

# Deepening Students' Understanding in Middle School Mathematics

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## **Notes**

### Teacher Questioning Strategies

1. What types of questions do teachers ask in class?
2. Why is it important to plan questions prior to a lesson?
3. How can teachers effectively order the questions they ask in class?

### Elevating Student Responses

1. How do the questions that teachers ask affect the responses that students give?
2. How can teachers help students to give more meaningful responses?

### Writing in Mathematics

1. Why should students write in mathematics?
2. How is writing in mathematics different from writing in other content areas?
3. What constitutes an effective argument or justification?
4. How can teachers support students' writing in mathematics?

## Teacher Questioning Strategies

Task:

My cat Fluffy went to the veterinarian, and he put her on a diet. Fluffy can only eat  $\frac{4}{5}$  of a cup of diet cat food each day. One bag of diet cat food contains 18 cups of food. How many days will one bag of diet cat food last?

To read about a similar task, see Smith, M., Bill, V., and Raith, M.L. (2018). "Promoting a Conceptual Understanding of Mathematics." *Mathematics Teaching in the Middle School*, 24(1), 36-43.

Task: \_\_\_\_\_ Unit: \_\_\_\_\_ Standards: \_\_\_\_\_


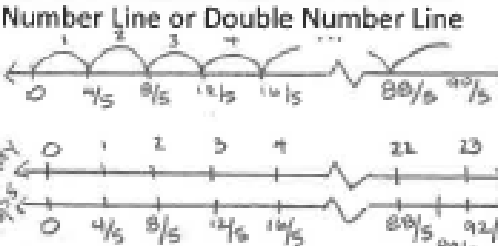
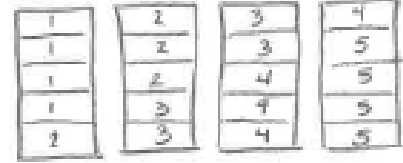
Mathematical Goal/Essential Understanding: \_\_\_\_\_

Anticipated Solution Paths	Which students?	Questions to Ask	Connections and Key Points	Order to Share and Discuss
		<i>Assessing:</i>  <i>Advancing:</i>		
		<i>Assessing:</i>  <i>Advancing:</i>		
		<i>Assessing:</i>  <i>Advancing:</i>		
		<i>Assessing:</i>  <i>Advancing:</i>		

Task Launch:

Formative Assessment: (Show Me/Hinge Question/Exit Ticket)

Task: Fluffy the CatUnit: Division with FractionsStandards: 6.NS.A.1Mathematical Goal/Essential Understanding: (1) Dividing a whole number by a fraction less than 1 will give a quotient that is greater than the dividend.(2) The remainder in a division problem  $a \div \frac{b}{c}$  is a portion of  $b$ .

Anticipated Solution Paths	Which students?	Questions to Ask	Connections and Key Points	Order to Share and Discuss
<b>Table or Repeated Addition</b> 		<i>Assessing:</i> Why are you adding? When do you know when to stop adding? <i>Advancing:</i> What can you do with the remainder? Can you write an equation to go with your solution?	Multiplication as repeated addition (or division as repeated subtraction)	1
<b>Number Line or Double Number Line</b> 		<i>Assessing:</i> What do the numbers on the number line represent? What do the jumps represent? <i>Advancing:</i> What can you do with the remainder? Can you write an equation to go with your solution? How does this relate to the table [or repeated addition]?	Connection to table or repeated addition	2
<b>Partitioning Wholes/Counting Out <math>\frac{4}{5}</math></b> 		<i>Assessing:</i> What do your boxes represent? Why did you divide your boxes the way you did? What were you counting? <i>Advancing:</i> Can you write an equation to go with your solution?	Part/whole relationships	3
<b>Expressions and/or Equations</b> $18 \div \frac{4}{5} = \square$ $\frac{4}{5} \times \square = 18$ $18 \times 5 = 90$ $90 \div 4 = 22 \frac{1}{2}$ <div style="display: flex; justify-content: space-around; font-size: small;"> <div>sections of wholes</div> <div>parts each day</div> <div>days</div> </div>		<i>Assessing:</i> What does the 18 represent? What does the $\frac{4}{5}$ represent? How did you know to divide/multiply? <i>Advancing:</i> How are the equations related? Can you see your equation in one of the visual representations?	Connection to visual representations; connection between division and multiplication; remainder as part of the quotient	4

## **Elevating Student Responses**

Reflection Questions to Consider after Reading the Hypothetical Conversations:

1. How would you characterize the responses that students are giving?
2. Are they the types of responses that you would like to see in your class? Why or why not?
3. How are the two conversations alike? How are they different?

### **Hypothetical Conversation #1**

Modern Middle School

Teacher: Ms. Minnow

#### A Conversation

Ms. Minnow: Darwin, tell us about your solution to the problem.

Darwin: I got \$56.

Ms. Minnow: How did you get \$56?

Darwin: I multiplied 4 by 14 and got \$56.

Ms. Minnow: Does anyone have questions for Darwin?

Ms. Minnow: Albert, you solved it differently. Tell us how you solved it.

Albert: I got \$56.

Ms. Minnow: How did you get \$56?

Albert: I multiplied 4 by 6 and got 24 and then 4 by 8 and got 32. When you add them you get \$56.

Ms. Minnow: Good. Does anyone have questions for Albert?

## Hypothetical Conversation #2

Modern Middle School

Teacher: Ms. Minnow

### A Conversation

Ms. Minnow: As you listen to your classmates' solutions, I want you to consider how your solution is the same as or different from what others have done. Darwin, tell us about your solution to the problem.

Darwin: I got \$56.

Ms. Minnow: How did you get \$56?

Darwin: I multiplied 4 by 14 and got \$56.

Ms. Minnow: Let's consider that a bit more. What does the 4 represent?

Darwin: The 4 is for the 4 people.

Ms. Minnow: What about the 14?

Darwin: I got the 14 from the cost of the items that they bought. They bought a ticket for \$8 and a snack pack for \$6. \$8 plus \$6 is \$14.

Ms. Minnow: So then why did you multiply the 4 by \$14?

Darwin: There were 4 people. The 4 people each paid \$14 total...\$8 for the ticket and \$6 for the snack pack. So 4 people times \$14 per person is \$56.

Ms. Minnow: Can you show that to us using an expression or equation?

Darwin: [writes  $4(\$8+\$6)=4(\$14)=\$56$ ]

Ms. Minnow: How did you know to add 8 and 6 first?

Darwin: The order of operations says we do the parentheses first, so I added the ticket and the snack pack first and got the total that each person paid. Then I multiplied by the 4 people and the answer is \$56.

Ms. Minnow: Does anyone have questions for Darwin?

Ms. Minnow: Albert, you solved it differently. Tell us how you solved it.

Albert: I got \$56.

Ms. Minnow: Remember, Albert, that I want you to think about how your solution is the same as or different than what others have done. Your answer of \$56 is just like Darwin's answer, but did you solve the problem in a different way from Darwin?



Albert: I didn't get 14 like Darwin. I multiplied 4 by 6 and got 24 and then 4 by 8 and got 32. When you add them you get \$56.

Ms. Minnow: Can you show us what you did using an expression or equation?

Albert: [writes  $4(\$8)+4(\$6)=\$32+\$24=\$56$ ]

Ms. Minnow: What does the 4 represent?

Albert: The 4 is the 4 people.

Ms. Minnow: Ok. How is your solution different from Darwin's? [points to Darwin's solution]

Albert: Darwin added the money things first. He added the ticket and the snack pack and then multiplied by the 4 people. I multiplied the 4 people by the ticket price to get \$32 and then the 4 people by the snack pack price to get \$24.

Ms. Minnow: Are you saying that the total amount paid for tickets for 4 people was \$32?

Albert: Yes, and the total amount paid for snack packs for 4 people was \$24. Then when you add the two costs, the \$32 and the \$24, you get \$56 that was paid.

Ms. Minnow: Albert, if we begin with Darwin's  $4(\$8+\$6)$ , how does that relate to your  $4(\$8)+4(\$6)$ ?

Albert: Well, that is the distributive property.

Ms. Minnow: What is the distributive property?

Albert: That is when you take the 4 and distribute it to everything inside the parentheses.

Ms. Minnow: So does it matter whether you use the order of operations, like Darwin did, or the distributive property, like Albert did? Sonia, what do you think?

Sonia: I think you get the same answer. Like, you can use the order of operations or the distributive property. But sometimes you can't do the parentheses first.

Ms. Minnow: What do you mean you can't do the parentheses first?

Sonia: When there is an  $x$ .

Ms. Minnow: Can you give us an example of what you mean?

Sonia: Like, if you don't know the ticket price, you could do  $4(x+6)$ . And we don't know  $x$ , so we can't add  $x$  and 6 first and get a number. That means we have to use the distributive property. Then we would have  $4x+24$ .

Ms. Minnow: Hypatia, can you explain to us whether or not Sonia's way will always work?

Hypatia: I know it will always work because it is the distributive property. We showed that  $a(b+c)$  is always  $ab+ac$  last week using the tiles. It is just that Darwin knew what the numbers were, so he added first. Albert chose to use the distributive property. It is the same thing.

	<b>Webb's Depth of Knowledge</b>				
<b>Revised Bloom's Taxonomy</b>		<b>DOK Level 1 (DOK-1): Recall and Reproduction</b>	<b>DOK Level 2 (DOK-2): Basic Skills and Concepts</b>	<b>DOK Level 3 (DOK-3): Strategic Thinking and Reasoning</b>	<b>DOK Level 4 (DOK-4): Extended Thinking</b>
	<b>Remember (Bloom 1)</b>	Recall, recognize, and locate basic facts, ideas, and principles			
	<b>Understand (Bloom 2)</b>	Describe/explain how (explain the steps required for specified algorithms)	Specify and explain relationships (explain why the procedure for a specified algorithm is reasonable)	Explain strategies and reasoning processes for solving tasks for which procedures have not been specified	Explain how concepts or ideas specifically relate to other content domains or concepts
	<b>Apply (Bloom 3)</b>	Apply an algorithm or formula	Solve routine problems applying multiple concepts or decision points	Use concepts to solve non-routine problems	Select or devise an approach among many alternatives to solve a novel problem
	<b>Analyze (Bloom 4)</b>	Retrieve information from a table or graph to answer a question	Compare and contrast figures or data	Generalize a pattern	Gather, analyze, and organize information
	<b>Evaluate (Bloom 5)</b>			Verify reasonableness of results	Draw and justify conclusions
	<b>Create (Bloom 6)</b>	Brainstorm ideas, concepts, or perspectives related to a topic or concepts	Generate conjectures or hypotheses based on observations or prior knowledge	Formulate an original problem	Design a model to inform and solve real-world, complex, or abstract situations

From Simpson, A., Mokalled, S., Ellenburg, L. A., and Che, S. M. (2014/2015). "A Tool for Rethinking Questioning." *Mathematics Teaching in the Middle School*, 20(5), 294-302.

## Framework of Cognitive Complexity

### Comprehension

Thinking Levels/ Listening Roles	Explanation	Examples
Short answer	Very, very brief response; often an answer to a closed question	What did you get?
Brief statement	A little more information than an answer but not very rich	How did you get that?
Describe	A rich verbalization of thinking	How did you get that?
Elaborate or clarify	Adding more information to make things clearer	What did you mean by that?
Represent	Showing thinking in one or more ways (e.g., concrete objects, pictures, symbols, etc.)	What would that thinking look like in a picture or manipulatives?
Translate	Communicating in words or in other ways	Would you explain that thinking in your own words?
Compare	Determining whether or not strategies, ideas	Is ____'s thinking the same or different than ____'s thinking?

### Connection

Thinking Levels/ Listening Roles	Explanation	Examples
Relate	Determining how strategies, ideas, or representations are similar or different	How is ____'s thinking the same or different than ____'s thinking?
Discern patterns	Recognizing and describing patterns across ideas, strategies, and representations	What are you noticing as you think about all these...?
Discern structure	Recognizing and describing structures across ideas, strategies, and representations	What are you noticing about the way these things fit together?
Reason	Explaining why the thinking is mathematically sensible	Why do you think your thinking about your idea is true (or strategy works or representation is accurate)?
Transfer	Using thinking in a new situation or context	Can you try this in a new situation?
Challenge or support	Agreeing or disagreeing	Do you agree or disagree?

Consensus

Thinking Levels/ Listening Roles	Explanation	Examples
Justify	Explaining why someone else's thinking is mathematically sensible or not	Why do you think someone else's idea is true (or strategy works or representation is accurate)?
Prove	Justifying truth, workability, or accuracy within a large domain	How do you know it is true (or will work or is accurate) in all cases?
Refine	Stating, solving, or representing more efficiently	Can you think of a more efficient way?
Generalize	Stating the net result of a proof precisely	How would you state what you have shown to be always true (to always work or to be always accurate)?

From Bahr, D., and Bahr, K. (2017). "Engaging All Students in Mathematical Discussions." *Teaching Children Mathematics*, 23(6), 350-359.

## Writing in Mathematics

Sample Rubrics:

Rating	2	1	0
Words	An explanation in words about <i>how</i> to find the solution <i>and</i> “because” statements explaining <i>why</i> are provided for each step.	An explanation in words about <i>how</i> to find the solution is given, but there is not an explanation of <i>why</i> for each step.	An explanation in words is <i>not</i> given.
Symbols	An expression showing <i>how</i> to find the solution is given, <i>and</i> each part is labeled with what the expression represents ( <i>why</i> ).	An expression showing <i>how</i> to find the solution is given, but it is not labeled with what each part represents ( <i>why</i> ).	No expression is given.
Picture	A labeled picture is used to show how to find the solution. The picture matches the description of words and/or symbols.	A picture is given, but it is not labeled or does not match the description in words or symbols.	No picture is given.

From Lepak, J. (2014). “Enhancing Students’ Written Mathematical Arguments.” *Mathematics Teaching in the Middle School*, 20(4), 212-219.

RACE:

**R**eword: Reword the question into a statement to begin your answer.

**A**nswer: Answer the question that you were asked to answer.

**C**ite: Cite examples from your previous mathematical learning (properties, theorems, etc.) that relate to your answer and to your explanation.

**E**xplain: Explain how you arrived at your answer (your thinking), and how what you cited relates to your answer.

From Cioe, M., King, S., Ostien, D., Pansa, N., and Staples, M. (2015). “Moving Students to ‘the Why?’.” *Mathematics Teaching in the Middle School*, 20(8), 484-491.

Acronym	Score Point	1	0
C	Calculations	<ul style="list-style-type: none"> <li>Calculations show mathematical ideas involved.</li> <li>Answer includes calculations and/or tables, graphs, or pictures.</li> </ul>	<ul style="list-style-type: none"> <li>No work is shown.</li> <li>Some work is missing.</li> </ul>
L	Labels	<ul style="list-style-type: none"> <li>Calculations are correctly labeled.</li> </ul>	<ul style="list-style-type: none"> <li>No labels are included.</li> <li>Items are incorrectly labeled.</li> </ul>
E	Evidence	<ul style="list-style-type: none"> <li>Calculations support the decision made.</li> <li>Evidence is provided for all parts of the problem.</li> </ul>	<ul style="list-style-type: none"> <li>Calculations do not support the decision made.</li> <li>Evidence is missing for some part of the problem.</li> </ul>
A	Answers the question	<ul style="list-style-type: none"> <li>Answers the question asked</li> <li>The answer is accurate.</li> </ul>	<ul style="list-style-type: none"> <li>The answer is inaccurate.</li> <li>The answer does not answer the question being asked.</li> </ul>
R	Reasons why	<ul style="list-style-type: none"> <li>Procedure is identified.</li> <li>Procedure and what it means is explained.</li> <li>Clear understanding is shown of content ideas and concepts.</li> </ul>	<ul style="list-style-type: none"> <li>Mathematical reasoning is not given for the procedure, or the explanation is not given.</li> <li>The response shows confusion about content ideas and concepts.</li> </ul>

From Cioe, M., King, S., Ostien, D., Pansa, N., and Staples, M. (2015). "Moving Students to 'the Why?'" *Mathematics Teaching in the Middle School*, 20(8), 484-491.