# Session 18
# More or Less? Using Learning Progressions to Support Student Understanding of Comparison

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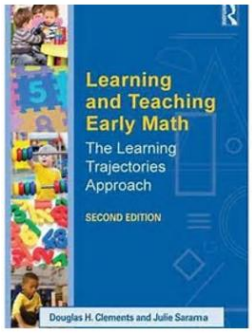
Learning Trajectories: Why Use Them?

https://www.youtube.com/watch?v=OSL6x4iBpCk

Dr. Doug Clements




Learning trajectories are important for several reasons. One of the main things is we know that mathematics education is more effective if it's based in a child centered approach, especially in the early years. Now does not that mean that the child rules everything? No. What it means is that that the mathematics should be from the children, of the children. Learning trajectories ensures that because it's based on the mathematics as it naturally grows within children, so it makes it developmentally appropriate by definition.

The other reason is it's more effective and efficient. Why? Because when kids follow these kinds of learning trajectories we know from developmental research that each level of thinking forms of foundation from which kids develop the next level of thinking. So it's natural then that education that's built on that is building the firmest foundation for kids as they grow up.

We know that those kinds of levels of thinking are more essential for the sequence of kids learning than is for instance an adult inspired sequence of activities where you say, “Oh they must learn this before they learn this before they learn this.” A logical analysis of what the mathematics is isn't as effective as is looking at how that mathematics grows from within children.

# Learning Trajectory Developmental Levels for "Comparing Numbers"

Comparing sets is a critical skill for children as they determine whether one set is larger than another. Pre-kindergartners can learn to use matching to compare sets or to create equivalent sets. The ability to compare sets with fluency develops over the course of several years. With instruction and number experience, most children develop foundational understanding of number relationships of more, less, and the same. Most children follow a natural developmental progression in learning to compare numbers with recognizable levels. This developmental path can be described as part of a learning trajectory.

| **Level** | **Level Name** | **Description** | **Notes** (e.g., "What language would you use with your students?") |
|---|---|---|---|
| 1 | Object Corresponder | At this early level a child puts objects into one-to-one correspondence, but with only intuitive understanding of equivalence. The child does not attend to cardinality of the sets. For example, a child may know that each carton has a straw but does not necessarily know there are the same number of straws and cartons. | |
| 2 | Perceptual Comparer | The child can compare sets that contain the same items but are quite different in quantity (e.g., one group is at least twice the amount of the other group) and know that one set has more objects than the other set. | |
| 3 | Comparer of Similar Items by Subitizing | The child can compare the same items in sets of 1-4 items by subitizing (i.e., just by looking at the sets). For example, compare 3 bears and 2 bears. | |
| 4 | Comparer of Dissimilar Items | The child can match small equal sets of dissimilar items, such as comparing 4 bears and 4 chairs, and shows that they are the same amount. | |
| 5 | Matching Comparer | The child compares same-sized (equal) or different-sized (unequal) sets of 1-6 objects by matching. The child can tell if the result is "the same" if there are no unmatched items or "more" if there are extra items. For example, a child gives one toy bone to every dog and determines if there are the same number of dogs and bones, or if there are extra dogs or bones. | |
| 6 | Counting Comparer within 5 | The child begins to compare sets by counting. Initially, the child is not always accurate when comparing different objects, for example a child might think that 3 soccer balls are more than 5 golf balls. The child can make accurate comparisons, but only when objects are about the same size and the sets are small (about 1–5 objects).<br>The child is solid at this level when he/she can accurately count to compare two sets and say which is larger even if the sets contain dissimilar items. | |

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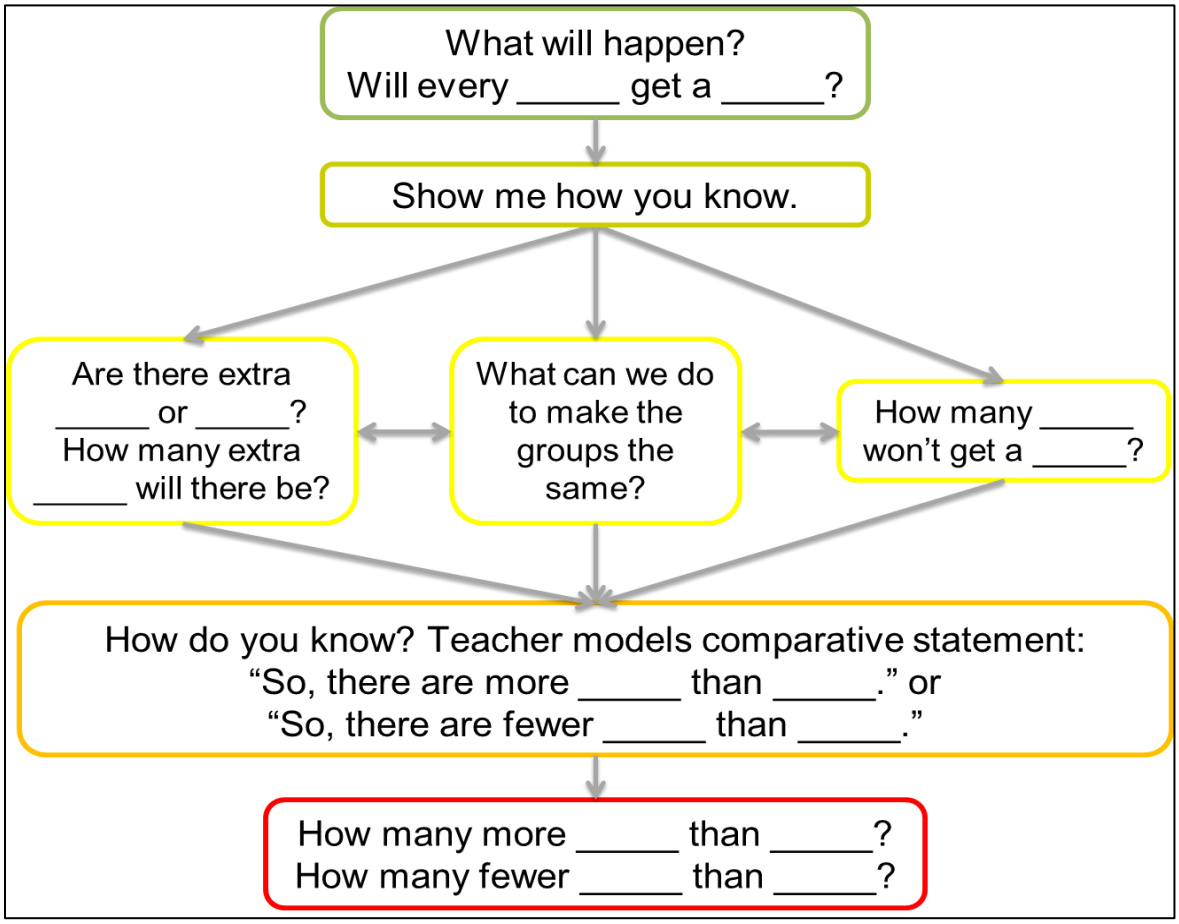
|---|---|---|---|
| 7 | Counting Comparer within 10 | The child compares sets within 10 by counting, the objects can be dissimilar (i.e., they do not need to be the same type of item or the same size). | |
| 8 | Comparer within 20 by Using Ten | The child compares sets of objects within 20 and uses ten as a benchmark. The child also begins to compare sets of objects by coordinating quantity with the relative position of numbers on a mental number path.<br><br>For example, when comparing 15 pillows and 8 children, the child reasons that 15 is more than 8 because 15 is more than ten and 8 is less than ten.<br><br>Another example; when comparing 13 pennies and 17 pennies, the child realizes that both numbers are composed of 10 ones and some extra ones, and reasons that 17 is greater because 7 ones is more than 4 ones. | |
| 9 | Place Value Comparer | As numbers get bigger, the child starts to anchor to the decades and can compare numbers using place value understanding. The child also extends his/her mental number path to compare larger quantities.<br><br>For example, a child at this level can explain that "63 is more than 59 because six tens is more than five tens," while simultaneously knowing that they do not need to compare the ones. A child might also reason that 63 is greater than 59 because it is more than 60 and further along on the number path. | |

Adapted from: Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

# Scaffolding Questions to Support Movement on the Comparing Numbers Trajectory




**Level 3:** The child can compare the same items in sets of 1-4 items by subitizing (i.e., just by looking at the sets). For example, compare 3 bears and 2 bears.

**Level 4:** The child can match small equal sets of dissimilar items, such as comparing 4 bears and 4 chairs, and shows that they are the same amount.

**Level 5:** The child compares same-sized (equal) or different-sized (unequal) sets of 1-6 objects by matching. The child can tell if the result is "the same" if there are no unmatched items or "more" if there are extra items. For example, a child gives one toy bone to every dog and determines if there are the same number of dogs and bones, or if there are extra dogs or bones.

**Level 6:** The child begins to compare sets by counting. Initially, the child is not always accurate when comparing different objects, for example a child might think that 3 soccer balls are more than 5 golf balls. The child can make accurate comparisons, but only when objects are about the same size and the sets are small (about 1–5 objects).
The child is solid at this level when he/she can accurately count to compare two sets and say which is larger even if the sets contain dissimilar items.

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**Comparing Quantities in Natural Settings**

Practice opportunity

**Example 1: There are six bears and four chairs.**
**Manipulatives: bears and chairs**

Marisol and Luis are putting one bear on each chair. There are six bears and four chairs. If each bear wants to sit on a chair, what is going to happen?

**Example 2: There are 11 milk cartons and 7 straws.**
**Manipulatives: milk cartons and straws**

Isabella and Jackson are getting ready the milks and straws for snack time. They put 11 milk cartons on the table. They count the straws. There are 7 straws. If each milk carton needs a straw, what's going to happen?

**Example 3: There are 5 children at the birthday party. There are 8 balloons.**
**Manipulatives: friends and balloons**

Donovan, Tristan, Ada, Sophie, and Viera all went to a birthday party. There are eight balloons for the five friends to take home after the party. If each child takes a balloon home after the party, what's going to happen?


Resources:

Hudson, T. (1983). Correspondences and numerical differences between disjoint sets. Child Development, 54, 94-90.

Student Achievement Partners. Coherence Map. Adapted from: Achieve the Core Coherence Map: http://achievethecore.org/coherence-map/

Richardson, K. (1999). Developing number concepts: Counting, comparing, and pattern. Dale Seymour Publications: Parsippany, NJ.

Richardson, K. (2012). How children learn number concepts: A guide to the critical learning phases. Math Perspectives: Bellingham, WA.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.



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