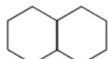


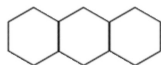


A Professional Learning Model to Build Cross-Grade Mathematics Learning Communities

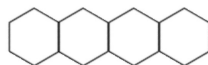
Hexagon Problem



Term 2



Term 3



Term 4

- ▷ Use colour to illustrate how you see the **perimeter** growing.
- ▷ What does Term 1 look like?
- ▷ How would you determine the perimeter of the 100th term?
- ▷ How would you determine the perimeter of any term?

What are some other strategies that your students might use to approach this problem?

Hey look! There's a math problem on your table... You should give it a try!

Resource Folder
<https://goo.gl/n3t6CE>



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@rhondahever

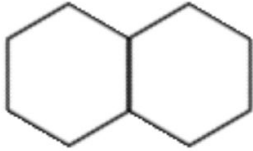


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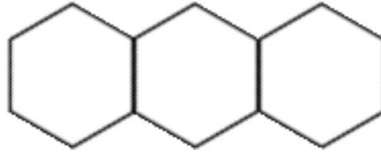


Hexagon Problem

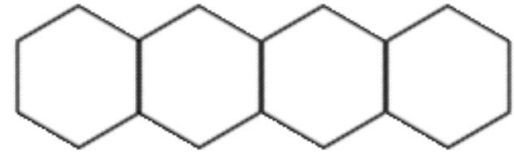
Grades 6 - 8



Term 2



Term 3



Term 4

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Tasked with designing a professional learning experience that would address transitions in Mathematics

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What we know...

Secondary claims too
many gaps are created
in Elementary

Elementary claims
Secondary teaches
only traditionally

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What we know...

Elementary does
too much show and
tell and not enough
math harvesting

Secondary does too
much math telling
and not enough
show and tell

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What we know...

These 2 panels
never communicate

Students are
expected to
transition smoothly

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PD Model - Objectives

1. Build cross-divisional relationships

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Build Cross Divisional Relationships

K - 6 Feeder School → **7/8 Feeder School** → **Secondary School**

Grade 6 Team

- 2 Grade 6 Teachers
- 1 Special Education Resource Teacher
- Admin

Grade 7/8 Team

- 1 Grade 7 Teacher
- 1 Grade 8 Teacher
- 1 Special Education Resource Teacher
- Admin

Grade 9 Team

- 2 Grade 9 Academic Teachers
- 1 Special Education Resource Teacher
- Admin

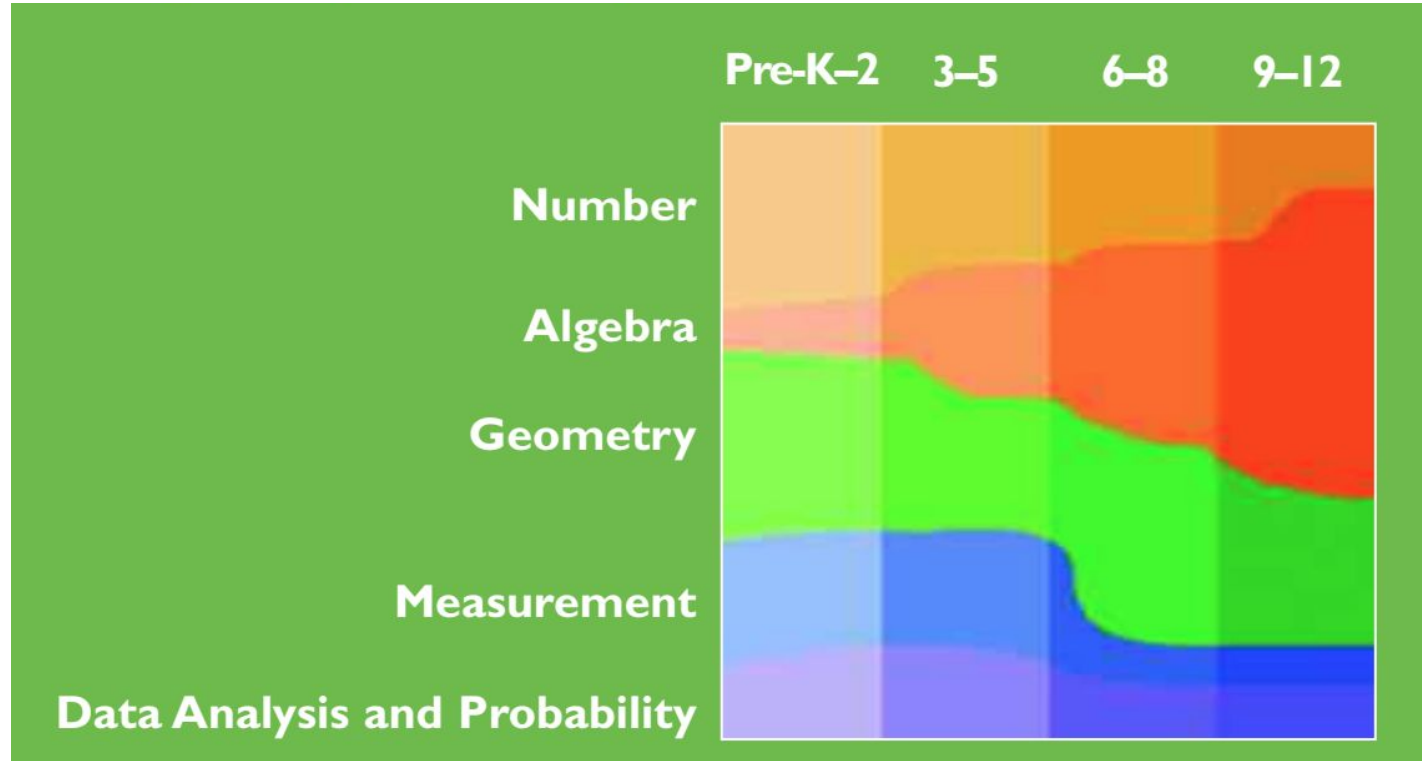


PD Model - Objectives

1. Build cross-divisional relationships
2. Understand the cognitive development of the algebraic progression
3. Clear understanding of grade level expectations and how they are built on in previous/future years



Model - Why Algebraic Reasoning?



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“The 12-year-old who enters Grade 7 is a baby, mathematically speaking. But that same person four years later, upon entering Grade 11, is expected to be acquainted with many of the most important mathematical ideas that exist.”

*~ The Importance of Transition Years, Grades 7-10, in School Mathematics. Usikin,
The University of Chicago School Mathematics Project, 2004.*

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Recursive / Functional Thinking

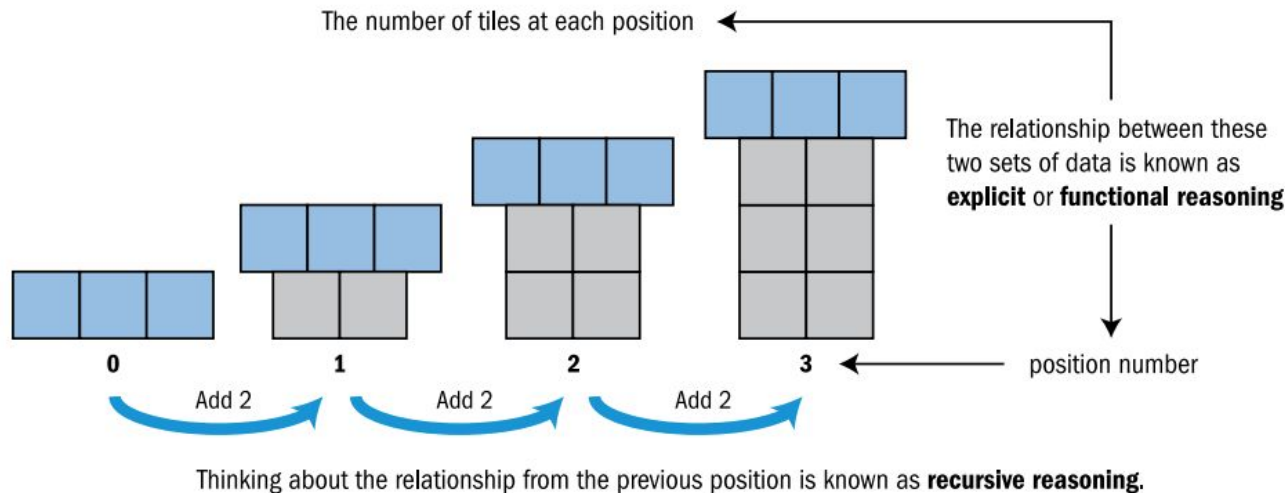


Figure 1: Two Ways of Thinking About Growing Patterns

PAYING ATTENTION TO ALGEBRAIC REASONING

Support Document for Paying Attention to Mathematics Education

K-12

Student Achievement Division

August 2014

WHAT WORKS? Research into Practice

A research-into-practice series produced by a partnership between the Student Achievement Division and the Ontario Association of Deans of Education

Research Monograph #55

Exploring the Power of Growing Patterns

By Dr. Ruth Beatty
Lakehead University

Why do we teach growing patterns and what do patterns have to do with algebra?

Research Tells Us

- Patterning activities are a powerful tool for introducing young children to sophisticated algebraic concepts.
- Even very young students can develop an understanding of functions.
- For older students, working with linear patterns can serve as an effective introduction to graphs of functions.
- How patterns are presented and discussed is the key to their effective use.

Two Ways of Thinking About Growing Patterns

Consider a typical growing patterning activity: Jesse (a Grade 6 student) was easily able to describe the pattern in Figure 1 as "Start with three and add two each time." When asked by his teacher how many tiles would be in the next position, he confidently stated, "Well, you have 9 tiles, so 11 for the next position." However, when Jesse was asked to predict the 100th position, the process proved to be too laborious. "That's too big! Maybe... 102 tiles?"

The number of tiles at each position

The relationship between these two sets of data is known as **explicit** or **functional reasoning**

position number

Thinking about the relationship from the previous position is known as **recursive reasoning**.

RUTH BEATTY is an Associate Professor in the Faculty of Education at Lakehead University. Her research interests include investigating how children of all ages develop algebraic thinking, the affordance of visual representations in mathematics and the connections between Anishinaabe ways of knowing and Western mathematics curricula.

The Student Achievement Division is committed to providing teachers with current research on instruction and learning. The articles and questions contained in these monographs are, however, those of the authors and do not necessarily reflect the policies, views, or directions of the Ontario Ministry of Education or the Student Achievement Division.

support every child
reach every student

Ontario



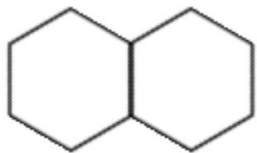
PD Model - Objectives

1. Build cross-divisional relationships
2. Understand the cognitive development of the algebraic progression
3. Clear understanding of grade level expectations and how they are built on in previous/future years
4. Differentiated Instruction - how one question can be used to address a variety of different learning needs

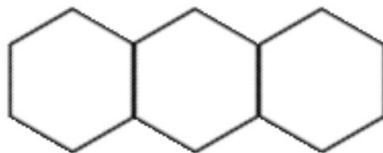


Hexagon Problem

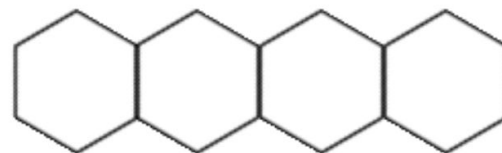
Grades 6 - 8



Term 2

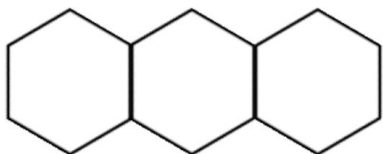


Term 3

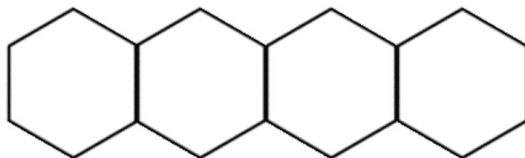


Term 4

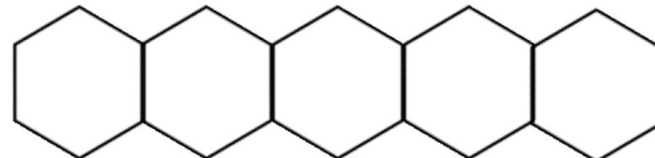
Grade 9 Academic



Term 1



Term 2

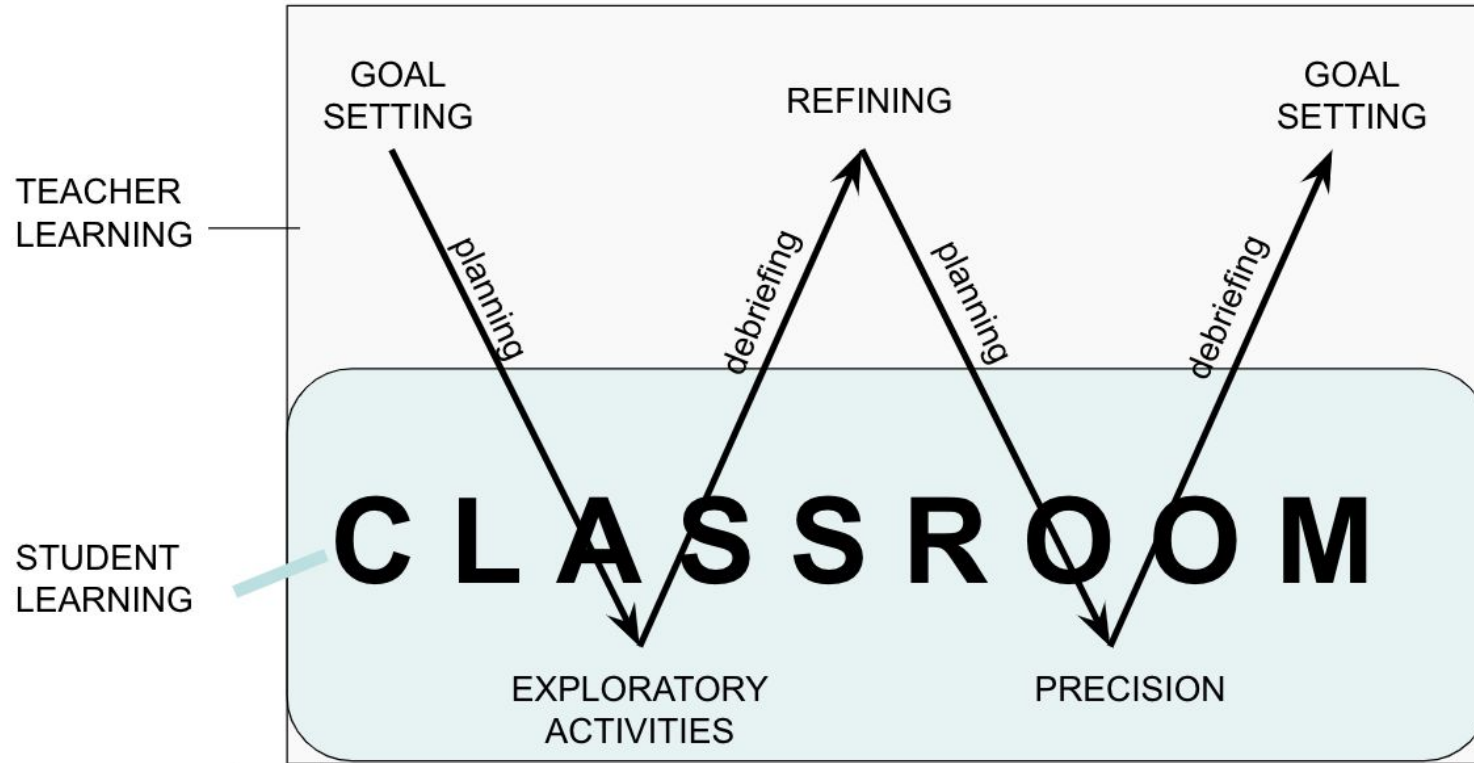


Term 3

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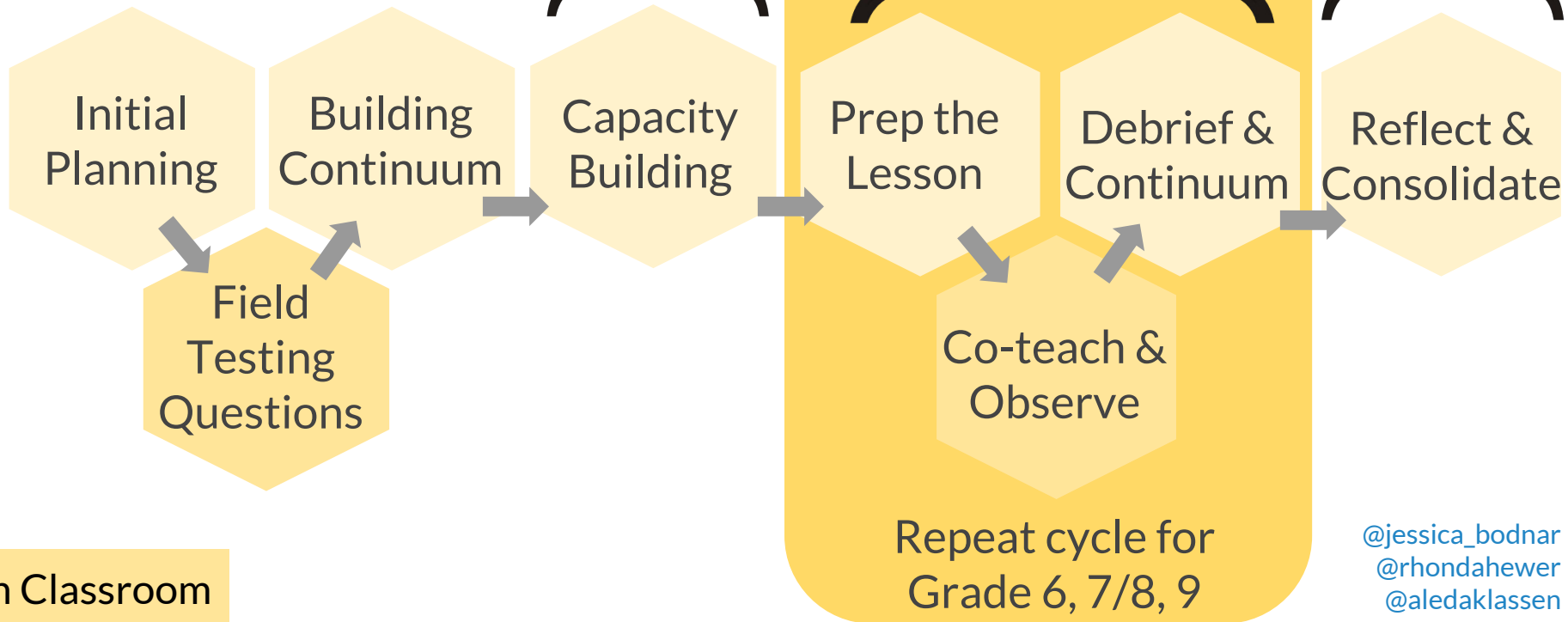
W Model - Cathy Bruce



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@aledaklassen



Out of Classroom

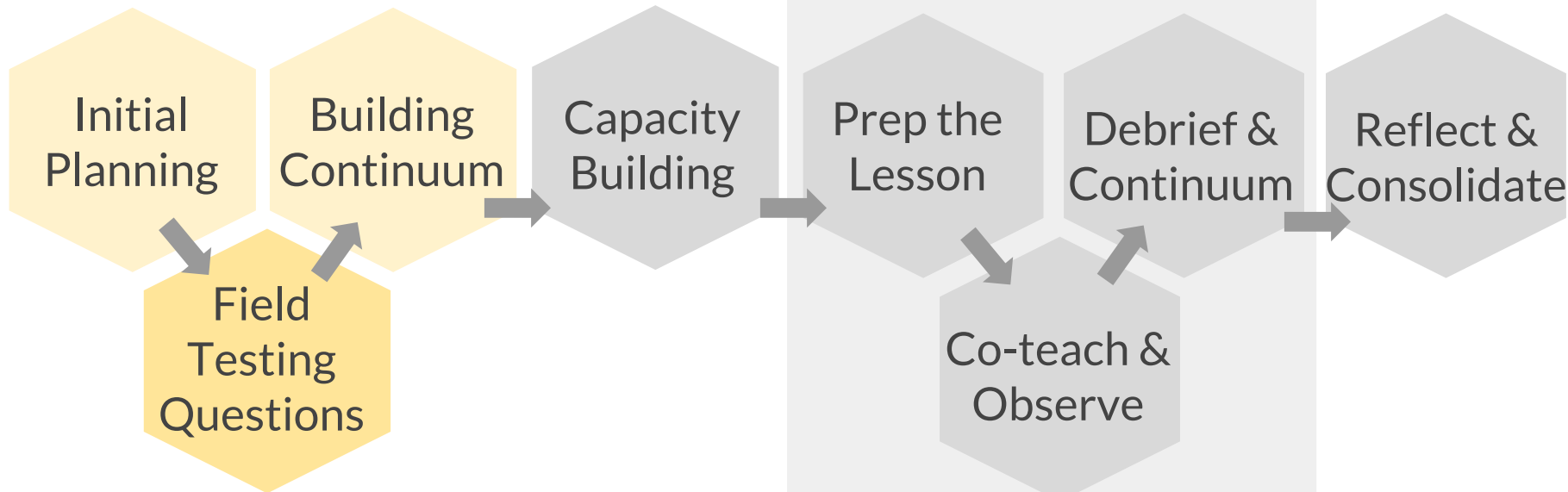


In Classroom

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Out of Classroom

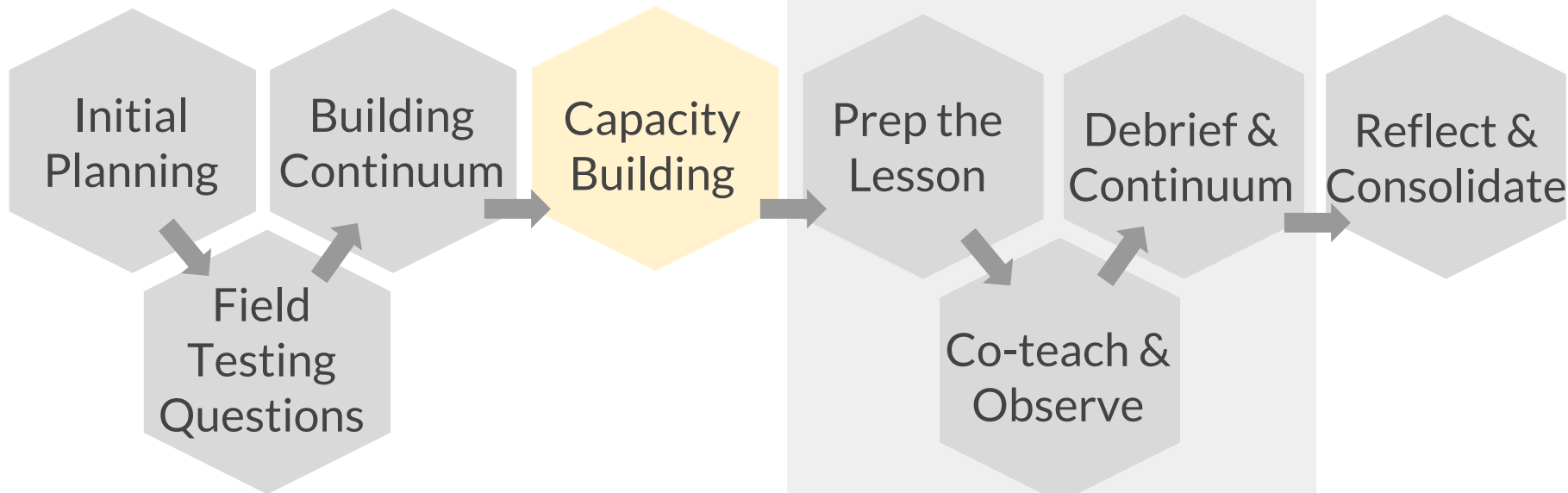


In Classroom

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Out of Classroom



In Classroom

Repeat cycle for
Grade 6, 7/8, 9

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Capacity Building

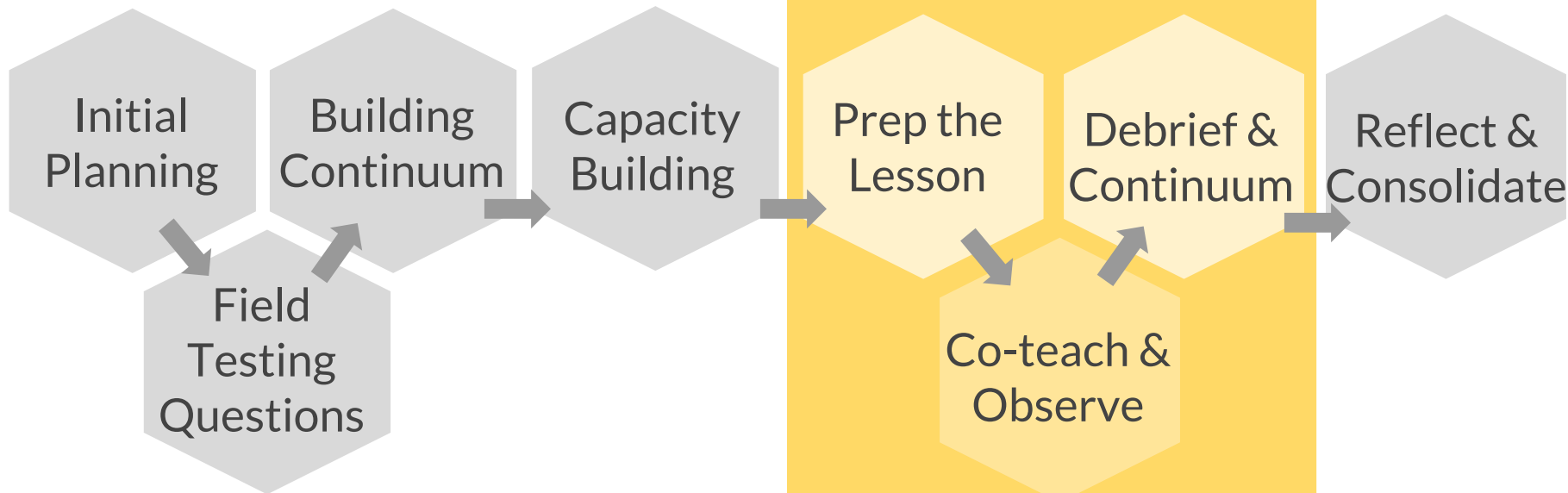


- Build community
- Do the problem
- Dig into the math
- Explore the continuum
- Logistics

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Out of Classroom



In Classroom

Repeat cycle for
Grade 6, 7/8, 9

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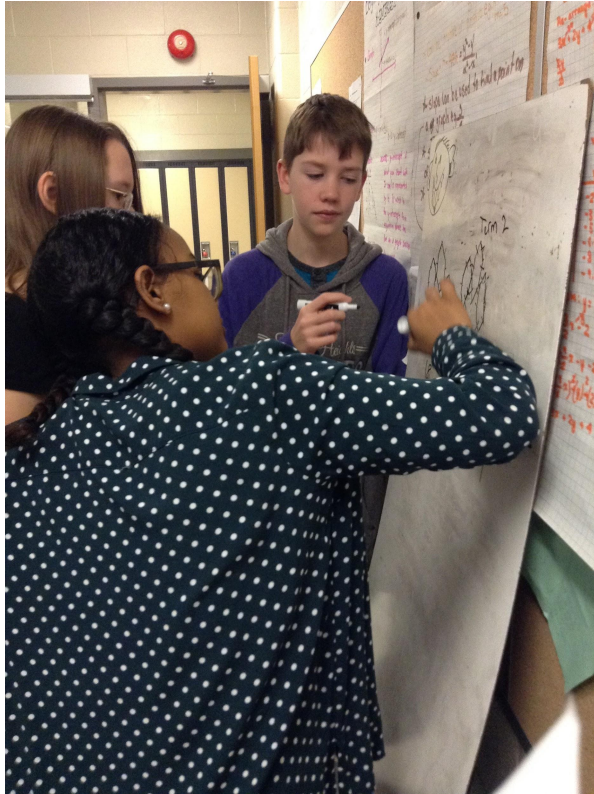
PD Model - Objectives

1. Build cross-divisional relationships
2. Understand the cognitive development of the algebraic progression
3. Clear understanding of grade level expectations and how they are built on in previous/future years
4. Differentiated Instruction - how one question can be used to address a variety of different learning needs
5. We all own these students

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Who are we going in to meet?



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PD Model - Objectives

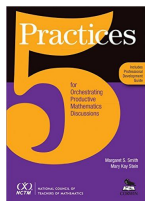
1. Build cross-divisional relationships
2. Understand the cognitive development of the algebraic progression
3. Clear understanding of grade level expectations and how they are built on in previous/future years
4. Differentiated Instruction - how one question can be used to address a variety of different learning needs
5. We all own these students
6. Exposure to pedagogical strategies

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5 Practices

1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting



Monitoring Tool

Mathematical Goal:

ANTICIPATED STRATEGY / MODEL / APPROACH	NOTES	Group Names	Order
<p>Term 1: $1 + 2(0+1)$ Term 2: $1 + 3(1+1)$ Term 3: $1 + 4(2+1)$</p> <p>$1 + 4n + 1$ OR $4n + 2$</p>			
<p>Term 1: $3 + 4 \times 0$ Term 2: $3 + 4(1)$ Term 3: $3 + 4(2)$</p> <p>$3 + 4(n - 1) + 3$</p>			
<p>Term 1: $6 + 4 \times 0$ Term 2: $6 + 4(1)$ Term 3: $6 + 4(2)$</p> <p>$6 + 4(n - 1)$</p>			
<p>Term 1: $6 - 2(0)$ Term 2: $6 - 2(1)$ Term 3: $6 - 2(2)$</p> <p>$6n - 2(n - 1)$</p>			
<p>Term 1: $5 + 4(0)$ Term 2: $5 + 4(1)$ Term 3: $5 + 4(2)$</p> <p>$5 + 4(n - 2) + 5$</p>			
<p>Term 1: $1 + 2(0)$ Term 2: $1 + 2(1)$ Term 3: $1 + 2(2)$</p> <p>$1 + 2n + 2n + 1$</p>			

Monitoring Tool

Mathematical Goal:

ANTICIPATED STRATEGY / MODEL / APPROACH	NOTES	Group Names	Order
<p>Pattern that has a constant perimeter</p> <p>$P = 6$</p>			
<p>Pattern that increases at a slower rate</p> <p>$P = 10 + 2n$</p>	<p>Term 1: 12 Term 2: $12 + 4 - 2$ Term 3: $12 + 2(4) - 2(2)$... Term x: $12 + (n - 1)(4) - (n - 1)(2)$</p>		
<p>Pattern that has the same perimeter</p> <p>$P = 10 + 4n$</p>	<p>Term 1: 14 Term 2: $14 + 5 - 1$ Term 3: $14 + 5 - 1 + 5 - 1$... Term x: $14 + (n - 1)(5 - 1)$</p>		
<p>Pattern that increases at a faster rate</p> <p>$P = 12 + 6n$</p>	<p>Term 1: 18 Term 2: $18 + 6$ Term 3: $18 + 6 + 6$... Term x: $18 + (n - 1)(6)$</p>		

o: Selecting Goals and Task

What is the mathematical goal for the students in your class?

By the end of the this activity my students will know and understand...

Does there need to be any changes to the prompts?

- Use colour to illustrate how you see the **perimeter** growing.
- What does Term 1 look like?
- How would you determine the perimeter of the 100th term?
- How would you determine the perimeter of any term?

1: Anticipating

Which solutions/strategies do you anticipate your students using?

Are there any common misconceptions you anticipate?

Which strategies/solutions will be the most useful in addressing the mathematics goal?

2: Monitoring

Role 1:

- ▷ Host Teacher
- ▷ Co-Teacher = Consultant
- ▷ Introduce lesson
- ▷ Ask students questions to guide their thinking
- ▷ Provide hints and extensions
- ▷ Participate in Consolidation huddle
- ▷ Consolidate learning

Role 2:

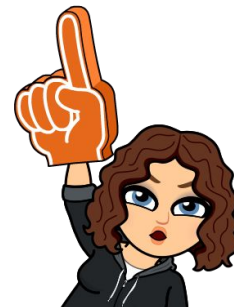
- ▷ Classroom Teachers
- ▷ Use Monitor Tracking sheet
- ▷ Cruise the room and observe different groups
- ▷ Select groups and pieces of work for consolidation of the mathematical goal
- ▷ Say nothing to guide students
- ▷ Participate in consolidation huddle

Role 3:

- ▷ SERTs, Admin, Itinerant Teacher, LS staff
- ▷ Observe one group of students for the entire period
- ▷ Document student thinking
- ▷ Say nothing to guide students
- ▷ One person needs to be a timer
 - 35 minutes – Intro and task
 - 5 minutes – consolidation huddle
 - 20 minutes - consolidation

Preparing for Game Time

- ▶ How will you **introduce** the problem?
Activate prior knowledge (without telling them what to do).
- ▶ How will you **check** for understanding of what the task is asking?
- ▶ How will students be **grouped**?
Which groups will have an adult sitting with them?
- ▶ What **tools/manipulatives** will be useful to have available?
- ▶ What will students **work on**?
(VNPS, chart paper, 11x17 paper, mini-whiteboards etc.)
- ▶ Any **other items** that need to be prepared?



LET'S GO!



Co-teaching



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@rhondahever
@aledaklassen

Consolidation Huddle (5 minutes)

3: Selecting

- Was there a common misconception that needs to be addressed?
- Which student work will you use?
- Which parts of the student work?
- Purposefully choose work that will advance mathematical thinking

4: Sequencing

- Which student(s) will speak?
- In what order will student work be presented?
- How will work be shared?

Debrief

ORDER

- ▷ **Role 1:**
 - Teacher
 - Co-Teacher (Consultant)
- ▷ **Role 2:**
 - Classroom Teachers
- ▷ **Role 3:**
 - SERTs, Admin, Itinerant Teachers

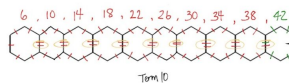
Lesson/Student Thinking Reflection

- ▷ What worked well overall?
- ▷ What was challenging overall?
- ▷ What were some examples of student thinking that confirmed your thinking or surprised you?
- ▷ What are possible next steps for the host teacher?
- ▷ What would you change about the task for your students?
- ▷ What would you change about the structure of the lesson for your students?



RECURSIVE THINKING

Extend the pattern:
skip counting



"I notice that the totals add four each time with the pattern 6, 10, 14, 18, 22, ..."

Number Pattern

Extend the pattern:
repeated addition

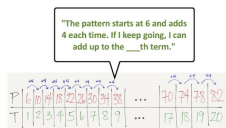


Table of Values

FUNCTIONAL THINKING

Extend the pattern
across the table.

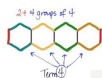
Term #	Perimeter
0	2 $4 \times 1 + 2$
1	6 $4 \times 2 + 2$
2	10 $4 \times 3 + 2$
3	14 $4 \times 4 + 2$
4	18 $4 \times 5 + 2$
5	22 $4 \times 6 + 2$
6	26 $4 \times 7 + 2$
100	402 $4 \times 100 + 2$

"I saw a relationship between the term number and the perimeter and it helped me make a prediction."

Table of Values

MULTIPLICATIVE
THINKING

Create an explicit
pattern rule



"I noticed that there are the term number groups of sides in each term plus the extra two on the end. I made an equation with those numbers."

Algebraic Equation

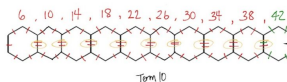
MULTIPLICATIVE
THINKING

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RECURSIVE THINKING

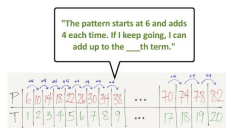
Extend the pattern:
skip counting



"I notice that the totals add four each time with the pattern 6, 10, 14, 18, 22, ..."

Number Pattern

Extend the pattern:
repeated addition



"The pattern starts at 6 and adds 4 each time. If I keep going, I can add up to the ____th term."

Table of Values

FUNCTIONAL THINKING

Extend the pattern
across the table.

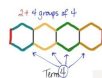
Term #	Perimeter
0	2
1	6
2	10
3	14
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5	22
6	26
100	402

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Table of Values

MULTIPLICATIVE THINKING

Create an explicit
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"I noticed that there are the term number groups of sides in each term plus the extra two on the end. I made an equation with those numbers."

Algebraic Equation

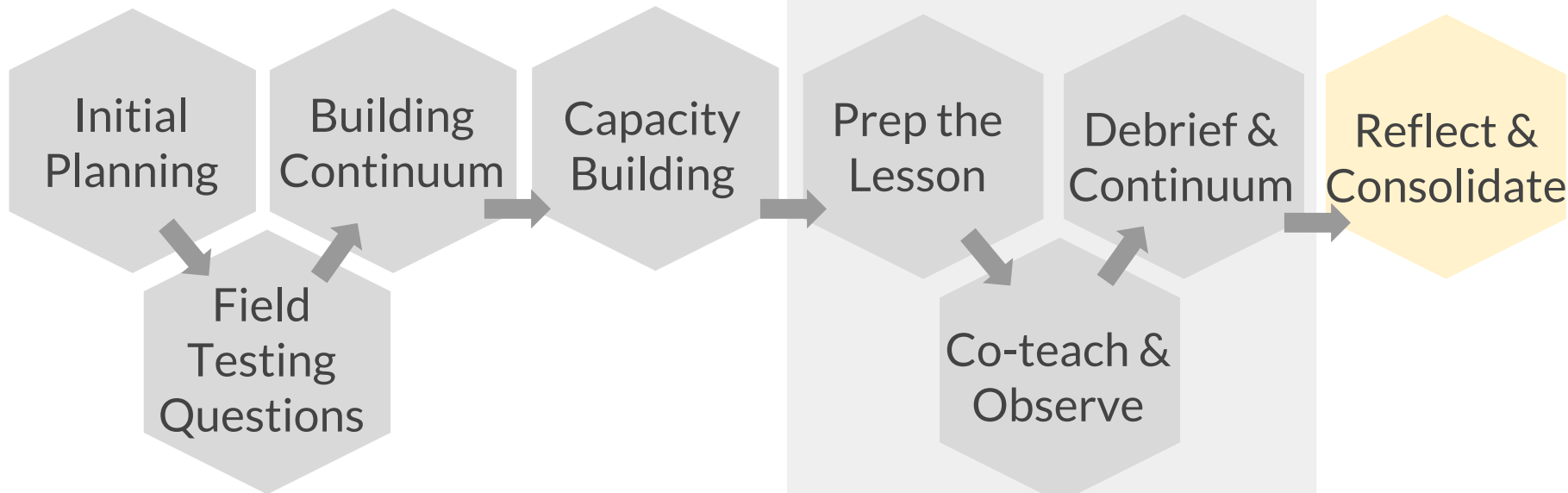
MULTIPLICATIVE THINKING

Where would you place the student work along the continuum?

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Out of Classroom



In Classroom

Repeat cycle for
Grade 6, 7/8, 9

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Reflect and Consolidate



GRADE 9 TRANSITIONS FEEDBACK	
QUESTION	TRANSCRIPT
EXPECTATIONS What did you come to this process expecting? Were these expectations met?	<ul style="list-style-type: none">No idea what to expectThought it would be more about transitioning kids in math (what are they getting in grade 6, 7 & 9?)Thought there would be more of this:<ul style="list-style-type: none">What can we be doing to fill in those gaps without changing grades? Because there is no room to move to a different level?
SURPRISES What surprised you throughout this process?	<ul style="list-style-type: none">Pleasantly surprised. Thought some of the students weren't going to solve the problem. Some solved it & I was really surprised. They were making more than I thought. Wasn't sure to take risks on the problem. They were making more than I thought. And it was a great use there were so

GRADE 7/8 TRANSITIONS FEEDBACK	
QUESTION	TRANSCRIPT
EXPECTATIONS What did you come to this process expecting? Were these expectations met?	<ul style="list-style-type: none">Connections with teachers at different schools - met going - metKnowing where my students have come from and where they are going - metLearn about what is being taught and have opportunity to talk to these teachers about what they saw, what they think - happened during breaks - would have liked more less structured time - math was way as secondary
SURPRISES What surprised you throughout this process?	<ul style="list-style-type: none">How you could use this same problem in 4 different gradesHow open minded secondary was and positive about the situationHow every teacher could teach a lesson after an hour of planning
FEEDBACK ON PROCESS What feedback would you give about the process?	<ul style="list-style-type: none">Tough to do the problem because I wasn't in that area of study at that timeTough to do the problem because I wasn't in that area of study at that time

GRADE 6 TRANSITIONS FEEDBACK	
QUESTION	TRANSCRIPT
EXPECTATIONS What did you come to this process expecting? Were these expectations met?	<ul style="list-style-type: none">Thought Gr7 would show gap between Grade 6 and Grade 7 teacher wants me to doSee where the students are goingDifferent methods of teachingPhysical space and lack of furniture around in the room, no posters or anchor charts in the room with so many people...
SURPRISES What surprised you throughout this process?	<ul style="list-style-type: none">Lack of materials beyond Grade 6Physical space was challenging, no room to move sometimesNot a lot of talk in higher gradesThe wooden desk with built in chair where crazyKids are so big and with the furniture we couldn't move aroundAll had phonesChromebooks with each kid

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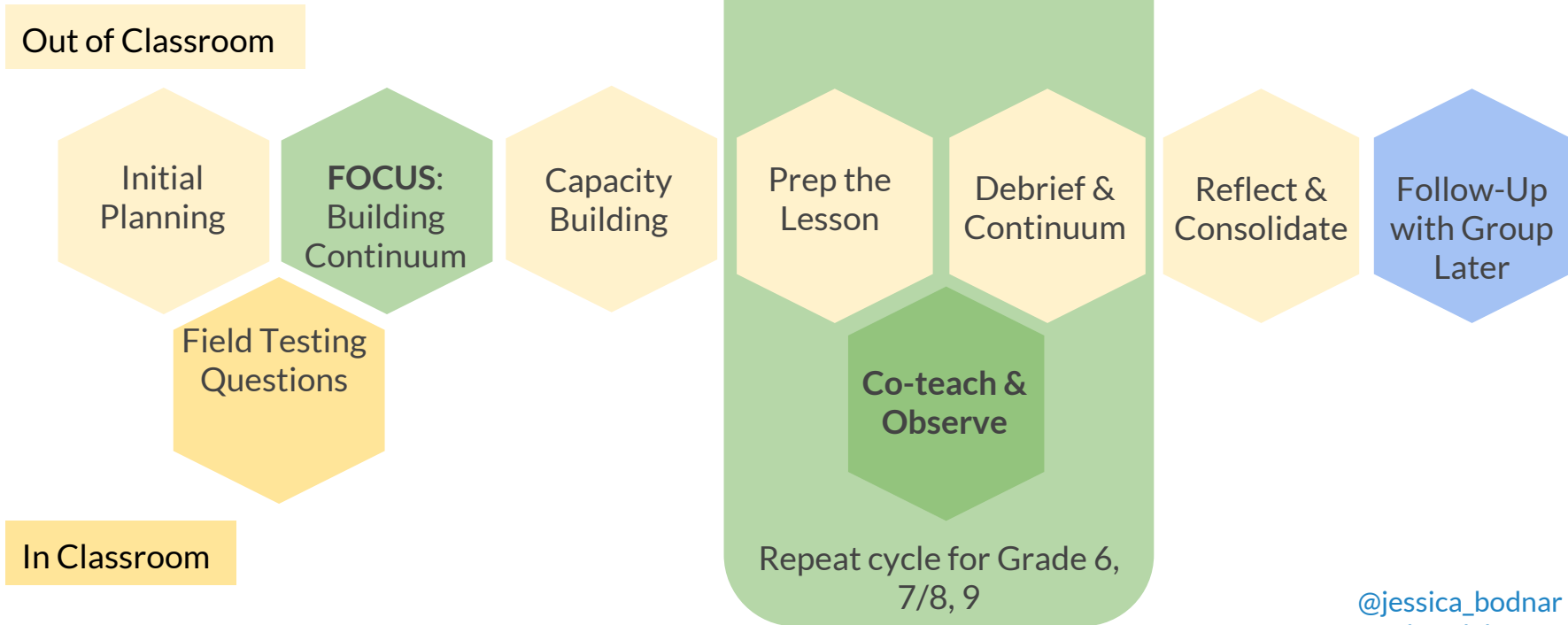


Teacher Perspectives: Video

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Our Learning



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"It was amazing to see where this problem would go in Grade 9 and how quickly the complexity jumps."

~ Grade 7 Teacher



@rhondaheuer

"I was surprised by how you could use the same problem in four different grades."

~ Grade 8 Teacher



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"It was helpful to learn what other schools are doing. Without this structure it's impossible to see it. Getting us to see lessons was helpful."

~ Grade 9 Teacher

"I liked watching the students do the math - how they did it was different from what I expected."

~ Grade 9 Teacher



@aledaklassen

"We didn't know graphing had anything to do with algebra and I went back and it's totally in the curriculum."

~ Grade 6 Teacher