Teaching without Telling: Designing Tasks for Fractions

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NCTM 2019, San Diego, CA

Comparing fractions that have the same denominator

Learning goal: The denominator specifies the partial unit. The numerator specifies the number of those partial units. If two fractions have the same denominator (i.e., same partial unit), the one with the larger numerator (i.e., more partial units) is the larger fraction.

Task 1. This bar is 1-unit long.

Create a bar that is $\frac{3}{8}$ of a unit long.

Create a bar that is $\frac{5}{8}$ of a unit long.

Which of the bars you created is longer?

Task 2. Which fraction is larger $\frac{4}{7}$ or $\frac{6}{7}$? Justify your answer. (Use the bars only if needed.)

Task 3. Which fraction is larger $\frac{17}{40}$ or $\frac{7}{40}$? Justify your answer.

Comparing unit fractions

Learning goal: The denominator specifies how many times the unit fraction will measure the unit. A unit fraction that measures the unit more times must be the smaller. That is, to fit more equal parts in a unit, those parts have to be smaller.

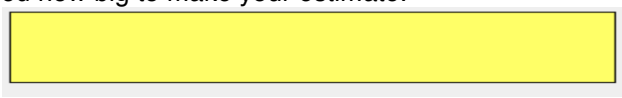
Step 1: Estimation strategies for comparing unit fractions

Task 1. This bar is one unit long. Estimate where to cut the bar to create a piece that is $\frac{1}{5}$ of a unit long.

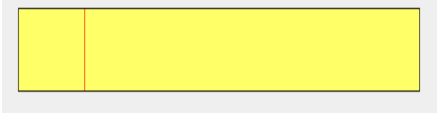
How can you check to see if your estimate is accurate? Check your estimate. If it is too big or too small, create a new estimate and check it out. Continue until your estimate is accurate.

Task 2. This bar is one unit long. Estimate where to cut the bar to create a piece that is $\frac{1}{7}$ of a unit long.

- Explain how you decided how big to make your estimate.



Task 3. The bar below is one-unit long. The line marked on the bar shows $\frac{1}{6}$ of a unit.



- Estimate where you would cut the bar to create a piece that is $\frac{1}{5}$ of a unit.
- Check to see if your estimate is accurate. If your estimate is not close, make a second estimate and see if it is closer.
- Is the part you estimated for $\frac{1}{5}$ larger or smaller than the $\frac{1}{6}$ of a unit marked on the unit bar? Explain why.

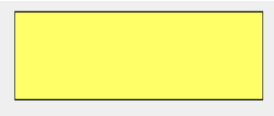
Task 4. Which is larger, $\frac{1}{13}$ or $\frac{1}{9}$? Explain.

Improper Fractions (i.e., fractions greater than one)

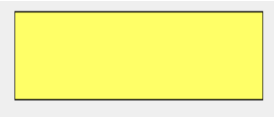
Learning goals: Quantities bigger than a unit can be expressed as $\frac{a}{b}$ of a unit where a > b > 0. Consistent with the concept of (proper) fraction- as-measure, $\frac{1}{b}$ measures the unit b times, and $\frac{1}{b}$ measures the quantity a times. An improper fraction $\frac{a}{b}$ of a unit can be created from the unit by partitioning the unit into b parts, pulling out one part, and iterating it a times (the same for proper fractions).

Step 1: Developing a Notion of Improper Fractions

Task 1. The bar below is $\frac{1}{7}$ of a unit long. Create a new bar that is 5 times as long. How long is the new bar?



Task 2. The bar below is $\frac{1}{5}$ of a unit long. Create a new bar that is 8 times as long. How long is the new bar?

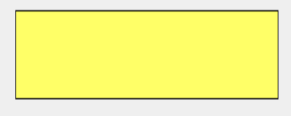


[After the student gives eight-fifths as an answer] Explain why "eight-fifths" is an appropriate way to describe the length of the bar you created.

Next, we introduce the student the term "improper fraction/fraction greater than one" and how to write the fraction.

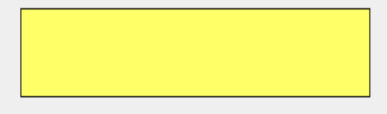
Task 3. The bar below is $\frac{1}{6}$ of a unit long. Create a new bar that is 7 times as long. How long is the new bar? (Write your answer as a fraction - proper or improper).

Task 4. The bar below is $\frac{5}{6}$ of a unit long. Create a new bar that is twice as long. How long is the new bar? (Write your answer as a fraction - proper or improper)

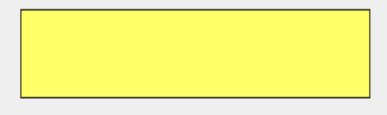


Step 2: Creating Improper Fractions

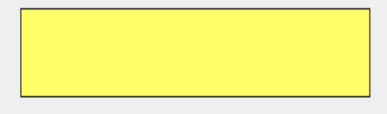
Task 5. The bar below is 1unit long. Create a bar that is $\frac{7}{5}$ of a unit long.



Task 6. The bar below is $\frac{5}{6}$ of a unit long. Create a bar that is $\frac{15}{6}$ of a unit long.



Task 8. The bar below is $\frac{4}{3}$ of a unit long. Create a bar that is $\frac{20}{3}$ of a unit long.

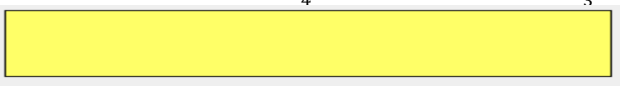


Recursive Partitioning

Learning goals: Students will abstract that $\frac{1}{b}$ of $\frac{1}{a} = \frac{1}{ab}$. That is, because $\frac{1}{a}$ iterates a times to the unit, and $\frac{1}{b}$ of $\frac{1}{a}$ iterates b times to $\frac{1}{a}$, $\frac{1}{b}$ of $\frac{1}{a}$ iterates ab times to the unit. Thus, $\frac{1}{b}$ of $\frac{1}{a} = \frac{1}{ab}$. Further, they will develop the reverse concept needed to make $\frac{1}{a}$ from $\frac{1}{ab}$.

Step 1: Recursively partitioning the unit and determining the measure of subpart.
Task 1. The bar below is 1-unit long. Partition the unit bar into halves. Next, partition one of the parts into thirds. Pull out one subpart. What fraction of the unit is the subpart?

Step 2: Taking a part of a part and determining its measure.
Task 2. The bar below is $\frac{1}{4}$ of a unit long. Pull out $\frac{1}{3}$ of the bar. What fraction of the unit is this new bar?



Task 3. [Use bar representation only if needed.] Imagine you have a bar that is $\frac{1}{3}$ of a unit long and you pulled out a subpart that was $\frac{1}{5}$ of the bar. What fraction of the unit did you pull out?

Task 4. What is $\frac{1}{4}$ of $\frac{1}{6}$?

Step 2: Obtaining $\frac{1}{mn}$ from $\frac{1}{n}$ with Fraction Bars
Task 5. If you had a bar that was $\frac{1}{5}$ of a unit long and wanted to make a bar that was $\frac{1}{10}$ of a unit long,
Would taking $\frac{1}{3}$ of the bar work?
Would taking $\frac{1}{5}$ of the bar work?
What fraction of the bar would give you a bar that is $\frac{1}{10}$ of a unit?

Task 6. If you had a bar that was $\frac{1}{6}$ of a unit long and wanted to make a bar that was $\frac{1}{18}$ of a unit, what would you need to do with the bar?

Task 7. $\frac{1}{40}$ is what fraction of $\frac{1}{8}$?

The task sequences of this presentation were created based on the findings of the research project, "Measurement Approach to Rational Numbers" (see references).

References

Kara, M., Simon, M. A., & Placa, N. (2018). An empirically-based trajectory for fostering abstraction of equivalent fraction concepts: A study of Learning Through Activity. In Martin A. Simon (Ed.), Learning Through Activity: studying and promoting reflective abstraction of mathematical concepts, *Journal of Mathematical Behavior, 52,* 134-150.

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