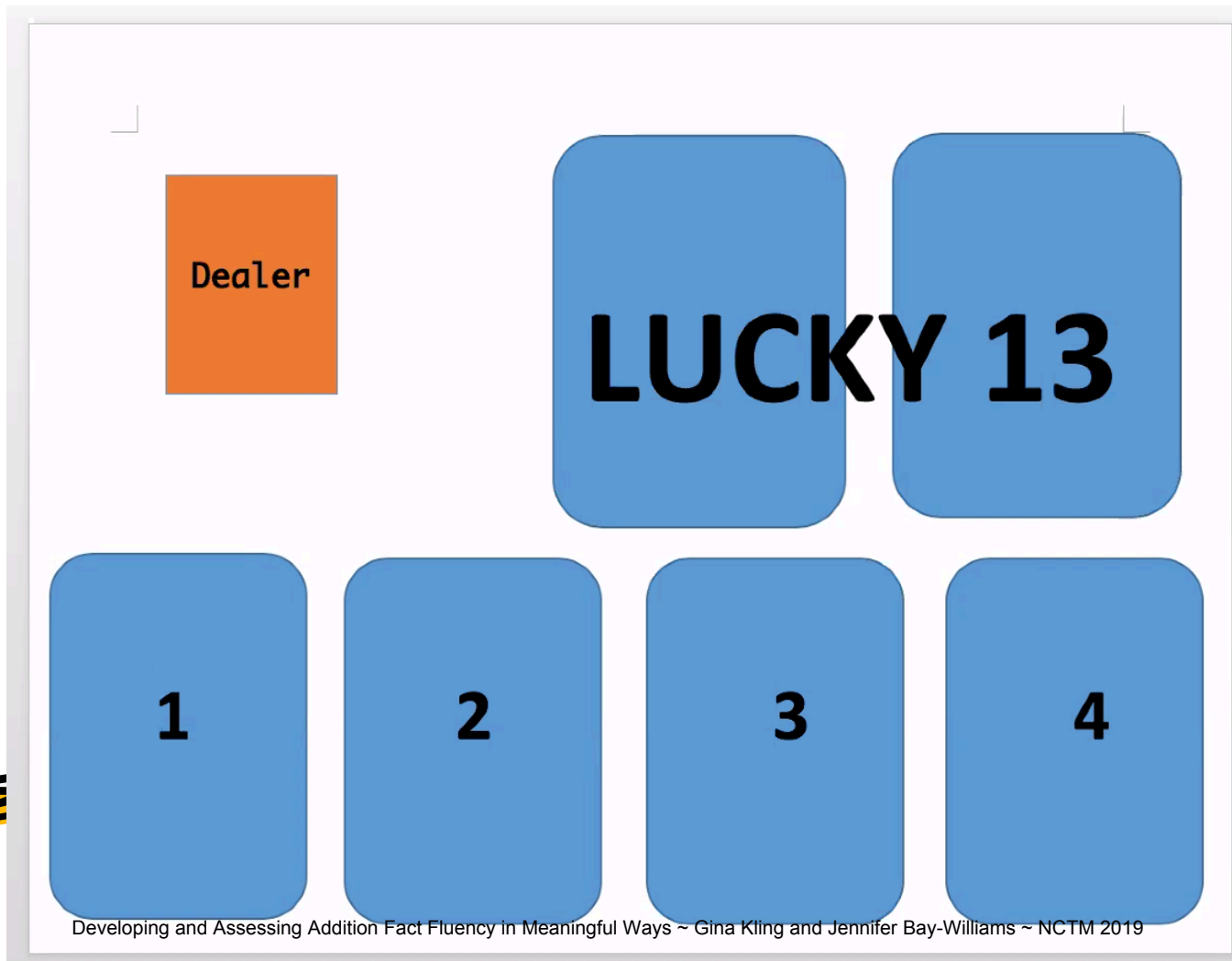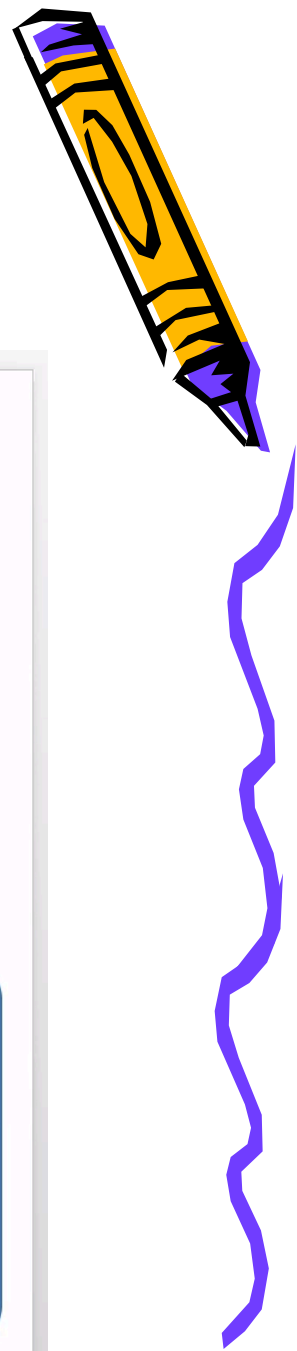Form a group of 2–4 and use one deck of cards (omitting face cards and using aces as ones, jacks as zeros). Each player turns over 5 cards. At the same time, each player selects 2 cards which, when added together, result in a sum as close to Lucky 13 as possible. Players find how far their sum is from Lucky 13 and record that difference as their score (e.g., if the two cards add to 11, the score is 2 because 11 is 2 away from 13). Players discard the two cards that were used and draw two new cards. Repeat for 5 rounds. The lowest total score wins!

Salute!

Played in a group of three with a deck of cards (omitting face cards and using aces as ones, queens as zeros). Two players draw a card without looking at it and place it on their foreheads facing outward (so the others can see it). The player with no card tells the sum. The other two players determine the value of their cards. Once both players have done so, they look at their cards and then players rotate roles before starting the next hand. Adaptations include restricting cards used (e.g., addition facts using only the numbers 1 through 5) or focusing on multiplication/division.

Games as Meaningful Practice

Developing and Assessing Addition Fact Fluency in Meaningful Ways ~ Gina Kling and Jennifer Bay-Williams ~ NCTM 2019

Meaningful Practice

Benefits of Games:

- Are engaging

- Provide opportunities for strategy discussion and assessment

- Lend to differentiation

- Can be targeted practice or general practice
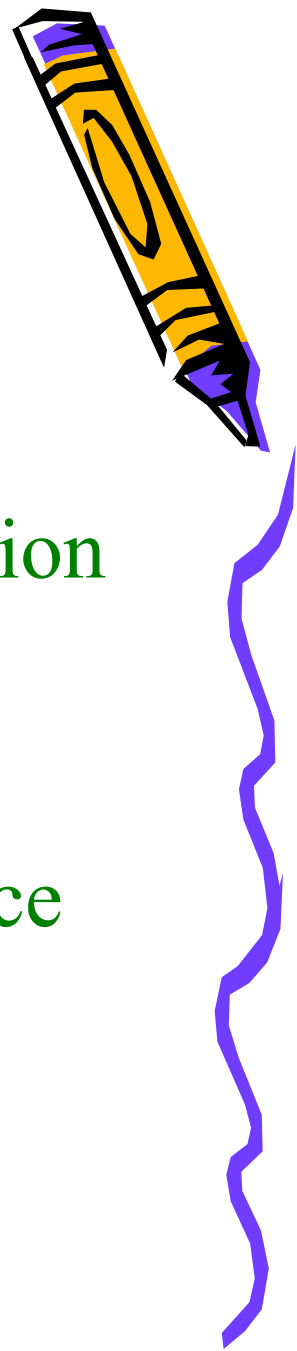
Meaningful Practice

Making SURE Your Games are Effective:

- **S**equence developmentally (for example, playing combinations of ten games before exploring making ten strategies)

- **U**ndo any time pressure

- **R**equire think-aloud

- **E**nsure students solve different problems
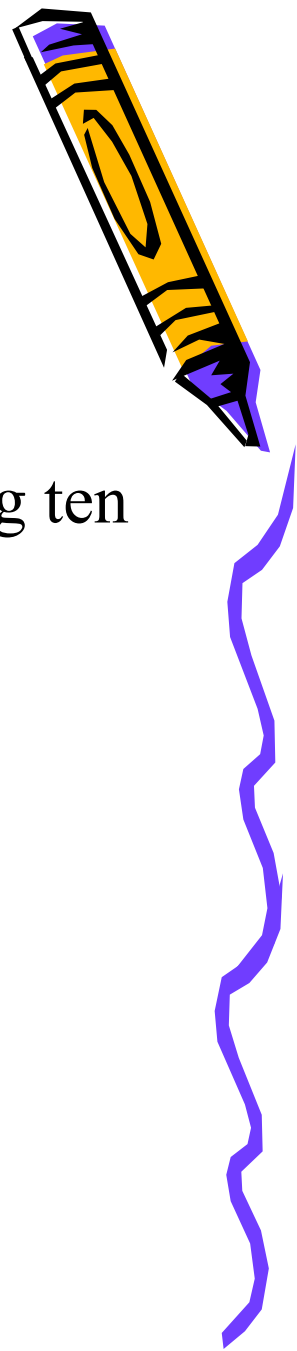
Would you Rather…

OR

(1) 19 + 1 =	(10) 2 + 4 =	(19) 10 + 2 =	(28) 1 + 3 =
(2) 9 + 4 =	(11) 9 + 1 =	(20) 5 + 3 =	(29) 5 + 4 =
(3) 5 + 4 =	(12) 15 + 1 =	(21) 14 + 3 =	(30) 2 + 2 =
(4) 8 + 4 =	(13) 16 + 1 =	(22) 5 + 3 =	(31) 11 + 1 =
(5) 14 + 4 =	(14) 16 + 1 =	(23) 7 + 2 =	(32) 14 + 1 =
(6) 15 + 3 =	(15) 8 + 4 =	(24) 5 + 1 =	(33) 7 + 4 =
(7) 14 + 4 =	(16) 15 + 2 =	(25) 9 + 2 =	(34) 6 + 4 =
(8) 15 + 1 =	(17) 9 + 1 =	(26) 2 + 1 =	(35) 4 + 2 =
(9) 14 + 2 =	(18) 15 + 4 =	(27) 12 + 1 =	(36) 10 + 1 =

MATH FACT **FLUENCY**

0 1 2 3 4 5 6

60+ Games and Assessment Tools to Support Learning and Retention

JENNIFER BAY-WILLIAMS and **GINA KLING**

Developing and Assessing Addition Fact Fluency in Meaningful Ways ~ Gina Kling and Jennifer Bay-Williams ~ NCTM 2019

Assessing Basic Fact Fluency

Procedural <u>fluency</u> is skill in carrying out procedures **flexibly**, **accurately**, **efficiently** and **appropriately**.

(NCTM, 2014; CCSSO, 2010; NRC, 2001)

Aspects of Fluency

- ☐ Flexibility
- ☐ Accuracy
- ☐ Efficiency
- ☐ Appropriate Strategy Use

Timed Tests

|			
(1) $19 + 1 =$	(10) $2 + 4 =$	(19) $10 + 2 =$	(28) $1 + 3 =$
(2) $9 + 4 =$	(11) $9 + 1 =$	(20) $5 + 3 =$	(29) $5 + 4 =$
(3) $5 + 4 =$	(12) $15 + 1 =$	(21) $14 + 3 =$	(30) $2 + 2 =$
(4) $8 + 4 =$	(13) $16 + 1 =$	(22) $5 + 3 =$	(31) $11 + 1 =$
(5) $14 + 4 =$	(14) $16 + 1 =$	(23) $7 + 2 =$	(32) $14 + 1 =$
(6) $15 + 3 =$	(15) $8 + 4 =$	(24) $5 + 1 =$	(33) $7 + 4 =$
(7) $14 + 4 =$	(16) $15 + 2 =$	(25) $9 + 2 =$	(34) $6 + 4 =$
(8) $15 + 1 =$	(17) $9 + 1 =$	(26) $2 + 1 =$	(35) $4 + 2 =$
(9) $14 + 2 =$	(18) $15 + 4 =$	(27) $12 + 1 =$	(36) $10 + 1 =$

Timed Testing: Issues

The issues with timed testing include:

1) Limitations as an assessment tool
2) Can impede progress when mastering facts
3) Psychological effects

Timed Tests: Alternatives
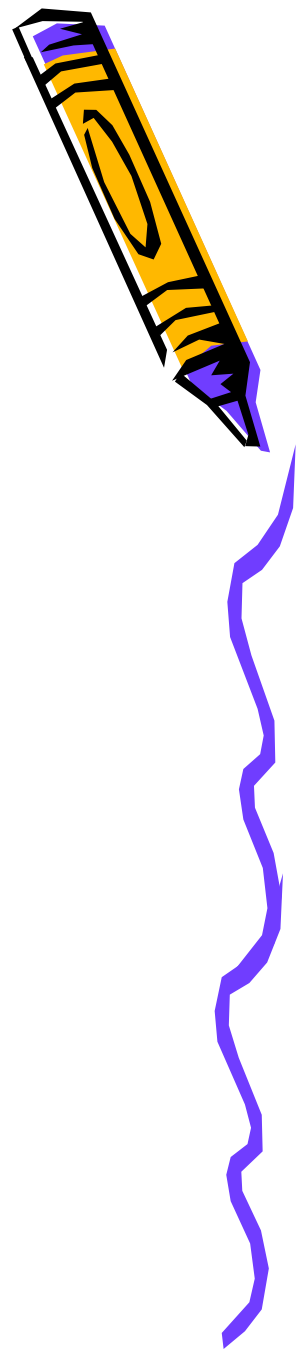
- ✓ Observation
- ✓ Interviews
- ✓ Writing prompts
- ✓ Strategy quizzes
- ✓ Self-assessment

Timed Tests: Alternatives
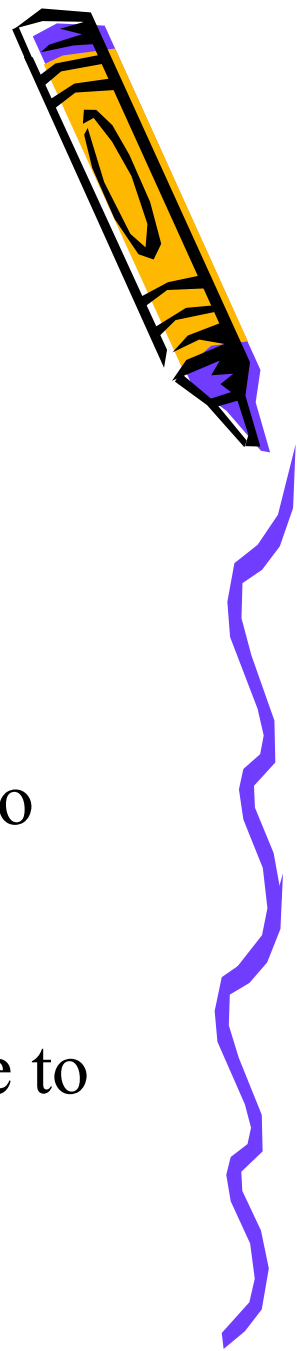
Interviews: "Riley" at the end of Grade 1

At first, Riley incorrectly answers that 8 + 7 = 17.

- Which components of fluency were you able to assess through the follow-up questions?

- How does this assessment experience compare to traditional facts assessments?

Timed Tests: Alternatives

Interviews: "Riley" at the end of Grade 2

Riley solves 5 + 9 in one second.

- Which components of fluency were you able to assess through the follow-up questions?

- How does this assessment experience compare to traditional facts assessments?

Timed Tests: Alternatives

Name: _____ Date: _____

(Ask students these facts verbally or
show cards or whiteboard with each fact,
one at a time. Record codes in each box.)

6 + 6	8 + 2	4 + 4	7 + 7
9 - 1	2 + 6	0 + 7	5 - 2
3 + 3	4 + 6	9 + 9	4 - 0
3 + 7	1 + 9	5 + 5	10 - 2

Strategy Codes
M = Models/objects used
CA = Counts all
CO = Counts on
S = Uses a strategy
R = Recall (just knows)

Mastery Code
* = Automatic

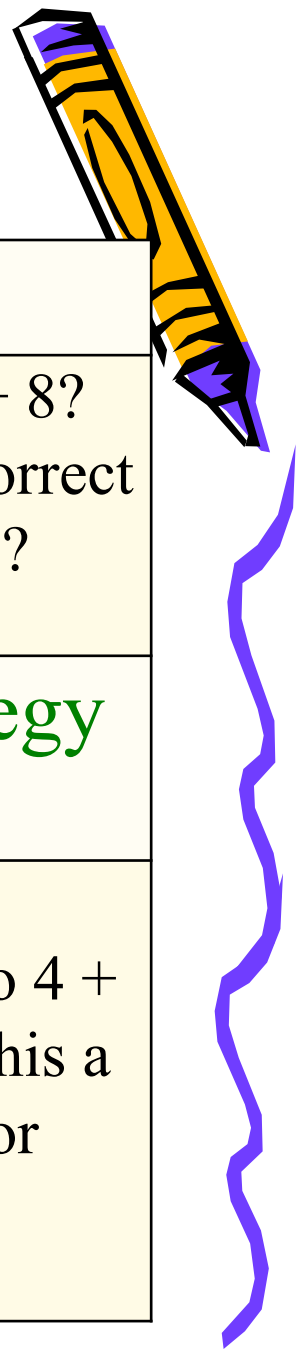**Strategy Codes** (record in space **to the right** of each fact as student explains)
C= Count On
S = Strategy   **ND**= Near Doubles   **M10**= Making 10   **P10** = Pretend-a-10
**O** = Other
K = Just Knew (Recall)

Interviews

| Flexibility	Accuracy
Solve 6 + 7 using one strategy. Now try solving it using a different strategy.	What is the answer to 7 + 8? How do you know it is correct (how might you check it)?
Efficiency	Appropriate Strategy Selection
For which facts did you **just know**? For which facts did you **use a strategy**?	Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.

Feedback to Parents
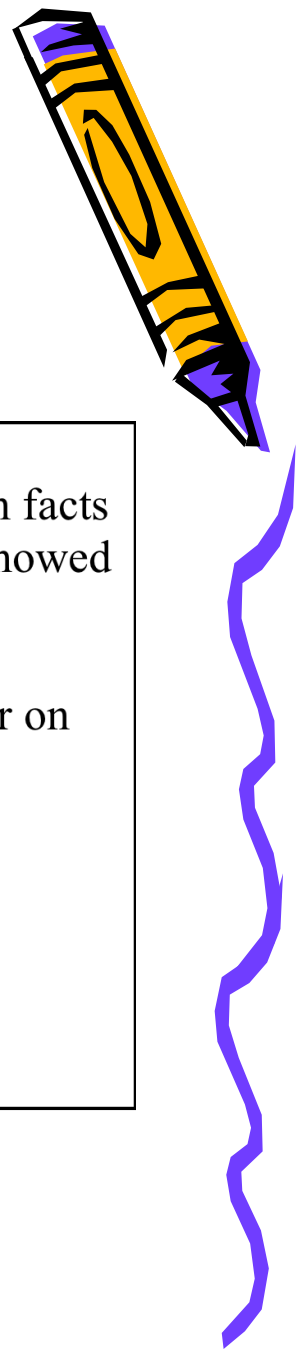
Although it is important for your child to practice **all** addition and subtraction facts this summer to maintain fluency, the end-of-year addition facts assessment showed your child,

_____, will benefit from focusing in particular on the following addition facts:
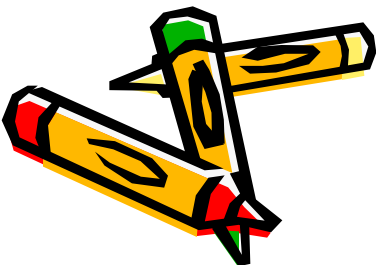
Observation Checklist

Addition Facts Fluency Chart   Date:_____   Game:_____

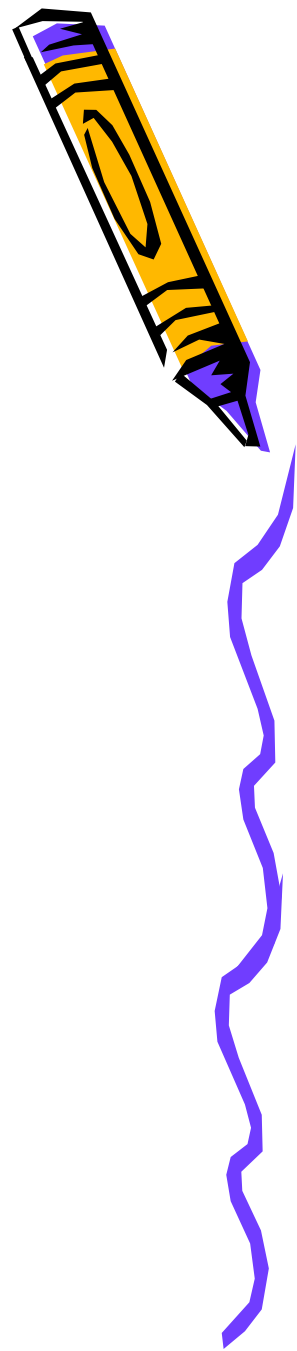| Student	Models and counts all	Counts on	Derived Fact	Recall (double or combo of 10)	Recall	Comments

Aspects of Fluency

☐ Flexibility

☐ Accuracy

☐ Efficiency

☐ Appropriate Strategy Use

Tests: Alternatives

Fact Strategy Quiz/Self-assessment

Solve these problems and tell how you figured them out.

4 + 5 = _____   Check one: ____ I used this strategy: _____
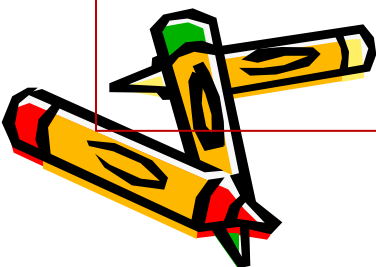   ____ I just knew.

10 + 6 = _____   Check one: ____ I used this strategy:_____
                                ____ I just knew.

6 + 2 = _____   Check one: ____ I used this strategy: _____
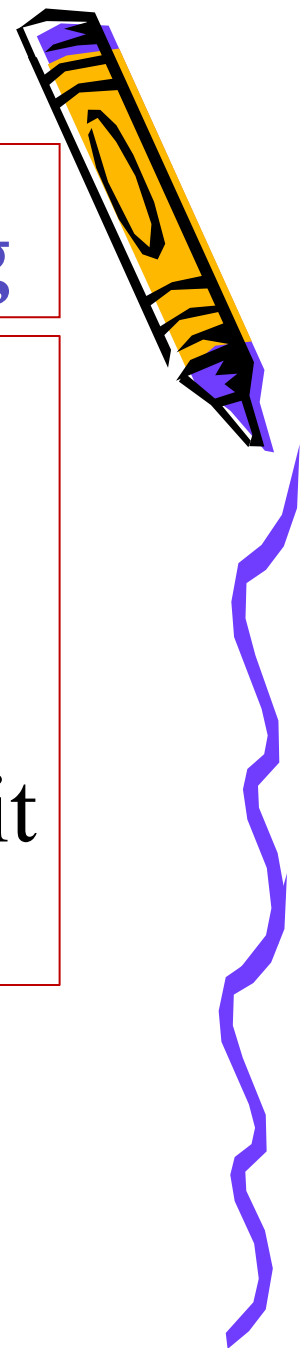   ____ I just knew.

Aspects of Fluency

- ☐ Flexibility
- ☐ Accuracy
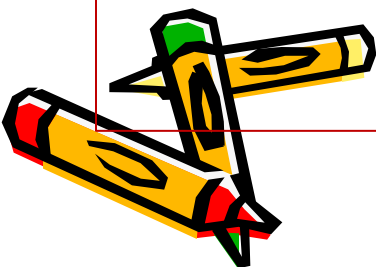- ☐ Efficiency
- ☐ Appropriate Strategy Use

Journal Writing

If your friend didn't know the answer to 4 + 5, how would you tell him to figure it out?
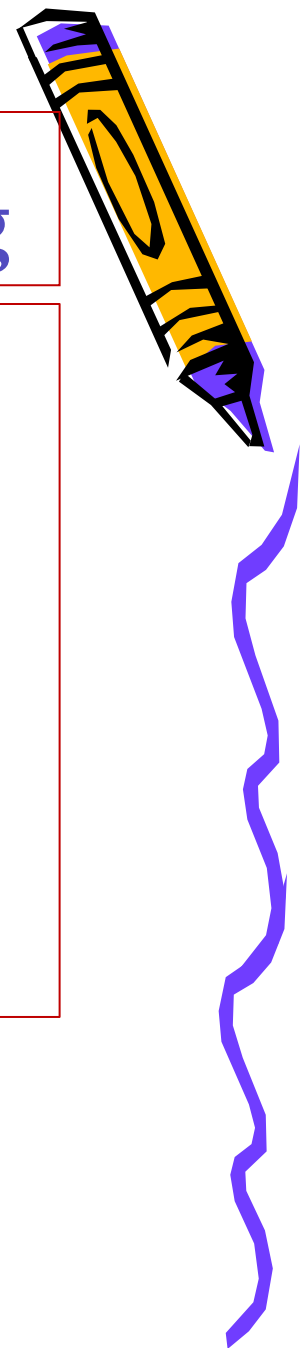
Aspects of Fluency

- ☐ Flexibility
- ☐ Accuracy
- ☐ Efficiency
- ☐ Appropriate Strategy Use

Journal Writing

Review the four student responses:

What might you infer about each child's level of fluency?

Timed Tests: Alternatives
Journal Writing Samples
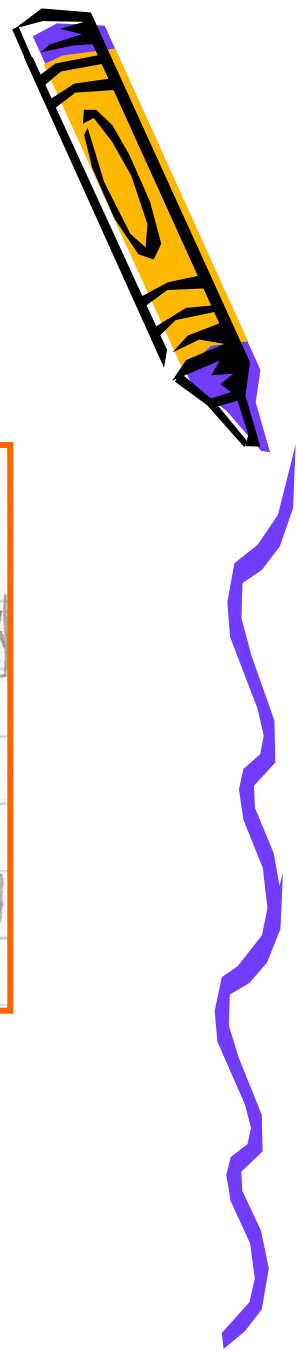
MAY 10, 2012
I would tell my friend
to take 5 and
count 4 in your hand

Timed Tests: Alternatives
Journal Writing Samples

I would tell my friend to start with 5 then add 2 then one more 2 and then you have 9.

Timed Tests: Alternatives
Journal Writing Samples

I would tell my friend to yoos a double plus 1. 4+4=8 so count 1 up. now you get your ancer

Timed Tests: Alternatives
Journal Writing Samples

I would tell my friend to tak away one number from ten. And that is nine. I know that five plas five equals ten.

Looking for More?

NCTM Articles in TCM

Assessing Basic Fact Fluency

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.

By Gina Kling and Jennifer M. Bay Williams

ENRICHING ADDITION AND SUBTRACTION FACT MASTERY THROUGH **Games**

By Jennifer M. Bay-Williams and Gina Kling

ASCD

GAMES AND TOOLS FOR TEACHING ADDITION FACTS

GAMES AND TOOLS FOR TEACHING MULTIPLICATION FACTS

MATH FACT FLUENCY

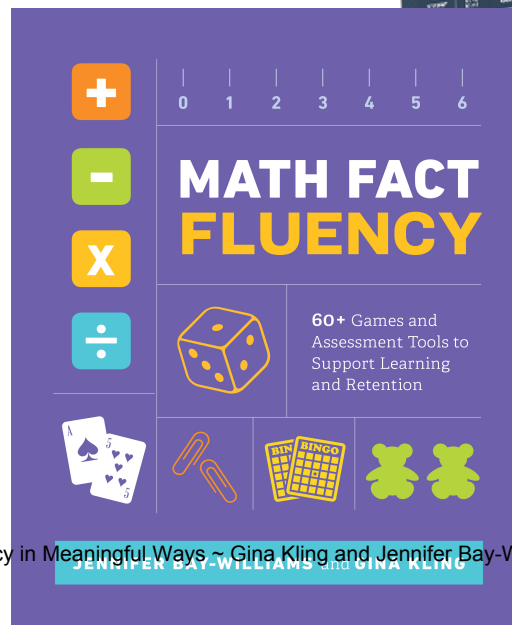60+ Games and Assessment Tools to Support Learning and Retention

JENNIFER BAY-WILLIAMS and GINA KLING

ASCD

+

NCTM

Developing and Assessing Addition Fact Fluency in Meaningful Ways ~ Gina Kling and Jennifer Bay-Williams ~ NCTM 2019

What Works with Addition Fact Fluency

Promising Results

Kling (2013) followed 30 children from 2 different schools, 4 different classrooms, in Kalamazoo, MI.; 21 had no exposure to timed testing or drill in the classroom in either 1st or 2nd grade. By the end of 2nd grade those 21 children demonstrated:

- automaticity with addition facts (solved within 3 seconds) 95% of the time.

- Strategy use (e.g., making ten) so quickly that it was impossible to distinguish between strategy use and "knowing from memory."
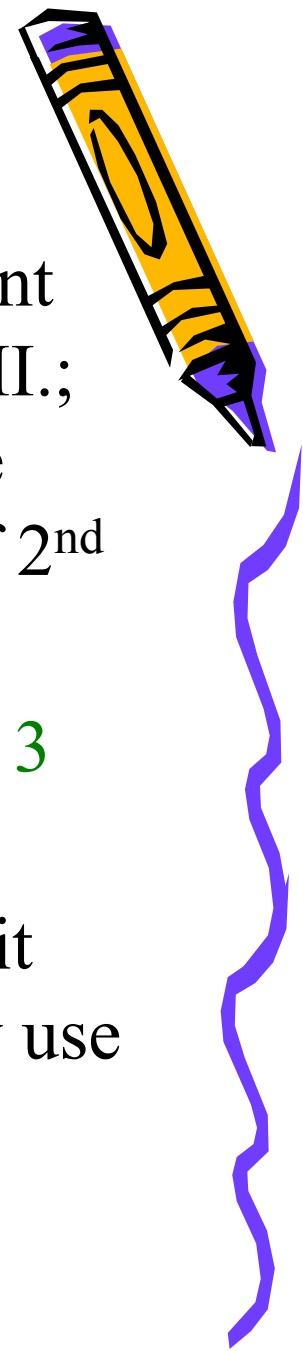
… and RETENTION

18 of the children were interviewed once more in the first week of 3rd grade *prior to any fact strategy review*.

- Children demonstrated automaticity 91% of the time.

- Were accurate and used strategies (not counting) 99.99% of the time.
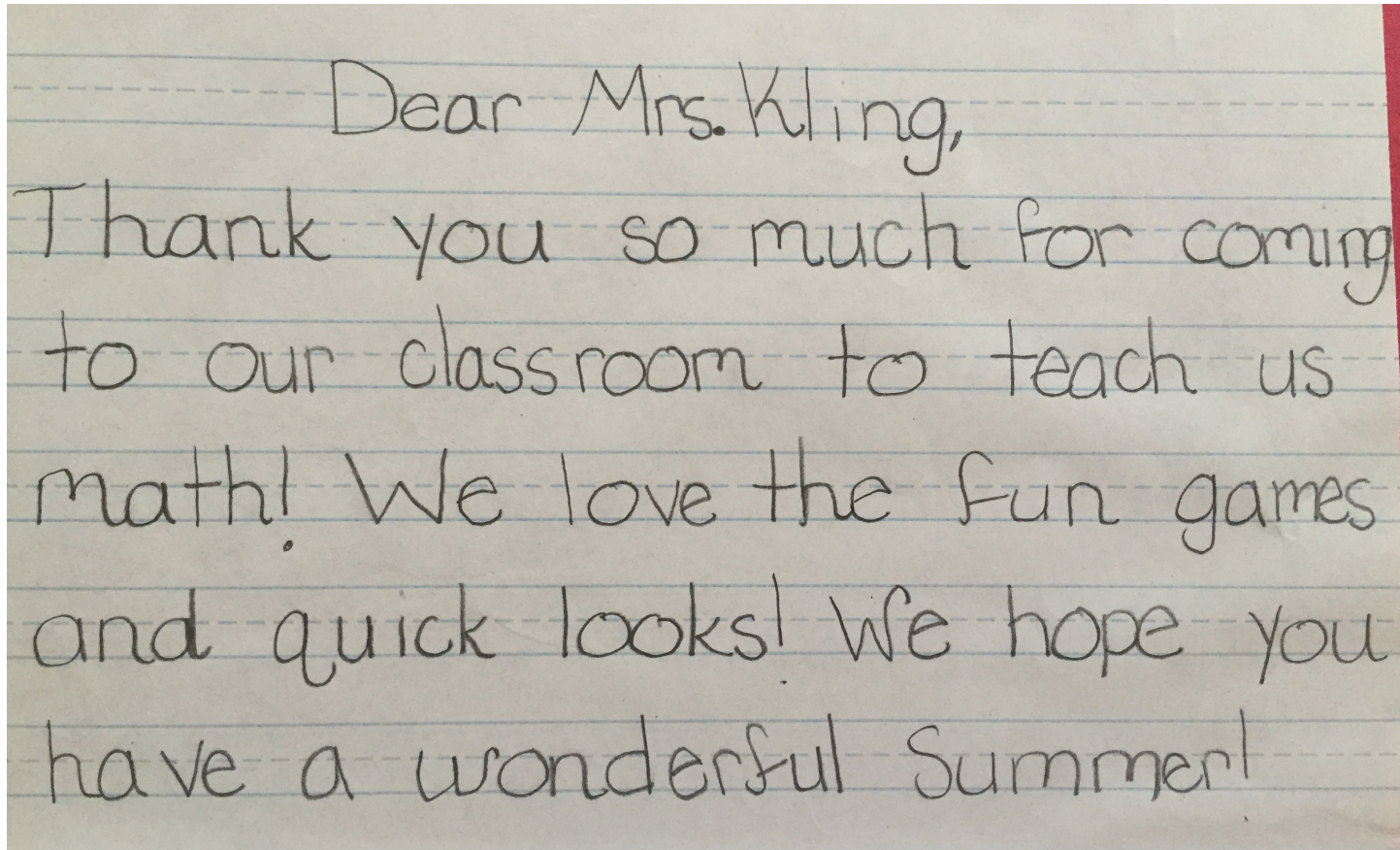
Conclusions

Basic facts instruction, practice, and assessment must truly encompass all four components of fluency:

- **Flexibility**
- **Accuracy**
- **Efficiency**
- **Appropriate strategy use**

Traditional approaches to teaching and assessing basic facts do not support these goals. But when *fluency* is the focus, children can achieve meaningful mastery of basic facts.
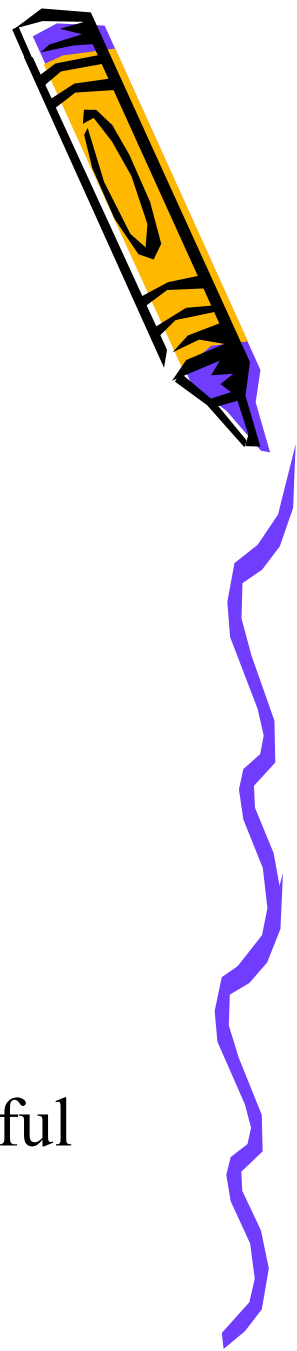
In conclusion: from the experts…

Dear Mrs. Kling,

Thank you so much for coming to our classroom to teach us math! We love the fun games and quick looks! We hope you have a wonderful Summer!

Bibliography

Bay-Williams, J. and G. Kling (2019). Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention. Association for Supervision and Curriculum Development (ASCD) and National Council of Teachers of Mathematics (NCTM), Alexandria, VA.

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Bay-Williams, J. and G. Kling (2014). Enriching Addition and Subtraction Fact Mastery through Games. *Teaching Children Mathematics* 21(4): 238-247.

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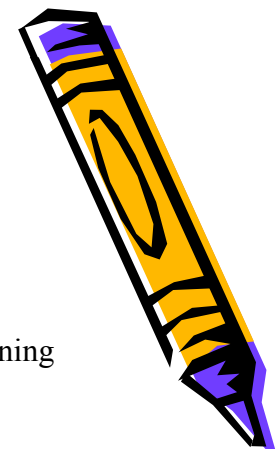
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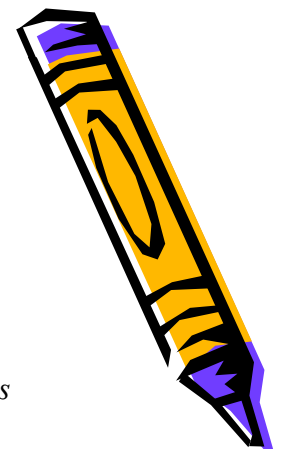
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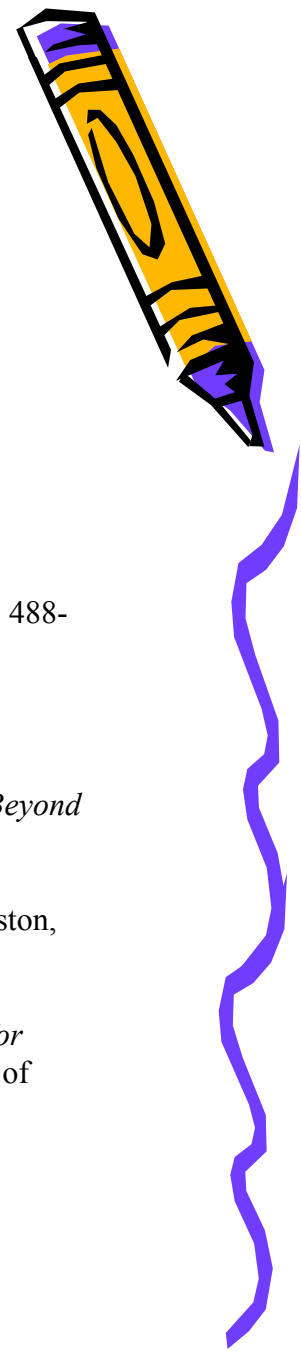
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