Possible mathematical ideas in children's representations of counting

PRINCIPLES OF COUNTING	
One-to-one correspondence	Each object counted is associated with only one number and each object must be counted.
Number sequence	The numeral sequence in the base ten system
Cardinality	The last number counted is the total number, and the total number contains within it all lesser numbers.
Abstraction principle	Counting doesn't depend on other qualities of the object like size, color, uniformity etc.
Order irrelevance	Counting objects in any order or grouping doesn't change the total number of objects
Other mathematical ideas	Verbal or written counting can indicate other mathematical ideas e.g. the understanding of the base-ten system, place value, increasing quantity etc.
STRATEGIES OF COUNTING	
Grouping	Grouping objects to count
Visual configurations	Arranging objects in various configurations as part of counting process e.g. in a line, in rows, in clusters, randomly.
Keeping track	Keeping track of how many objects have been counted so far, and which objects have or haven't been counted e.g. by touching, moving, flipping, grouping, slowing down etc.
Associating number with object	Using numbers to support counting. This refers to number as a quantity while counting.
UNDERSTANDINGS RELATED TO REPRESENTING COUNTING	
Counting can be shown on paper.	The understanding that the process and outcome of counting can be represented on paper.
Marks on paper can be counted independently	The understanding that marks drawn on the paper can be counted separately from the objects counted
Representations can show what objects are counted	The understanding that representations can show what objects are counted e.g. cubes, flowers, blocks etc.
The total number of objects in the collection determines the number in the representation	The understanding that the representation reflects the number of objects in the collection, and that the collection cannot change to match the representation.
Symbolic strategies for representation of counting	The understanding that symbols can be used to represent and count objects on paper including detailed drawings of objects; abstracted but analogous shapes like squares to show cubes; conventional systems like tally marks; numerals; tracing etc.
Utilizing space on paper	Manipulating size and spacing in representation to fit the paper.
Knowing how numerals look and how to write numerals	Ability to accurately follow conventions of writing numerals.

(Anantharajan (in press); Carpenter et. al., 2017; National Research Council, 2001)

Protocol for looking at children's representations of counting

What do you notice in the representation? *Describe what you see*	
What does it tell you about what the student might know and understand? *Student's understanding of principles and strategies of counting, and understandings about representation*	
What partial understandings might the student have? *Student's partial understanding of principles and strategies of counting, and understandings about representation*	
What does the representation <u>not</u> tell you about the student's thinking? *Principles and strategies of counting, and understandings about representation that are not apparent in the representation*	
What can you <u>not know for sure</u> based on the representation? *Principles and strategies of counting, and understandings about representation that may seem to be visible in the representation, but you can't know for sure*	
How can you verify your perceptions about the student's thinking? *As you did not observe the student count or represent, how can you verify your perception of mathematical ideas in the student's thinking?*	
What can you do next with this student? *Next steps in teaching*	

Additional resources:
- DREME Network, Development and Research in Early Math Education. www.dreme.stanford.edu
- Franke, M. L., Kazemi, E., & Turrou, A.C. (Eds.) (2018). *Choral Counting & Counting Collections: Transforming the PreK-5 Classroom*. Stenhouse Publishers. Portsmouth, New Hampshire.