

Mighty Misconceptions: Identifying and Addressing Error Patterns to Deepen Student Understanding



NCTM 2019

WE ALL MAKE MISTAKES!

- All mistakes are NOT equal.
- Error patterns reveal misconceptions.
- Misconceptions are learned – despite our best efforts.

EVIDENCE OF THINKING

- Look for correct and incorrect answers.
- Then look deeper! Are there patterns in the errors?
- Look for evidence of student thinking. Include this in assessments by asking for students to show their work and provide explanations.
- Form hypotheses and seek verification.

Fred M.

$$\begin{array}{r} 1. \quad 43 \\ \times 2 \\ \hline 86 \end{array}$$

$$\begin{array}{r} 2. \quad 31 \\ \times 4 \\ \hline 124 \end{array}$$

$$\begin{array}{r} 3. \quad \overset{1}{4}3 \\ \times 6 \\ \hline 308 \end{array}$$

$$\begin{array}{r} 4. \quad \overset{2}{3}5 \\ \times 5 \\ \hline 255 \end{array}$$

$$\begin{array}{r} 5. \quad \overset{2}{6}3 \\ \times 7 \\ \hline 561 \end{array}$$

$$\begin{array}{r} 6. \quad \overset{4}{5}8 \\ \times 6 \\ \hline 548 \end{array}$$

- Analyze Fred's paper.
- What do you notice?
- What do think may be the cause of his errors?
- How could you verify your hypothesis?



Preconceptions

Overgeneralization
s

Partial Conceptions

Conceptual
Misunderstandings

(Rose Tobey and Fagan, 2013)

PRECONCEPTIONS

Ideas students have
from previous
experiences, including
everyday interactions

$$583,214 > 962$$

Whole Numbers: the longer
number is the larger number

Not so for decimals!

$$\del{583.214 > 962}$$

OVERGENERALIZATIONS

Where is $\frac{3}{4}$ located on the number line?



Extending
information to
another context in
an inappropriate
way

Students sometimes have difficulty perceiving the unit on a number line diagram. When locating a fraction on a number line diagram, they might use the unit for the entire portion that is shown on the diagram. (CCSS Writing Team, 2011b, p. 3)

PARTIAL CONCEPTIONS

Using some correct and some incorrect ideas. This may result from difficulty generalizing or connecting concepts or distinguishing between two concepts.



27, 415

Rounding to the unit represented by the leftmost place is typically the sort of estimate that is easiest for students. Rounding to the unit represented by the place in the middle of a number may be more difficult for students (the surrounding digits are sometimes distracting).

(CCSS Writing Team, 2011a, p. 11)

CONCEPTUAL MISUNDERSTANDINGS

Content that students “learn” in school but have misinterpreted and internalized, which often goes unnoticed by the teacher. Students often make their own meaning out of what is taught.

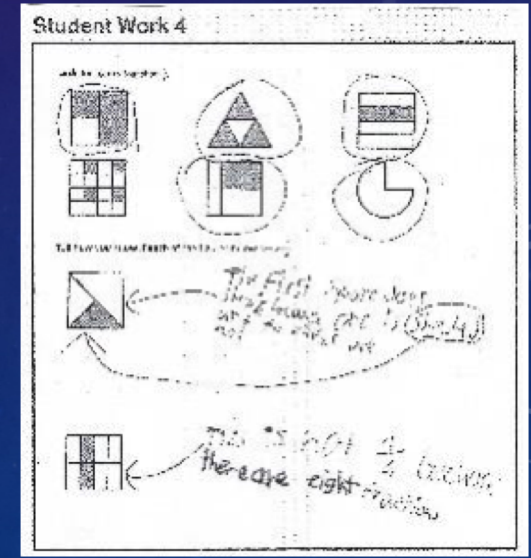
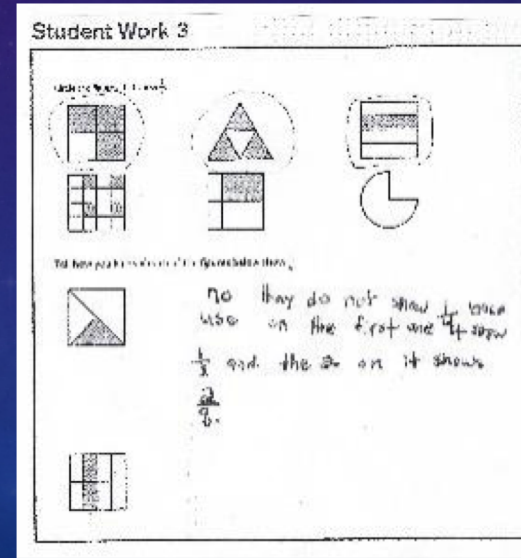
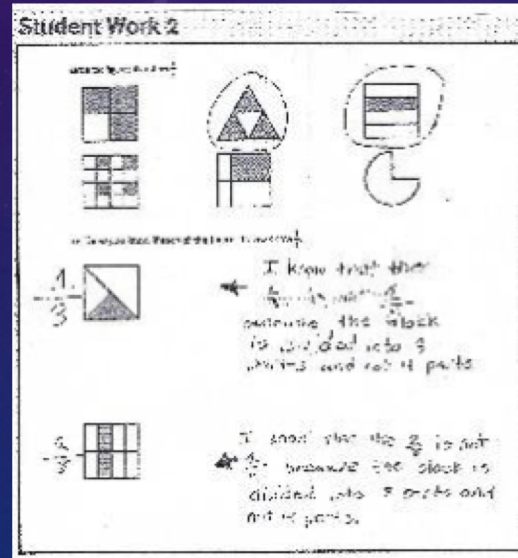
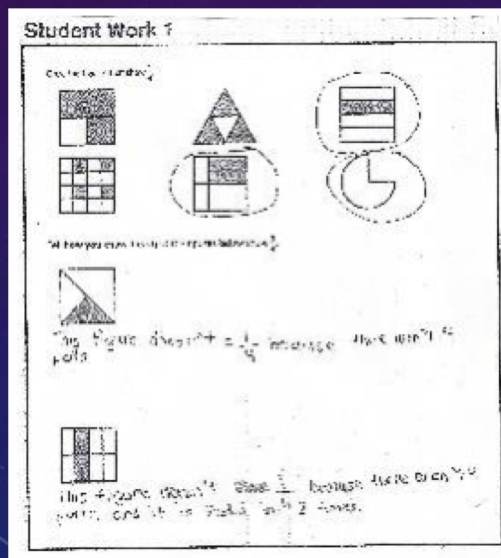
$$4 + 3 = 7$$

Students who only see equations written in one way often misunderstand the meaning of the equal sign and think that the “answer” always needs to be to the right of the equal sign. (CCSS Writing Team, 2011c, p. 10)



Rather than just scoring papers, we need to examine each student's paper diagnostically—looking for patterns, hypothesizing possible causes, and verifying our ideas. As we learn about each student, we will find that a student's paper is sometimes a problem or puzzle to be solved.
(Ashlock, 2010)

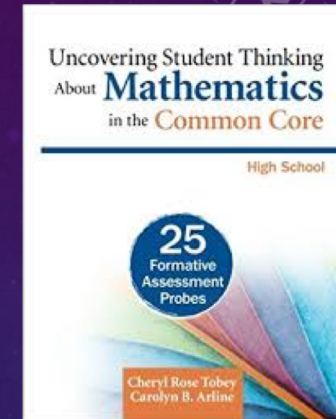
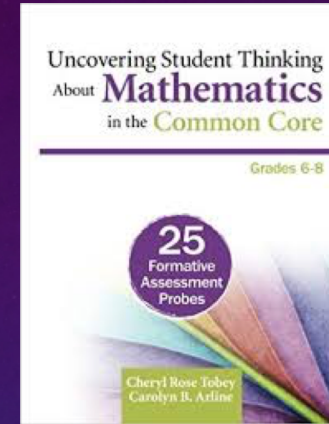
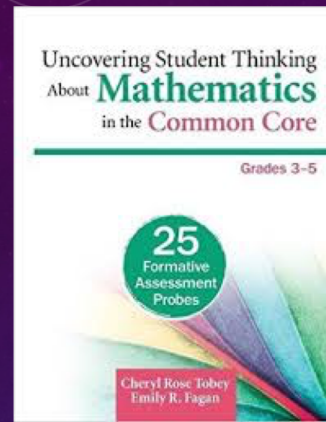
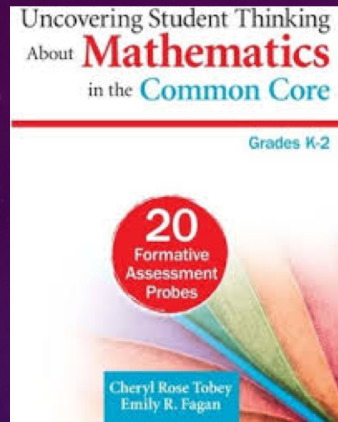
1. What can we learn about what these students know and can do mathematically?
2. What might we ask these students?
3. What might we do next?



Rubric 12: Providing Feedback to Guide Learning

What type of feedback does the candidate provide to focus students?

Level 1	Level 2	Level 3	Level 4	Level 5
<p>Feedback is unrelated to the learning objectives OR is developmentally inappropriate.</p> <p>OR</p> <p>Feedback contains significant content inaccuracies.</p> <p>OR</p> <p>No feedback is provided to one or more focus students.</p>	<p>Feedback is general and addresses needs AND/OR strengths related to the learning objectives.</p>	<p>Feedback is specific and addresses either needs OR strengths related to the learning objectives.</p>	<p>Feedback is specific and addresses both strengths AND needs related to the learning objectives.</p>	<p>Level 4 plus: Feedback for one or more focus students</p> <ul style="list-style-type: none">• provides a strategy to address an individual learning need OR• makes connections to prior learning or experience to improve learning.

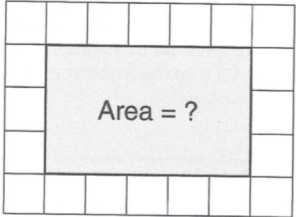


Our job as educators is to minimize the chances of students harboring misconceptions by knowing the potential difficulties students are likely to encounter, using assessments to elicit misconceptions and implementing instruction designed to build new and accurate mathematical ideas. (Rose Tobey & Fagan, 2013)

3.MD.C.7

Do students understand area as a two-dimensional measure of the number of square units that fill a space without gaps or overlaps?

Four students are working together to find the area of the shaded rectangle.



Area = ?

Aisha: I think the area is 35 square units.

Juanita: I think the area is 15 square units.

Karl: I think the area is 16 square units.

Samson: I think the area is 20 square units.

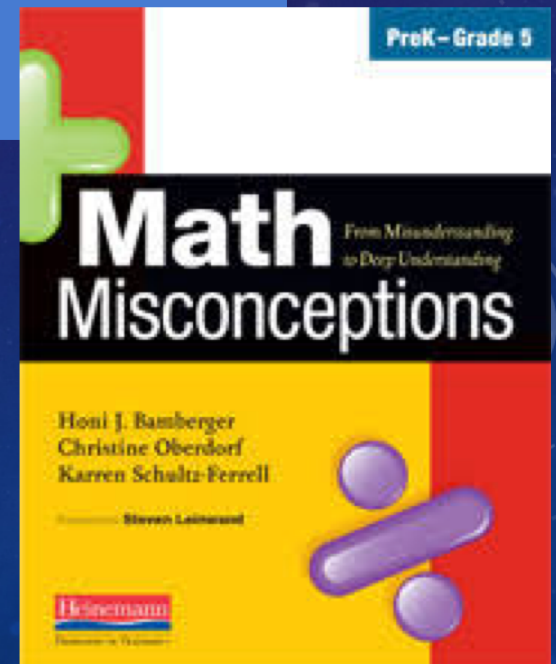
Aisha Juanita Karl Samson

Circle one:	Explain your choice. Include in your explanation why you disagree with the other students.
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Of course it is essential that teachers recognize and anticipate misconceptions and even understand the research findings that help to explain these misunderstandings, but it is the instructional tasks, the ongoing classroom discourse, and the embedded formative assessment—all components of good instruction and the activities that comprise the “Ideas for Instruction”—that make the real difference in student learning of mathematics. (Bamberger, Oberdorf, and Schultz-Ferrell, 2010)

Misconception

Applying whole number concepts to decimal fractions. For example, students ordering decimals by the number of digits rather than the value. Students may also align digits rather than decimal points when adding and subtracting decimals.





Thank you!

Jennifer R. Meadows, Ph.D.
Tennessee Tech University
jrmeadows@tntech.edu

