DEEPENING STUDENTS' UNDERSTANDING IN MIDDLE SCHOOL MATHEMATICS

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AGENDA

- Introduction
- Effective Teacher Questioning Strategies
- Elevating Student Responses
- Student Writing in Mathematics
- Conclusion

INTRODUCTION

- How do we know that the strategies presented today actually deepen student understanding?
  - Wide research base
  - Experiences of the presenter at two different schools
  - Anecdotal evidence from teachers

GOALS

- To plan for questioning based on a task
- To consider ways to elevate student responses in mathematics
- To describe strategies for enhancing students' written mathematical explanations, arguments, or justifications

TEACHER QUESTIONING STRATEGIES

What types of questions do teachers ask in class?

Why is it important to plan questions prior to a lesson?

How can teachers effectively order the questions they ask in class?

TEACHER QUESTIONING STRATEGIES

"Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (p. 35).

From National Council of Teachers of Mathematics. (2014). *Principles to Action: Ensuring Mathematical Success for All*. Reston, VA: National Council of Teachers of Mathematics.

TEACHER QUESTIONING STRATEGIES

"Just as small-group work and whole-class discussions can be powerful supports for student reading and sense making in mathematics classrooms, they also serve as an important support for student writing in mathematics classrooms. For instance, the third Standard for Mathematical Practice (MP3) calls for students to 'construct viable arguments and critique the reasoning of others.' Students who learn to do this through small-group and whole-group discussion may find it easier to lay out their 'viable arguments' and 'critique the reasoning of others' in writing. Similarly, the sixth Standard for Mathematical Practice is 'attend to precision.' Students who have opportunities to practice communicating precisely to others during small-group and whole-class discussion are better poised to convey this communication more precisely in writing."

From https://www.learner.org/courses/readwrite/mathematics/writing-mathematics/4.html

TEACHER QUESTIONING STRATEGIES

Assessing/Funneling Questions

The Positives	The Negatives
• Gathers information from students • Probes students to explain how they solve a problem (procedures) • Helps to scaffold student thinking	• Leads students to the answer using the teacher's way of thinking • Requires limited responses from students

TEACHER QUESTIONING STRATEGIES

Advancing/Focusing Questions

The Positives	The Negatives
• Makes students' mathematical thinking visible • Probes students to explain why they solved a problem in the way they did • Encourages students to make connections • Encourages reflection, explanation, justification, and argumentation • Extends the situation being discussed	• Can be time-consuming • Can sometimes be confusing if students are not explaining their thinking clearly

TEACHER QUESTIONING STRATEGIES

Consider these questions…assessing (funneling) or advancing (focusing)?

- What is the value of *y* in the equation?
- How could you prove that the solution is 24?
- How does your number line relate to the equation that Johnny has?
- What is the formula for finding the area of a triangle?
- How did you get your answer of 12?

TEACHER QUESTIONING STRATEGIES

"In mathematics classrooms, high-quality discussions support student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas" (p. 1).

From Smith, M.A., and Stein, M.K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

TEACHER QUESTIONING STRATEGIES

1. Complete the task that is given to you. Then consider other ways that students might complete the task.
2. After everyone at your table has completed the task, discuss with your peers how you approached the problem. Think about whether there are other ways that students could solve the problem.
3. Once you have considered the ways in which students might solve the problem, use the planning template to write questions that you can use to assess or advance student thinking.

TEACHER QUESTIONING STRATEGIES

Task:

My cat Fluffy went to the veterinarian, and he put her on a diet. Fluffy can only eat $\frac{4}{5}$ of a cup of diet cat food each day. One bag of diet cat food contains 18 cups of food. How many days will one bag of diet cat food last?

Task: ______________________________ Unit: ______________ Standards: ________________

Mathematical Goal/Essential Understanding: ____

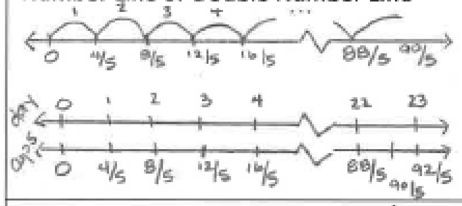
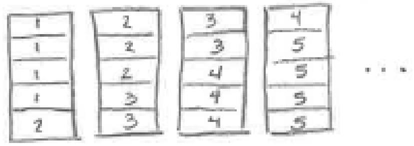
Anticipated Solution Paths	Which students?	Questions to Ask	Connections and Key Points	Order to Share and Discuss
		Assessing: *Advancing:*		
		Assessing: *Advancing:*		
		Assessing: *Advancing:*		
		Assessing: *Advancing:*		

From https://drive.google.com/file/d/1ZfO7F5GTkNEPQsYV-ZDz7EJp0tWG5UIh/view

Task Launch:
Formative Assessment: (Show Me/Hinge Question/Exit Ticket)

Task: Fluffy the Cat Unit: Division with Fractions Standards: 6.NS.A.1

Mathematical Goal/Essential Understanding: (1) Dividing a whole number by a fraction less than 1 will give a quotient that is greater than the dividend. (2) The remainder in a division problem $a \div \frac{b}{c}$ is a portion of b.

Anticipated Solution Paths	Which students?	Questions to Ask	Connections and Key Points	Order to Share and Discuss
Table or Repeated Addition day | cups used 1 | $\frac{4}{5}$ 2 | $\frac{8}{5}$ 3 | $\frac{12}{5}$ ⋮ | ⋮ $\frac{4}{5} + \frac{4}{5} = \frac{8}{5}$ $\frac{8}{5} + \frac{4}{5} = \frac{12}{5}$ ⋮		*Assessing*: Why are you adding? When do you know when to stop adding? *Advancing*: What can you do with the remainder? Can you write an equation to go with your solution?	Multiplication as repeated addition (or division as repeated subtraction)	1
Number Line or Double Number Line 		*Assessing*: What do the numbers on the number line represent? What do the jumps represent? *Advancing*: What can you do with the remainder? Can you write an equation to go with your solution? How does this relate to the table [or repeated addition]?	Connection to table or repeated addition	2
Partitioning Wholes/Counting Out $\frac{4}{5}$ 		*Assessing*: What do your boxes represent? Why did you divide your boxes the way you did? What were you counting? *Advancing*: Can you write an equation to go with your solution?	Part/whole relationships	3
Expressions and/or Equations $18 \div \frac{4}{5} = \square$ $\frac{4}{5} \times \square = 18$ $18 \times 5 = 90$ sections of wholes $90 \div 4 = 22\frac{1}{2}$ days (parts each day)		*Assessing*: What does the 18 represent? What does the $\frac{4}{5}$ represent? How did you know to divide/multiply? *Advancing*: How are the equations related? Can you see your equation in one of the visual representations?	Connection to visual representations; connection between division and multiplication; remainder as part of the quotient	4

TEACHER QUESTIONING STRATEGIES

For information on a similar lesson, see

Smith, M., Bill, V., and Raith, M.L. (2018). Promoting a Conceptual Understanding of Mathematics. *Mathematics Teaching in the Middle School. 24*(1), 36-43.

ELEVATING STUDENT RESPONSES

How do the questions that teachers ask affect the responses that students give?

How can teachers help students to give more meaningful responses?

ELEVATING STUDENT RESPONSES

Hypothetical Conversations

Read the two hypothetical conversations that are in your packet. For each conversation, consider the following questions:

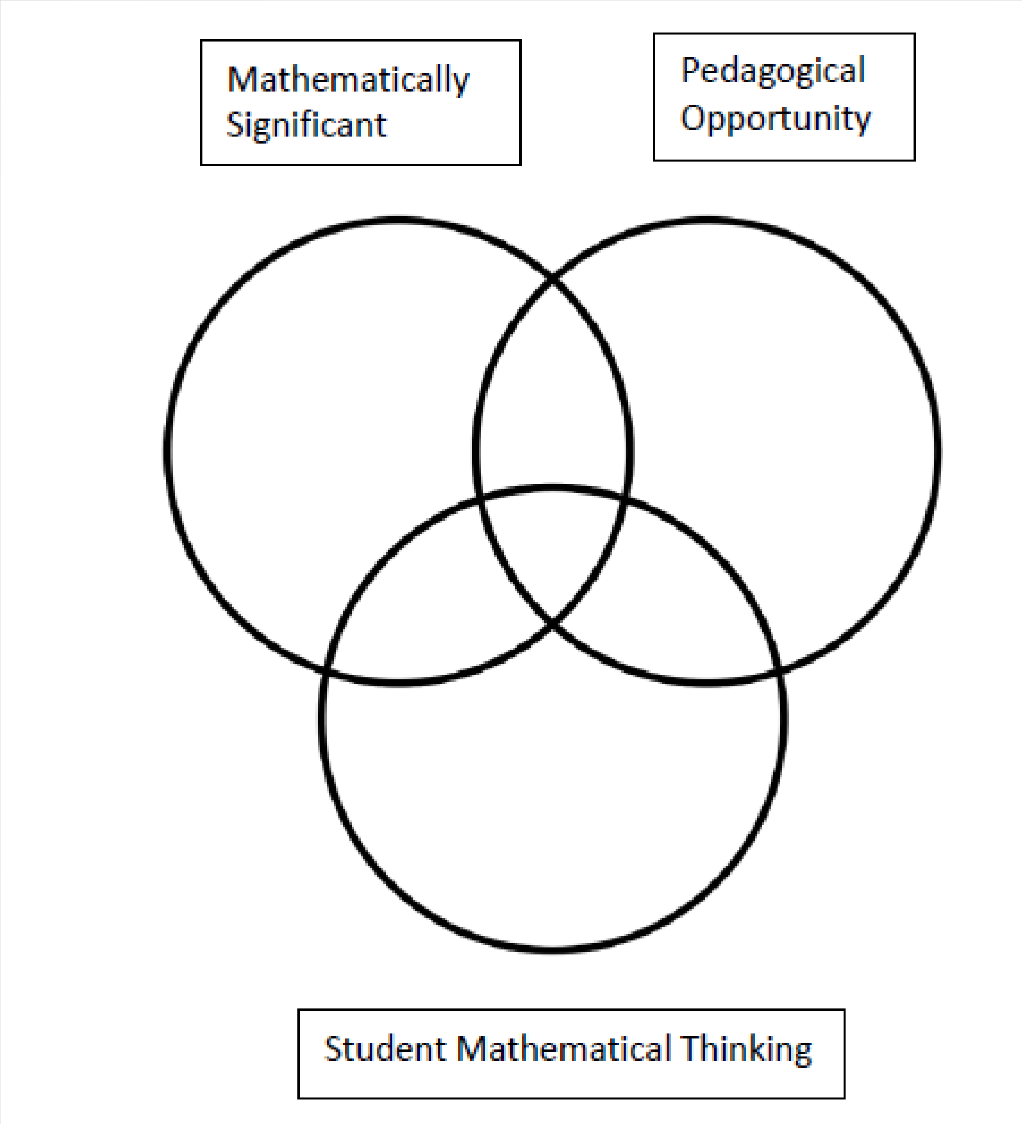
1. How would you characterize the responses that students are giving?
2. Are they the types of responses you would like to see in your class? Why or why not?
3. How are the two conversations alike? How are they different?

ELEVATING STUDENT RESPONSES

"MOST"—**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking

When mathematical opportunities present themselves in class (even if you had not planned for them), are you taking advantage of those opportunities to capitalize on student thinking?

From Leatham, K.R., Peterson, B.E., Stockero, S., and VanZoest, L.R. (2015). Conceptualizing Mathematically Significant Pedagogical Opportunities to Build on Student Thinking. *Journal for Research in Mathematics Education*, 46(1), 88-124.


Mathematically Significant
Pedagogical Opportunity
Student Mathematical Thinking

ELEVATING STUDENT RESPONSES

Classifying Teacher Questions

How can we classify the questions we ask? Do our questions (and our probing) really have an effect on student responses?

- Webb's Depth of Knowledge/Revised Bloom's Taxonomy

From Simpson, A., Mokailed, S., Ellenburg, L.A., and Che, S.M. (2014/2015). A Tool for Rethinking Questioning. *Mathematics Teaching in the Middle School*, *20*(5), 294-302.

- Framework of Cognitive Complexity

From Bahr, D., and Bahr, K. (2017). Engaging All Students in Mathematical Discussions. *Teaching Children Mathematics*, *23*(6), 350-359.

ELEVATING STUDENT RESPONSES

Webb's Depth of Knowledge/Revised Bloom's Taxonomy

	Webb's Depth of Knowledge				
		DOK Level 1 (DOK-1): Recall and Reproduction	DOK Level 2 (DOK-2): Basic Skills and Concepts	DOK Level 3 (DOK-3): Strategic Thinking and Reasoning	DOK Level 4 (DOK-4): Extended Thinking
	Remember (Bloom 1)	Recall, recognize, and locate basic facts, ideas, and principles			
axonomy	Understand (Bloom 2)	Describe/explain how (explain the steps required for specified algorithms)	Specify and explain relationships (explain why the procedure for a specified algorithm is reasonable)	Explain strategies and reasoning processes for solving tasks for which procedures have not been specified	Explain how concepts or ideas specifically relate to other content domains or concepts

ELEVATING STUDENT RESPONSES

Revised Bloom's Ta	**Apply (Bloom 3)**	Apply an algorithm or formula	Solve routine problems applying multiple concepts or decision points	Use concepts to solve non-routine problems	Select or devise an approach among many alternatives to solve a novel problem
	Analyze (Bloom 4)	Retrieve information from a table or graph to answer a question	Compare and contrast figures or data	Generalize a pattern	Gather, analyze, and organize information
	Evaluate (Bloom 5)			Verify reasonableness of results	Draw and justify conclusions
	Create (Bloom 6)	Brainstorm ideas, concepts, or perspectives related to a topic or concepts	Generate conjectures or hypotheses based on observations or prior knowledge	Formulate an original problem	Design a model to inform and solve real-world, complex, or abstract situations

ELEVATING STUDENT RESPONSES

Framework of Cognitive Complexity

Comprehension

Thinking Levels/ Listening Roles	Explanation	Examples
Short answer	Very, very brief response; often an answer to a closed question	What did you get?
Brief statement	A little more information that an answer but not very rich	How did you get that?
Describe	A rich verbalization of thinking	How did you get that?
Elaborate or clarify	Adding more information to make things clearer	What did you mean by that?
Represent	Showing thinking in one or more ways (e.g., concrete objects, pictures, symbols, etc.)	What would that thinking look like in a picture or manipulatives?
Translate	Communicating in words or in other ways	Would you explain that thinking in your own words?
Compare	Determining whether or not strategies, ideas	Is ____'s thinking the same or different than ____'s thinking?

ELEVATING STUDENT RESPONSES

Framework of Cognitive Complexity

Connection

Thinking Levels/ Listening Roles	Explanation	Examples
Relate	Determining how strategies, ideas, or representations are similar or different	How is ____'s thinking the same or different than ____'s thinking?
Discern patterns	Recognizing and describing patterns across ideas, strategies, and representations	What are you noticing as you think about all these…?
Discern structure	Recognizing and describing structures across ideas, strategies, and representations	What are you noticing about the way these things fit together?
Reason	Explaining why the thinking is mathematically sensible	Why do you think your thinking about your idea is true (or strategy works or representation is accurate)?
Transfer	Using thinking in a new situation or context	Can you try this in a new situation?
Challenge or support	Agreeing or disagreeing	Do you agree or disagree?

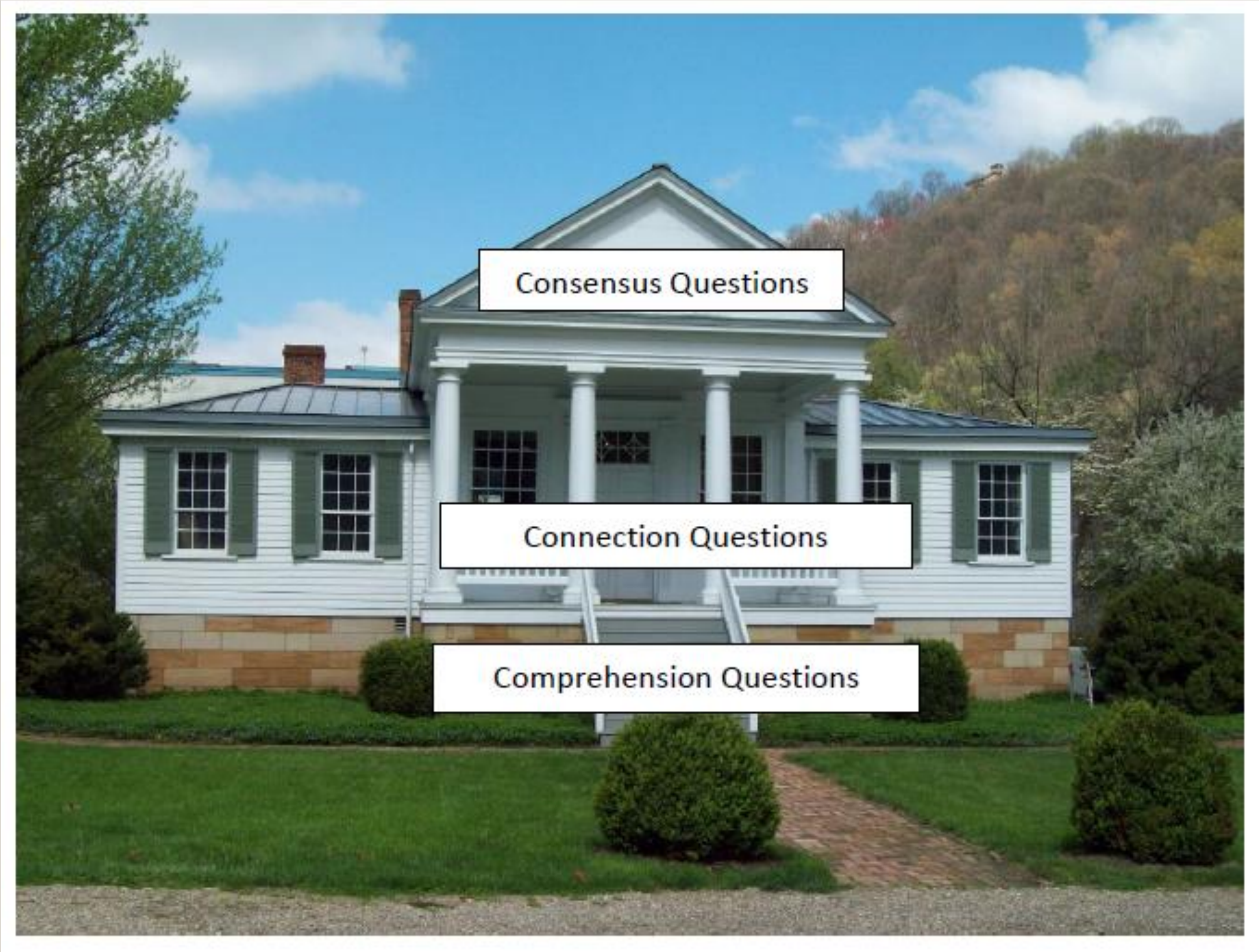
ELEVATING STUDENT RESPONSES

Framework of Cognitive Complexity

<u>Consensus</u>

Thinking Levels/ Listening Roles	Explanation	Examples
Justify	Explaining why someone else's thinking is mathematically sensible or not	Why do you think someone else's idea is true (or strategy works or representation is accurate)?
Prove	Justifying truth, workability, or accuracy within a large domain	How do you know it is true (or will work or is accurate) in all cases?
Refine	Stating, solving, or representing more efficiently	Can you think of a more efficient way?
Generalize	Stating the net result of a proof precisely	How would you state what you have shown to be always true (to always work or to be always accurate)?

ELEVATING STUDENT RESPONSES



ELEVATING STUDENT RESPONSES

- Prepare students for the types of questions that you will ask.
  - "I want you to think about the solution strategy that is most efficient and be prepared to explain why."
  - "I want you to think about how your strategy is similar to or different from the strategies that have already been presented."
  - "I want you to think about whether or not the methods that are presented will always work."
  - "I want you to think about patterns that you see in the solutions that are presented."
  - I want to see if you can explain how your strategy is related to the other strategies presented."

ELEVATING STUDENT RESPONSES

- Continue to push students as they answer questions to ensure that they are giving thorough explanations, arguments, and/or justifications.
  - "Explain to me what the 3 represents in your equation."
  - "Tell me why you divided the 12 by the 4."
  - "How did you know that you could re-order the numbers in your equation?"

WRITING IN MATHEMATICS

Why should students write in mathematics?

How is writing in mathematics different from writing in other content areas?

What constitutes an effective argument or justification?

How can teachers support students’ writing in mathematics?

WRITING IN MATHEMATICS

"A metacognitive framework was evident in the students' writings about their problem solving processes" (p. 242).

"Through written accounts of problem solving processes, these students demonstrated their mathematical reasoning" (p. 243).

From Pugalee, D. (2010). Writing, Mathematics, and Metacognition: Looking for Connections Through Students' Work in Mathematical Problem Solving. *School Science and Mathematics*. *101*, 236-245.

WRITING IN MATHEMATICS

How is writing in mathematics different from writing in other disciplines?

- Writing in mathematics is the process to help students improve their thinking abilities to learn and communicate mathematics.
- Writing serves as a tool for learning to organize, analyze, and communicate mathematical ideas as well as a way to display acquired information. Mathematical communication can be in words and/or symbols. Appropriate content vocabulary should be used at all times. The structure of the work shown should demonstrate the writer's thinking process and follow the historically accepted mathematical formatting protocol. Student writing should primarily be evaluated on the accuracy of the mathematical content.

From http://www.bcps.org/offices/lis/writing/secondary/wac_tech.html

WRITING IN MATHEMATICS

"Just as small-group work and whole-class discussions can be powerful supports for student reading and sense making in mathematics classrooms, they also serve as an important support for student writing in mathematics classrooms. For instance, the third Standard for Mathematical Practice (MP3) calls for students to 'construct viable arguments and critique the reasoning of others.' Students who learn to do this through small-group and whole-group discussion may find it easier to lay out their 'viable arguments' and 'critique the reasoning of others' in writing. Similarly, the sixth Standard for Mathematical Practice is 'attend to precision.' Students who have opportunities to practice communicating precisely to others during small-group and whole-class discussion are better poised to convey this communication more precisely in writing."

From https://www.learner.org/courses/readwrite/mathematics/writing-mathematics/4.html

WRITING IN MATHEMATICS

What are explanations, arguments, and justifications?

> "*Explanations* are mathematical arguments, not procedural summaries of the steps that were used to arrive at the answer. Explanations include justifications." (p. 150)

From Hattie, J., Fisher, D., and Frey, N. (2017). *Visible Learning for Mathematics: What Works Best to Optimize Student Learning*. Thousand Oaks, CA: Corwin.

WRITING IN MATHEMATICS

What are explanations, arguments, and justifications?

"Many opportunities for discussion and communication take place in most classrooms. Students may share a computational answer, disagree with an answer, list the steps in a procedure, explain a solution strategy, compare two strategies, or notice a pattern. Argumentation—in mathematics and other subject areas—goes beyond these types of communication. We view *mathematical argumentation* as a process of dynamic social discourse for discovering new mathematical ideas and convincing others that a claim is true. Within an instructional setting, justifications are part of mathematical arguments because students provide evidence and reasoning to convince others that their claim is valid. Sometimes students base their claims on generalizations or patterns that they notice" (p. 414).

From Rumsey, C., and Langrall, C.W. (2016). "Promoting Mathematical Argumentation." *Teaching Children Mathematics*, *22* (7), 413-419.

WRITING IN MATHEMATICS

What is argumentation?

- Generating cases—creating something to argue about
- Conjecturing—making bold claims
- Justifying—building a chain of reasoning
- Concluding—closure on truth or falsity (p. 4)

From Knudsen, J., Stevens, H. S., Lara-Meloy, T., Kim, H., and Shechtman, N. *Mathematical Argumentation in Middle School—The What, Why, and How. Thousand Oaks, CA: Corwin.*

WRITING IN MATHEMATICS

Research on argumentation and justification

Progression of mathematical reasoning:

1. Empirical stage—Students use examples to support a conjecture.
2. Preformal stage—Students use intuitions and partial/incomplete arguments
3. Formal stage—Students justify their work through theorems, axioms, and other accepted features of mathematics.

From Bieda, K. N., and Lepak, J. (2012). "Examples as Tools for Constructing Justifications." *Mathematics Teaching in the Middle School*, *17* (9), 520-523.

WRITING IN MATHEMATICS

Example:

Prove that the sum of two consecutive odd numbers will always be even.

WRITING IN MATHEMATICS

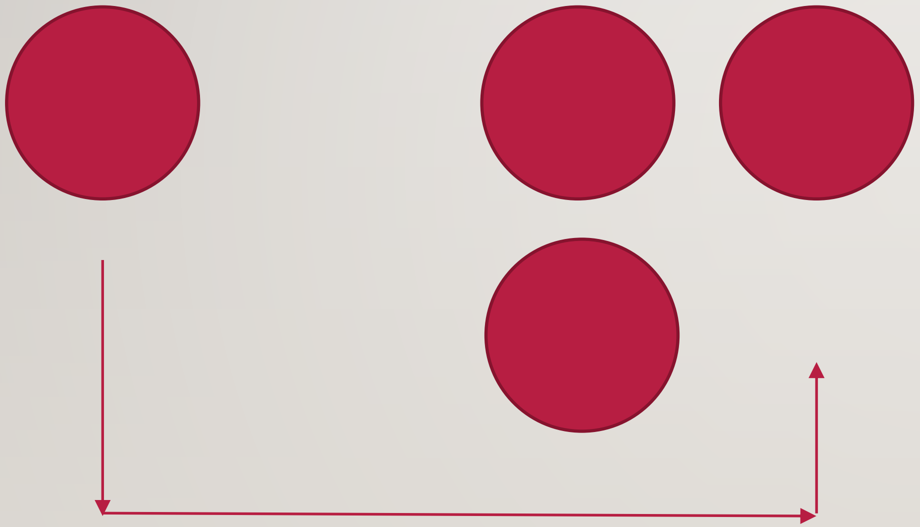
Empirical Stage:

$1 + 3 = 4$
$3 + 5 = 8$
$5 + 7 = 12$

All of the sums are even.

WRITING IN MATHEMATICS

Preformal Stage:



If you add one odd number to another odd number, you can move one circle from one group to the other group to make two full rows. If there are two full rows, then the result is even.

WRITING IN MATHEMATICS

Formal Stage:

Any odd number can be written as an even number plus 1. Even numbers are multiples of 2.

One odd number: $2n + 1$ Next consecutive odd number: $2n + 3$

To find the sum, we can add. $(2n + 1) + (2n + 3)$

$= (2n + 2n) + (1 + 3)$ [Commutative/Associative]

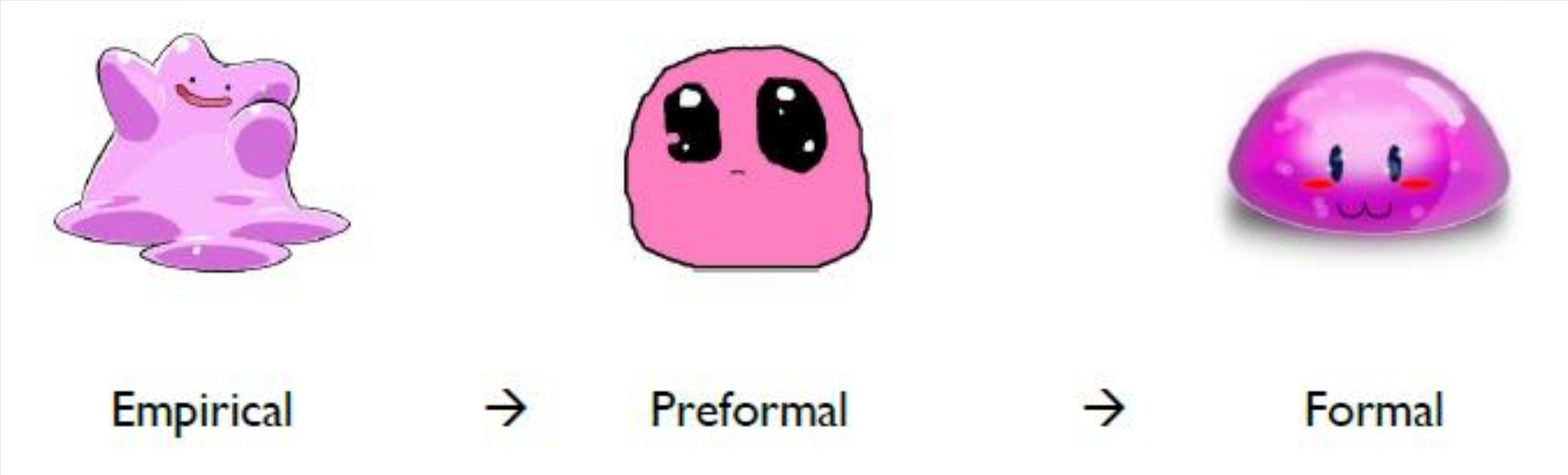
$= 4n + 4$ [Add. Prop. Of Equality]

$= 2(2n + 2)$ [Factoring]

Because all multiples of 2 are even, then sum is even.

WRITING IN MATHEMATICS

Where are most middle-school students in these stages?



WRITING IN MATHEMATICS

Where are most middle-school students in these stages?

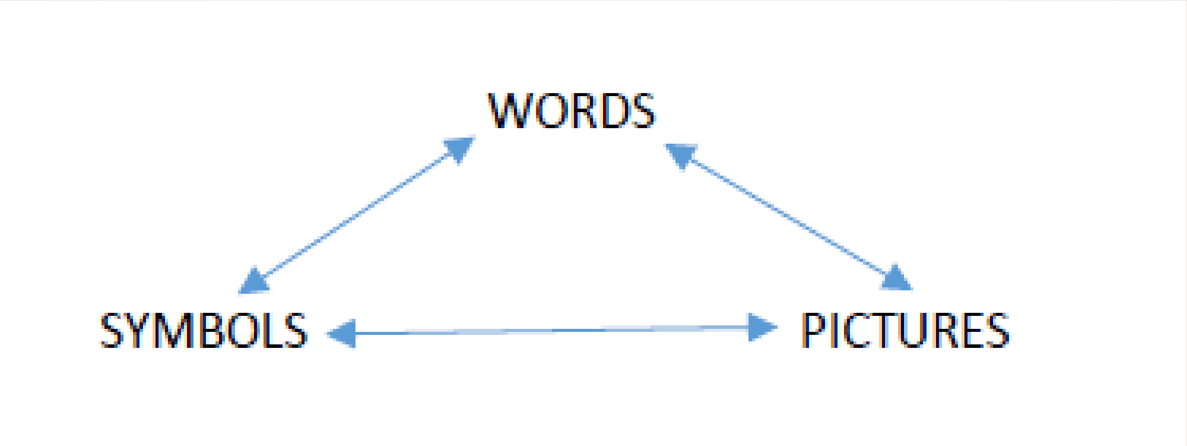
WRITING IN MATHEMATICS

How do we support students' development in writing in mathematics?

- Scaffold students' use of examples.
- Ask something general about something specific.
  - Examples: How do you know this is true? Is this true in all cases? Is there a relationship between these quantities? Is there a general statement that you could use to show this is true for every case?

WRITING IN MATHEMATICS

- Use a graphic organizer to support students' written explanations, arguments, and justifications.
  - Example:



From Lepak, J. (2014). "Enhancing Students' Written Mathematical Arguments." *Mathematics Teaching in the Middle School*, *20*(4), 212-219.

WRITING IN MATHEMATICS

- Have intentional conversations in class about what constitutes an effective explanation, argument, or justification.
- Use exemplar papers to show effective explanations, arguments, or justifications.
- Use peer review to help students refine and revise their arguments. (In this case, the audience is no longer the all-knowing teacher but is instead another individual in the class.)
- Use rubrics to help students evaluate their writing in mathematics.

WRITING IN MATHEMATICS

Sample Rubric:

Rating	2	1	0
Words	An explanation in words about *how* to find the solution *and* "because" statements explaining *why* are provided for each step.	An explanation in words about *how* to find the solution is given, but there is not an explanation of *why* for each step.	An explanation in words is *not* given.
Symbols	A expression showing *how* to find the solution is given, *and* each part is labeled with what the expression represents (*why*).	An expression showing *how* to find the solution is given, but it is not labeled with what each part represents (*why*).	*No* expression is given.
Picture	A labeled picture is used to show *how* to find the solution. The picture matches the description of words and/or symbols.	A picture is given, but it is not labeled or does not match the description in words or symbols.	*No* picture is given.

From Lepak, J. (2014). "Enhancing Students' Written Mathematical Arguments." *Mathematics Teaching in the Middle School*, *20*(4), 212-219.

WRITING IN MATHEMATICS

Sample Rubric:

RACE:

Reword: Reword the question into a statement to begin your answer.

Answer: Answer the question that you were asked to answer.

Cite: Cite examples from your previous mathematical learning (properties, theorems, etc.) that relate to your answer and to your explanation.

Explain: Explain how you arrived at your answer (your thinking), and how what you cited relates to your answer.

From Cioe, M., King, S., Ostien, D., Pansa, N., and Staples, M. (2015). "Moving Students to 'the Why?'." *Mathematics Teaching in the Middle School, 20*(8), 484-491.

WRITING IN MATHEMATICS

Sample Rubric:

Acronym	Score Point	1	0
C	**C**alculations	• Calculations show mathematical ideas involved. • Answer includes calculations and/or tables, graphs, or pictures.	• No work is shown. • Some work is missing.
L	**L**abels	• Calculations are correctly labeled.	• No labels are included. • Items are incorrectly labeled.
E	**E**vidence	• Calculations support the decision made. • Evidence is provided for all parts of the problem.	• Calculations do not support the decision made. • Evidence is missing for some part of the problem.
A	**A**nswers the Question	• Answers the question asked using a complete sentence (capitalization and punctuation). • Answer is accurate.	• Answer is inaccurate. • Answer does not answer the question being asked.
R	**R**easons Why	• Procedure is identified. • Procedure is explained and what it means. • Clear understanding is shown of content ideas and concepts.	• Mathematical reasoning is not given for the procedure, or the explanation is not given. • The response shows confusion about content ideas and concepts.

From Cioe, M., King, S., Ostien, D., Pansa, N., and Staples, M. (2015). "Moving Students to 'the Why?'." *Mathematics Teaching in the Middle School, 20*(8), 484-491.

FINAL REFLECTIONS/CONCLUSIONS

Food for Thought:

Hattie's Effect Size for One Year of Growth: +0.40

Effect Sizes:

Classroom Discussion: +0.82

Self-verbalization and Self-questioning: +0.64

Metacognitive Strategies: +0.69

From Hattie, J., Fisher, D., and Frey, N. (2017). *Visible Learning for Mathematics: What Works Best to Optimize Student Learning*. Thousand Oaks, CA: Corwin.

CONTACT INFORMATION

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