Link to Google slides: https://tinyurl.com/StructureTablesNCTM19

Connecting Function Representations by Looking for and Making Use of Structure in In-Out Tables

NCTM 2019

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William, 6th grade

Link to Google slides: https://tinyurl.com/StructureTablesNCTM19

William, 5th grade

“We’re learning about these things called ‘expressions.’

I don’t like them because they don’t seem finished.”

$$4 + 5 + 4$$

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William, 5th grade

"We're learning about these things called 'expressions.'

I don't like them because they don't seem finished."

4 + 5 + 4 = 13

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Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

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Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

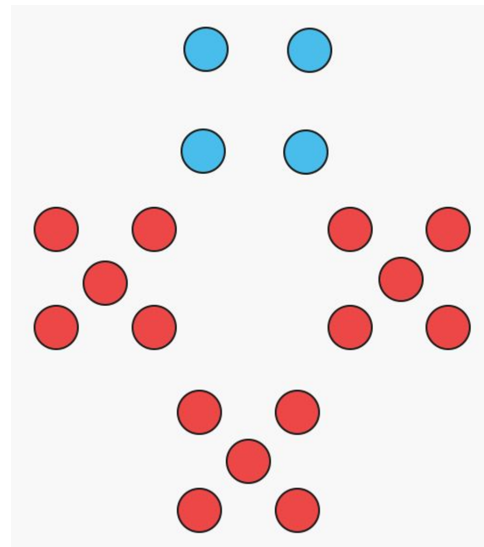
"groups of"

How might I make a picture of 4 plus 3 groups of 5?

Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

How might I make a picture of 4 plus 3 groups of 5?

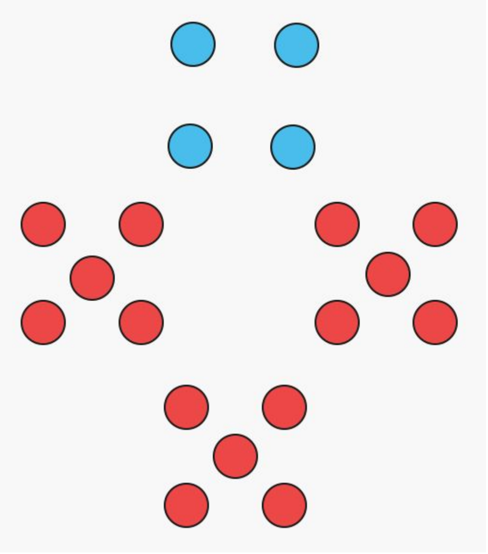


Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

"Writing for Structure"

The mathematical symbols accurately reflect the *structure* of the dot picture



Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

"Writing for Structure"

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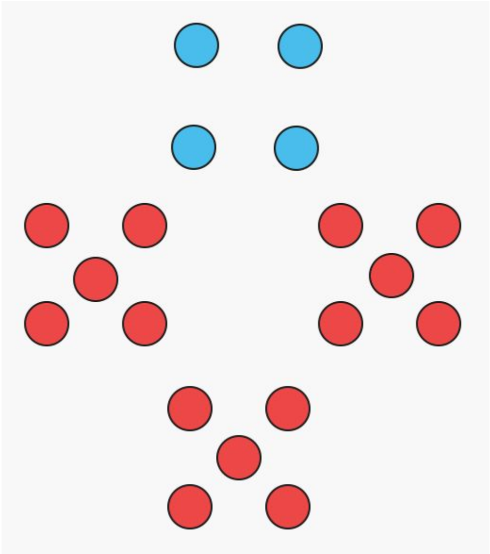
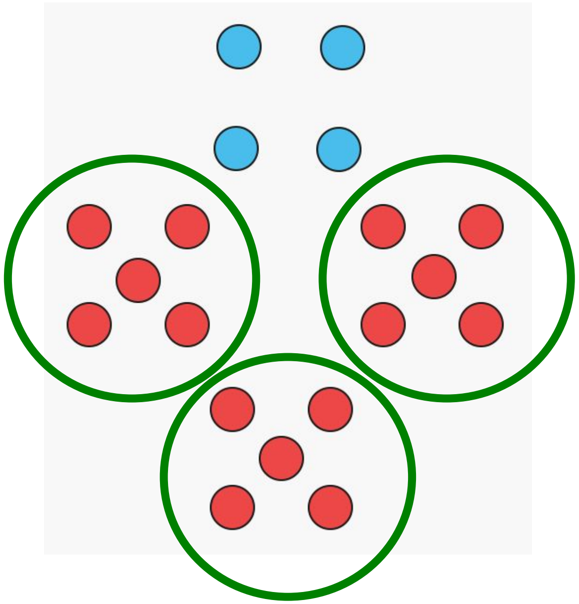


Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

"Writing for Structure"

The mathematical symbols accurately reflect the *structure* of the dot picture



Expressions have meaning beyond computation

$$4 + (3 \cdot 5)$$

"Writing for Structure"

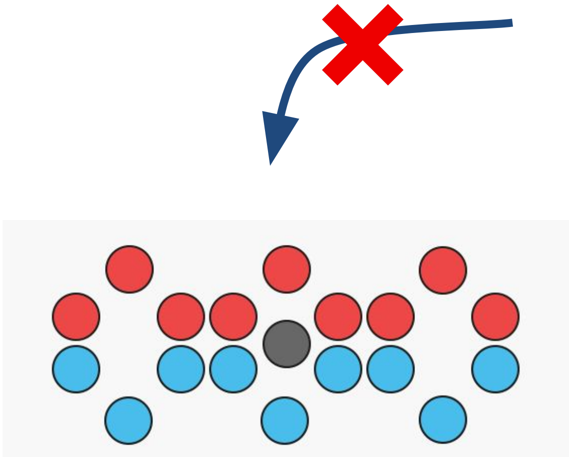
The mathematical symbols accurately reflect the *structure* of the dot picture

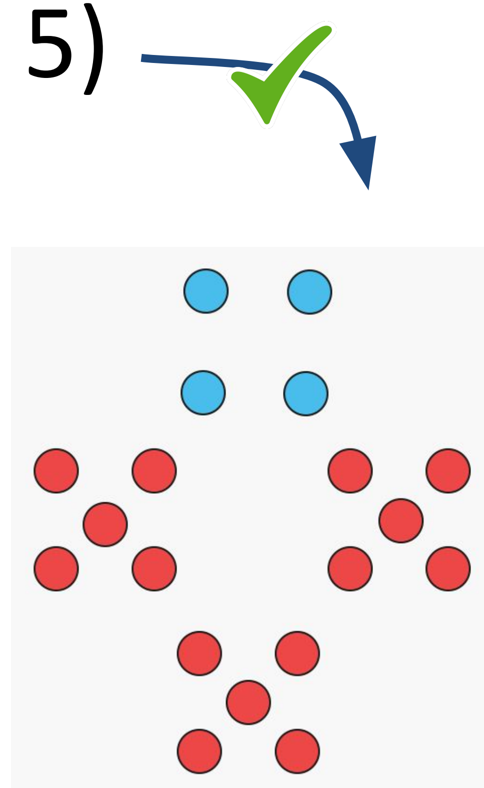
Make a picture that has 19 dots but that you would NOT describe as 4 + (3 x 5)

What expression would you write to describe your picture?

Writing for Structure

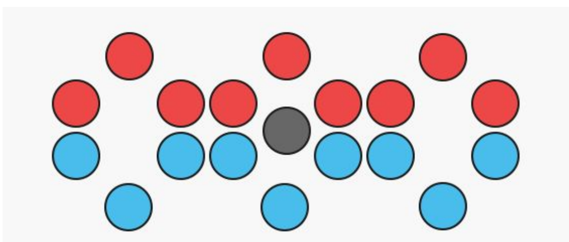
$$4 + (3 \cdot 5)$$

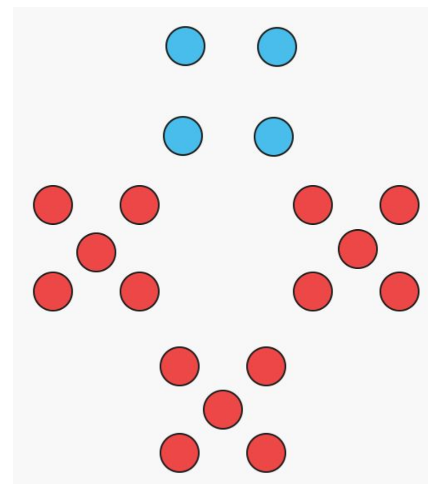




Writing for Structure

$$4 + (3 \cdot 5)$$





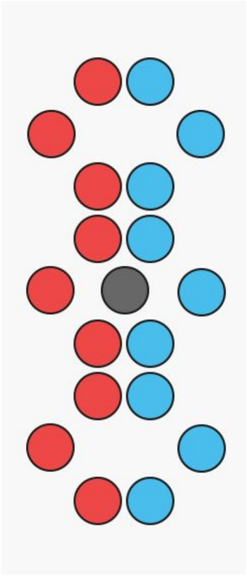
$$2 \cdot 9 + 1$$

$$3 \cdot 6 + 1$$

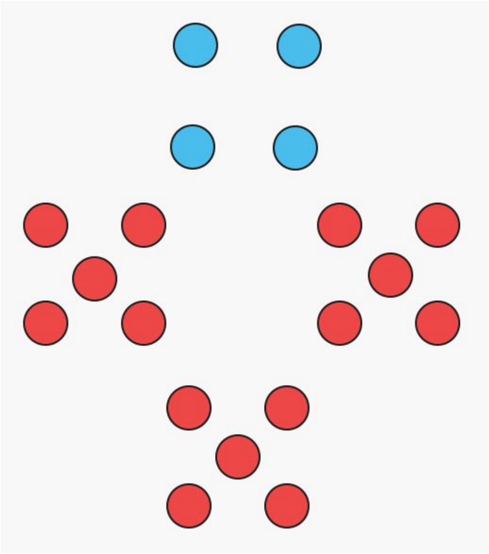
$$3 \cdot (3 + 3) + 1$$

Writing for Structure

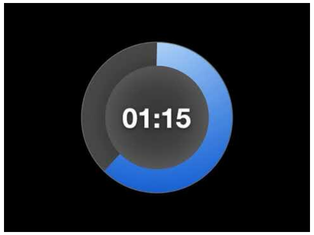
?????



$4 + (3 \times 5)$



Symbol Sense



Without calculating, what are some things you can say are true about these number patterns?

$(1 \cdot 2) + 7$	$5(\frac{1}{2})^1$	$1 + 2 \cdot 3$
$(2 \cdot 3) + 7$	$5(\frac{1}{2})^2$	$2 + 2 \cdot 4$
$(3 \cdot 4) + 7$	$5(\frac{1}{2})^3$	$3 + 2 \cdot 5$
$(4 \cdot 5) + 7$	$5(\frac{1}{2})^4$	$4 + 2 \cdot 6$
...	...	...

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Symbol Sense → **Arithmetic** understanding

I can *fluently* (accurately, flexibly, efficiently) **calculate the value** of expressions.

$(1 \cdot 2) + 7 = 9$	$5(\frac{1}{2})^1 = 2.5$	$1 + 2 \cdot 3 = 7$
$(2 \cdot 3) + 7 = 13$	$5(\frac{1}{2})^2 = 1.25$	$2 + 2 \cdot 4 = 10$
$(3 \cdot 4) + 7 = 19$	$5(\frac{1}{2})^3 = 0.625$	$3 + 2 \cdot 5 = 13$
$(4 \cdot 5) + 7 = 27$	$5(\frac{1}{2})^4 = 0.3125$	$4 + 2 \cdot 6 = 16$
...	...	...

Symbol Sense → **Algebraic** understanding

I can investigate and make sense of mathematical expressions *without calculating.*

The structure of the expression has meaning for me, and I can use that structure to solve problems, make conjectures, and investigate patterns and relationships.

$(1 \cdot 2) + 7$	$5(\frac{1}{2})^1$	$1 + 2 \cdot 3$
$(2 \cdot 3) + 7$	$5(\frac{1}{2})^2$	$2 + 2 \cdot 4$
$(3 \cdot 4) + 7$	$5(\frac{1}{2})^3$	$3 + 2 \cdot 5$
$(4 \cdot 5) + 7$	$5(\frac{1}{2})^4$	$4 + 2 \cdot 6$
…	…	…

Let’s Talk Tables

In	Out
0	2
1	5
2	8
3	11
4	14
...	...
n	$3n + 2$

Let's Talk Tables

In	Out
0	2
1	5
2	8
3	11
4	14
...	...
n	$3n + 2$

In	Out
0	2
1	7
2	18
3	35
4	58
...	...

Symbol Sense → **Arithmetic** understanding

I can *fluently* (accurately, flexibly, efficiently) **calculate the value** of expressions.

In	Out
0	2
1	5
2	8
3	11
4	14
...	...
n	$3n + 2$

In	Out
0	2
1	7
2	18
3	35
4	58
...	...

$\underline{3}\,x^2 + \underline{\quad}\,x + \underline{2}$

Symbol Sense → **Algebraic** understanding

I can investigate and make sense of mathematical expressions *without calculating.*

The structure of the expression has meaning for me,

and I can use that structure

to solve problems,
make conjectures,
and investigate
patterns and relationships.

In	Out
0	2
1	5 2 + 3
2	8
3	11
4	14
…	…

+3

Symbol Sense → **Algebraic** understanding

I can investigate and make sense of mathematical expressions *without calculating.*

The structure of the expression has meaning for me,

and I can use that structure

to solve problems,
make conjectures,
and investigate
patterns and relationships.

In	Out
0	2
1	2 + 3
2	2 + 8 2 · 3
3	11
4	14
...	...

+3
+3

Symbol Sense → **Algebraic** understanding

I can investigate and make sense of mathematical expressions *without calculating.*

The structure of the expression has meaning for me,

and I can use that structure

to solve problems,
make conjectures,
and investigate
patterns and relationships.

In	Out
0	2
1	$2 + 3$
2	$2 + 2 \cdot 3$
3	$2 + 3 \cdot 3$ / 11
4	$2 + 4 \cdot 3$ / 14
…	…
n	$2 + n \cdot 3$

+3
+3
+3
+3

Symbol Sense → **Algebraic** understanding

I can investigate and make sense of mathematical expressions *without calculating.*

The structure of the expression has meaning for me,

and I can use that structure

to solve problems,
make conjectures,
and investigate
patterns and relationships.

In	Out	
0	$2 + 0 \cdot 3$	+3
1	$2 + 1 \cdot 3$	+3
2	$2 + 2 \cdot 3$	+3
3	$2 + 3 \cdot 3$	+3
4	$2 + 4 \cdot 3$	
…	…	
n	$2 + n \cdot 3$	

Arithmetic Thinking

How can I calculate each step?

In	Out
0	2
1	5
2	8
3	11
4	14
...	...
n	$3n + 2$

Algebraic Thinking

What do all these numbers have in common?

In	Out
0	$2 + 0 \cdot 3$
1	$2 + 1 \cdot 3$
2	$2 + 2 \cdot 3$
3	$2 + 3 \cdot 3$
4	$2 + 4 \cdot 3$
...	...
n	$2 + n \cdot 3$

There are many ways to *write for structure*

Arithmetic Thinking

In	Out
0	2
1	5
2	8
3	11
4	14
...	...
n	$3n + 2$

Algebraic Thinking

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	$2 + \underbrace{3+\ldots+3}_{n}$

In	Out
0	$2 + 0 \cdot 3$
1	$2 + 1 \cdot 3$
2	$2 + 2 \cdot 3$
3	$2 + 3 \cdot 3$
4	$2 + 4 \cdot 3$
...	...
n	$2 + n \cdot 3$

Writing for structure: Choices teachers make

constant

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2 + 3+3+...+3

+3 is the change from one step to the next

n times

Rewrite the table in a way that reveals the structure

IN	OUT
0	7
1	12
2	17
3	22
4	27
...	...
n	

IN	OUT
0	
1	
2	
3	
4	
...	...
n	

Rewrite the table in a way that reveals the structure

IN	OUT
0	7
1	12
2	17
3	22
4	27
...	...
n	

IN	OUT
0	$7 + 0 \cdot 5$
1	$7 + 1 \cdot 5$
2	$7 + 2 \cdot 5$
3	$7 + 3 \cdot 5$
4	$7 + 4 \cdot 5$
...	...
n	$7 + n \cdot 5$

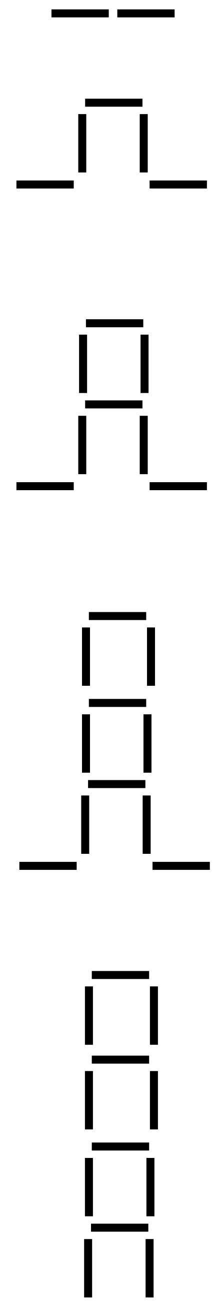
Choose one table and make a visual pattern that represents the structure

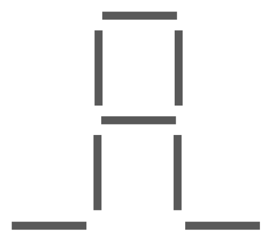
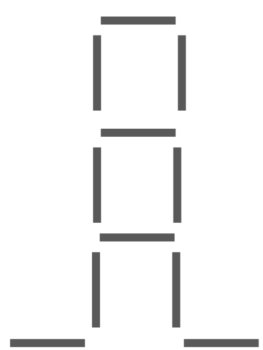
IN	OUT
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	$2 + \underbrace{3+\ldots+3}_{n}$

IN	OUT
0	$7 + 0 \cdot 5$
1	$7 + 1 \cdot 5$
2	$7 + 2 \cdot 5$
3	$7 + 3 \cdot 5$
4	$7 + 4 \cdot 5$
...	...
n	$7 + n \cdot 5$

Terry's Toothpick Pattern

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	$2 + \underbrace{3+\ldots+3}_{n}$



Step	# toothpicks	Visual pattern
0	2	
1	2 + 3	
2	2 + 3 + 3	
3	2 + 3 + 3 + 3	

What can *writing for structure* in tables help you see?

x	y
0	0+2
1	1+2
2	2+2
3	3+2
4	4+2

x	y
0	2
1	1+2
2	1+1+2
3	1+1+1+2
4	1+1+1+1+2

x	y
0	$3 \cdot 2^0$
1	$3 \cdot 2^1$
2	$3 \cdot 2^2$
3	$3 \cdot 2^3$
4	$3 \cdot 2^4$

x	y
0	$0 \cdot 3 + 2$
1	$1 \cdot 3 + 2$
2	$2 \cdot 3 + 2$
3	$3 \cdot 3 + 2$
4	$4 \cdot 3 + 2$

x	y
0	2
1	3+2
2	3+3+2
3	3+3+3+2
4	3+3+3+3+2

x	y
0	3
1	$3 \cdot 2$
2	$3 \cdot 2 \cdot 2$
3	$3 \cdot 2 \cdot 2 \cdot 2$
4	$3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

What can *writing for structure* in tables help you see?

x	y
0	0+2
1	1+2
2	2+2
3	3+2
4	4+2

x	y
0	2
1	1+2
2	1+1+2
3	1+1+1+2
4	1+1+1+1+2

x	y
0	$0 \cdot 3 + 2$
1	$1 \cdot 3 + 2$
2	$2 \cdot 3 + 2$
3	$3 \cdot 3 + 2$
4	$4 \cdot 3 + 2$

x	y
0	2
1	3+2
2	3+3+2
3	3+3+3+2
4	3+3+3+3+2

What can *writing for structure* in tables help you see?

x	y
0	0+2
1	1+2
2	2+2
3	3+2
4	4+2

x	y
0	2
1	1+2
2	1+1+2
3	1+1+1+2
4	1+1+1+1+2

x	y
0	$0 \cdot 3 + 2$
1	$1 \cdot 3 + 2$
2	$2 \cdot 3 + 2$
3	$3 \cdot 3 + 2$
4	$4 \cdot 3 + 2$

x	y
0	2
1	3+2
2	3+3+2
3	3+3+3+2
4	3+3+3+3+2

+3
+3
+3
+3

What can *writing for structure* in tables help you see?

$y = mx + b$

These *look* so different!

$y = a \cdot r^x$

x	y
0	$0 \cdot 3 + 2$
1	$1 \cdot 3 + 2$
2	$2 \cdot 3 + 2$
3	$3 \cdot 3 + 2$
4	$4 \cdot 3 + 2$

x	y
0	2
1	3+2
2	3+3+2
3	3+3+3+2
4	3+3+3+3+2

x	y
0	$3 \cdot 2^0$
1	$3 \cdot 2^1$
2	$3 \cdot 2^2$
3	$3 \cdot 2^3$
4	$3 \cdot 2^4$

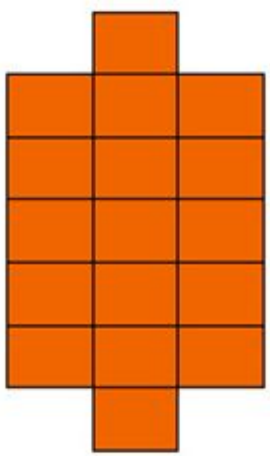
x	y
0	3
1	$3 \cdot 2$
2	$3 \cdot 2 \cdot 2$
3	$3 \cdot 2 \cdot 2 \cdot 2$
4	$3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

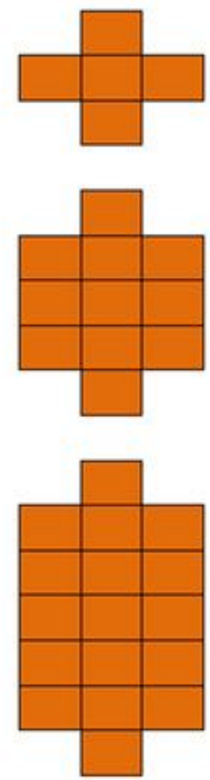
The commonalities between arithmetic and geometric sequences is revealed when *writing for structure*

A “structure-focused” introduction to In-Out tables...

Dot Talks ⟷ Pattern Talks

How did you determine the number of squares?





http://www.visualpatterns.org

Dot Talks	Pattern Talks

1. What can you know without calculating?

$(0 \cdot 3) + 1$

$(1 \cdot 3) + 1$

$(2 \cdot 3) + 1$

$(3 \cdot 3) + 1$

...

Dot Talks	Pattern Talks

1. What can you know without calculating?

2. Make an In-Out table and find the rule

$(0 \cdot 3) + 1$

$(1 \cdot 3) + 1$

$(2 \cdot 3) + 1$

$(3 \cdot 3) + 1$

...

In	Out
0	$(0 \cdot 3) + 1$
1	$(1 \cdot 3) + 1$
2	$(2 \cdot 3) + 1$
3	$(3 \cdot 3) + 1$
...	...
n	$(n \cdot 3) + 1$

Dot Talks	Pattern Talks

1. What can you know without calculating?
2. Make an In-Out table and find the rule
3. Can you rewrite to show the structure?

$(0 \cdot 3) + 1$

$(1 \cdot 3) + 1$

$(2 \cdot 3) + 1$

$(3 \cdot 3) + 1$

...

In	Out
0	$(0 \cdot 3) + 1$
1	$(1 \cdot 3) + 1$
2	$(2 \cdot 3) + 1$
3	$(3 \cdot 3) + 1$
...	...
n	$(n \cdot 3) + 1$

In	Out
0	7
1	12
2	17
3	22
...	...
n	

Rewrite the table in a way that reveals the structure

x	y
0	2
1	5
2	8
3	11
4	14

→

x	y
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3

x	y
0	7
1	12
2	17
3	22
4	27

→

x	y
0	
1	
2	
3	
4	

Connecting Representations--A Structured View

Table

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

n times

Graph

Try it!

Use the structure to graph these points *without calculating.*

Try to *show the structure* on the graph.

Connecting Representations--A Structured View

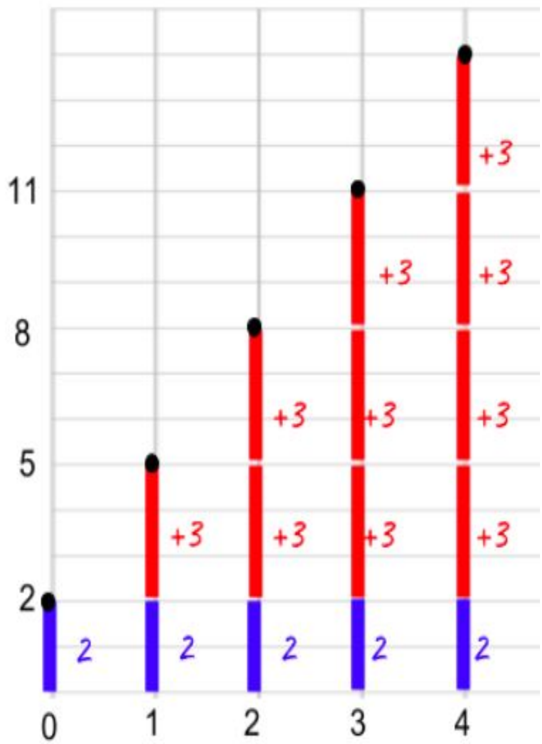
Table

constant

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

n times

Graph



Connecting Representations--A Structured View

Table

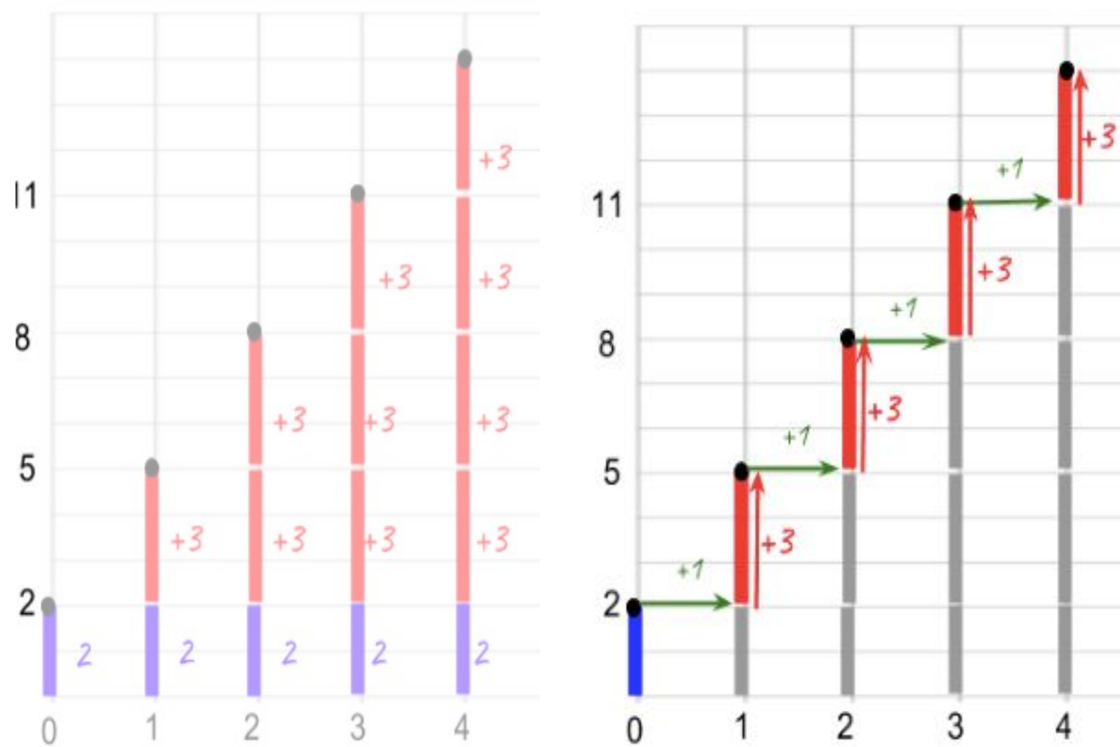
In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

constant

+1

n times

Graph



Connecting Representations--A Structured View

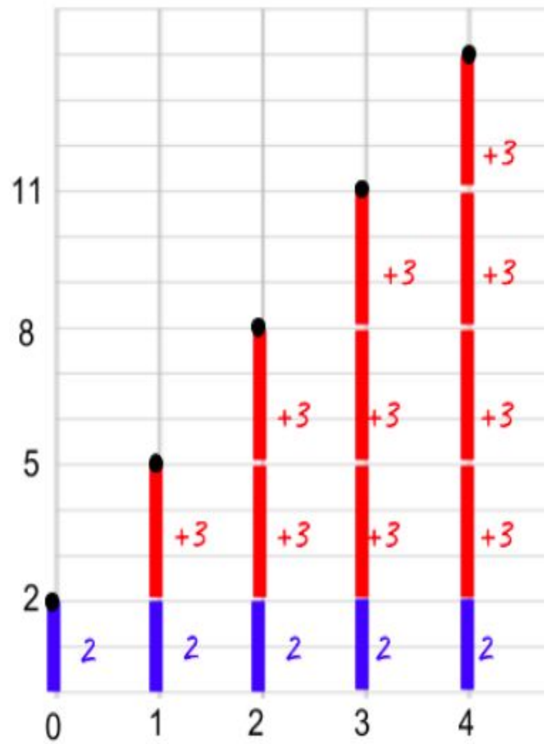
Table

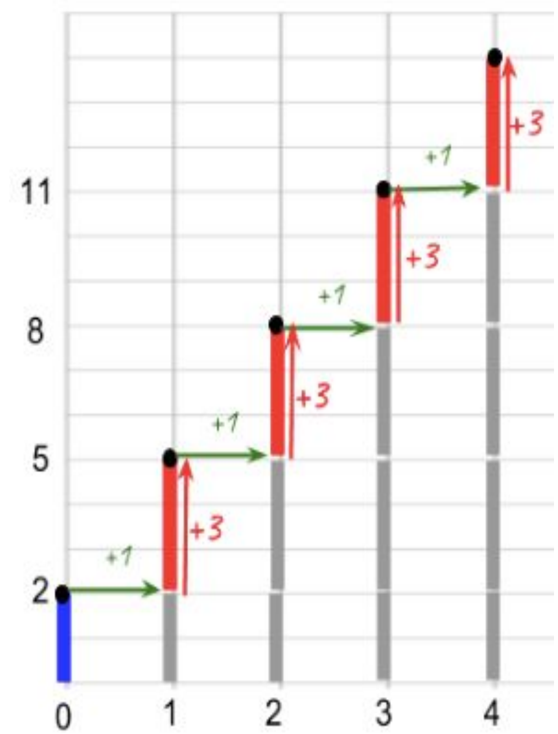
constant

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

n times

Graph





Connecting Representations--A structured view

Table

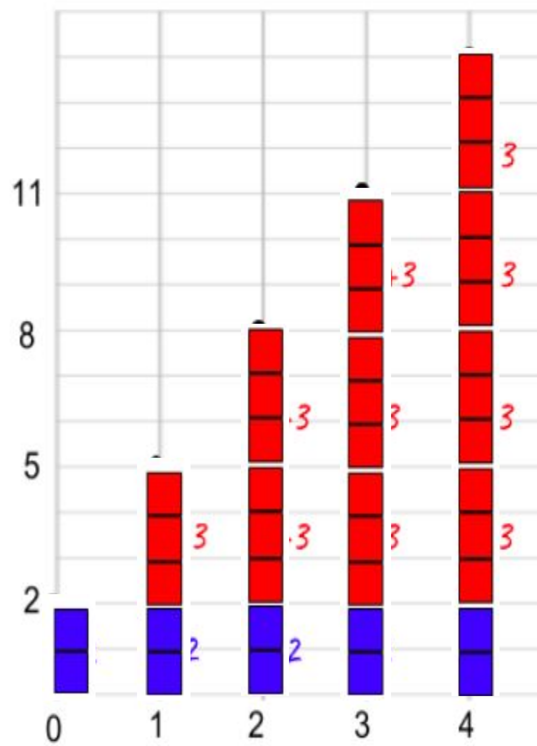
Graph

Visual Pattern

constant

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

n times



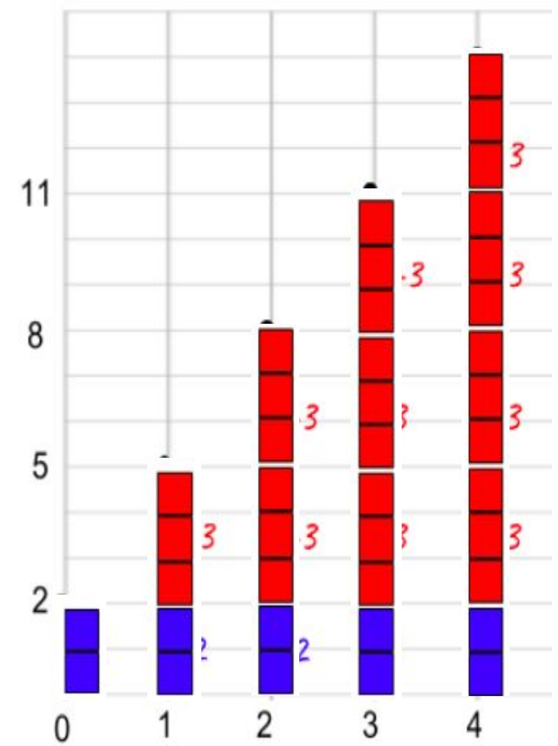
Connecting Representations--A structured view

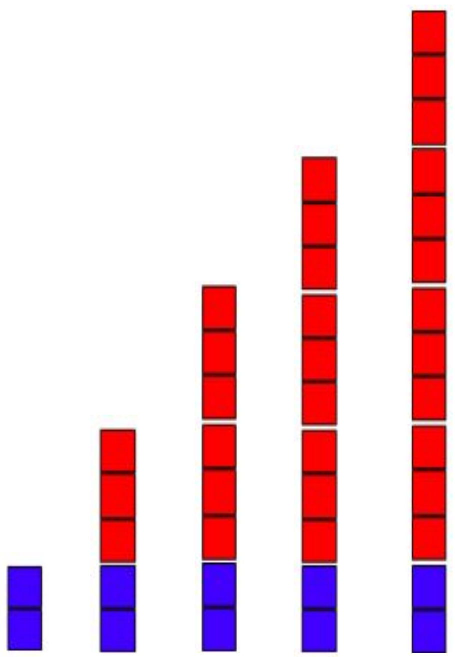
Table | **Graph** | **Visual Pattern**

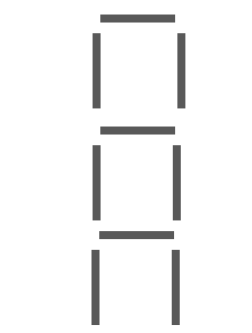
constant (→ 2)

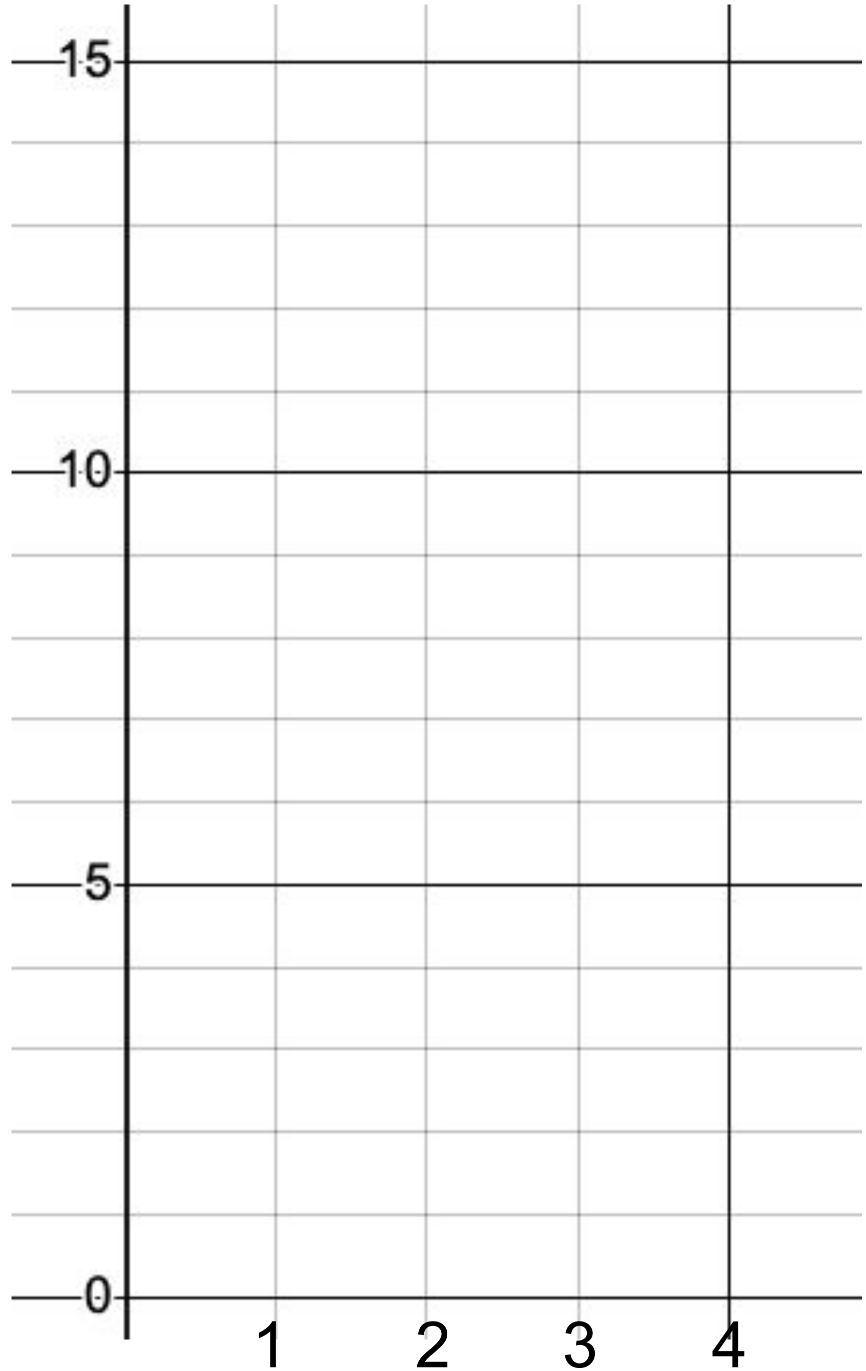
In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

n times





Step	# toothpicks	Visual pattern
0	2	
1	2 + 3	
2	2 + 3 + 3	
3	2 + 3 + 3 + 3	



Connecting Representations--A structured view

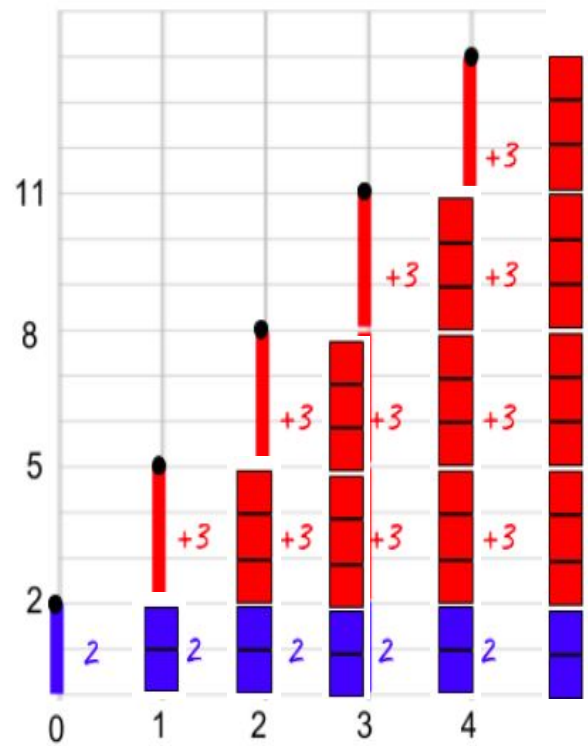
Table

constant

In	Out
0	2
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3
...	...
n	2+3+3+...+3

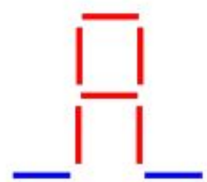
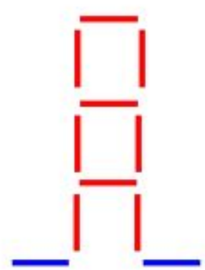
n times

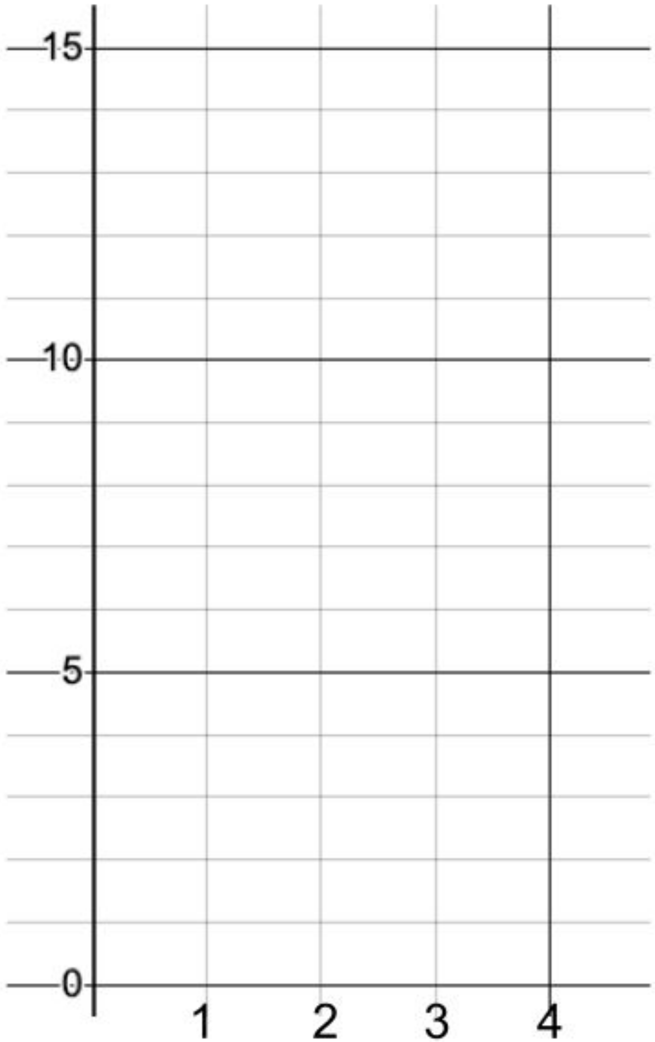
Graph



Visual Pattern

Connecting Representations using Structure

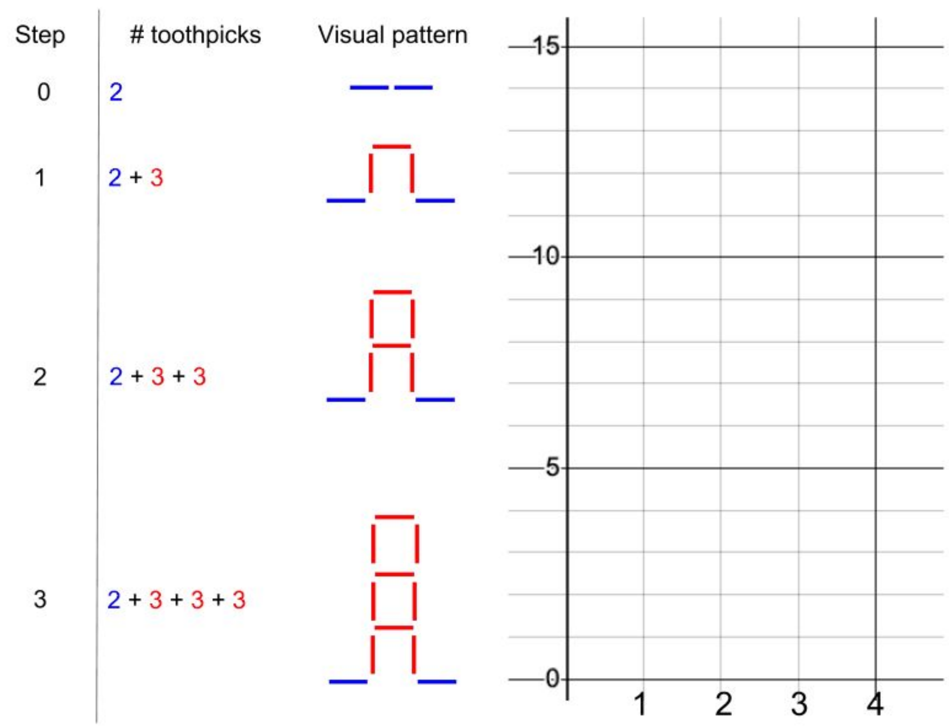
Step	# toothpicks	Visual pattern
0	2	
1	2 + 3	
2	2 + 3 + 3	
3	2 + 3 + 3 + 3	



Stop Motion Animation

Google Slides:
1 per move
(26 for this animation)

[Tall Tweets](#) to turn Google Slides into gif



Same or Different?

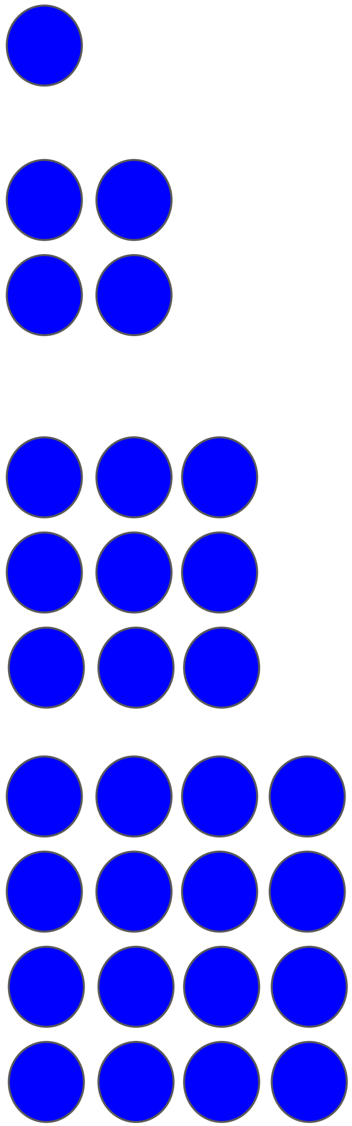
In	Out
0	2
1	$2 + 3$
2	$2 + 3 + 3$
3	$2 + 3 + 3 + 3$
4	$2 + 3 + 3 + 3 + 3$
...	...
n	

In	Out
0	2
1	$2 \cdot 3$
2	$2 \cdot 3 \cdot 3$
3	$2 \cdot 3 \cdot 3 \cdot 3$
4	$2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
...	...
n	

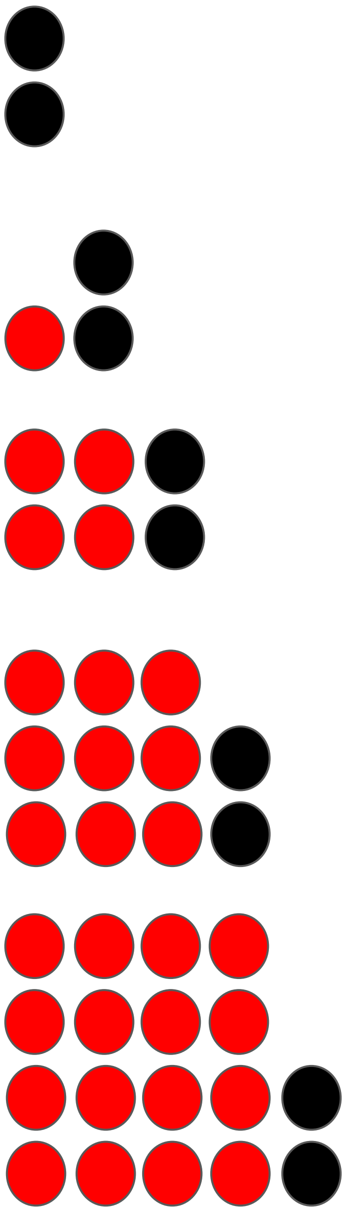
In	Out
0	$3 + 0^2$
1	$3 + 1^2$
2	$3 + 2^2$
3	$3 + 3^2$
4	$3 + 4^2$
...	...
n	$3 + n^2$

In	Out
0	$3 + 2 \cdot 0^2$
1	$3 + 2 \cdot 1^2$
2	$3 + 2 \cdot 2^2$
3	$3 + 2 \cdot 3^2$
4	$3 + 2 \cdot 4^2$
...	...
n	$3 + 2 \cdot n^2$

In	Out
0	0^2
1	1^2
2	2^2
3	3^2
4	4^2
...	...
n	n^2



In	Out
0	$0^2 + 2$
1	$1^2 + 2$
2	$2^2 + 2$
3	$3^2 + 2$
4	$4^2 + 2$
...	...
n	$n^2 + 2$



https://www.desmos.com/calculator/kr4quo5ocx

In	Out
0	0^2
1	1^2
2	2^2
3	3^2
4	4^2
...	...
n	n^2

In	Out
0	$2 \cdot 0^2 + 4 \cdot 0 + 3$
1	$2 \cdot 1^2 + 4 \cdot 1 + 3$
2	$2 \cdot 2^2 + 4 \cdot 2 + 3$
3	$2 \cdot 3^2 + 4 \cdot 3 + 3$
4	$2 \cdot 4^2 + 4 \cdot 4 + 3$
...	...
n	$2 \cdot n^2 + 4 \cdot n + 3$

Link to Google slides: https://tinyurl.com/StructureTablesNCTM19

Thank you for coming!

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Do you love multiplication and integers?!

3-4 pm TODAY
Sapphire KL
Minus times minus is plus: The reason for this we will discuss