NCTM ANNUAL MEETING & EXPOSITION 2019

April 3-6 | San Diego

Empowering the Mathematics Community

Mathematical Structure: what is it, why is it so important, and how do you teach it?

Amy Lucenta

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FOSTERING MATH PRACTICES

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@GraceKelemanik
#NCTMSD2019
#FosteringMPs

Goals

- Understand structural thinking as the ‘glue’ of mathematics
- Develop a structural thinking lens for teaching
- Explore how structural thinking supports students
- Identify instructional practices that support all students’ --including students with learning disabilities and English language learners-- development of structural thinking

How we will spend our time..

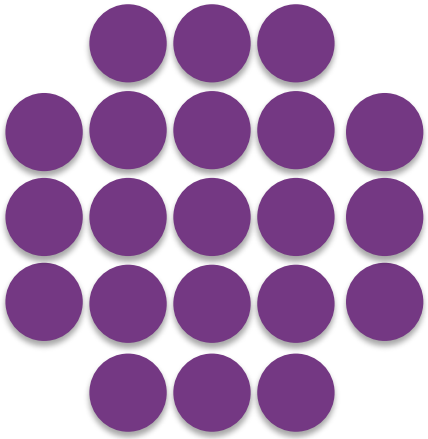
- Build common understanding of and language for structural thinking
- Consider trajectories of structural thinking
- Experience and reflect on an instructional routine that fosters structural thinking

How might you make connections between or among these 4 tasks?

Evaluate: 103 – 97

Solve for n

$n^2 + 4n = (n+2)^2 - 4$



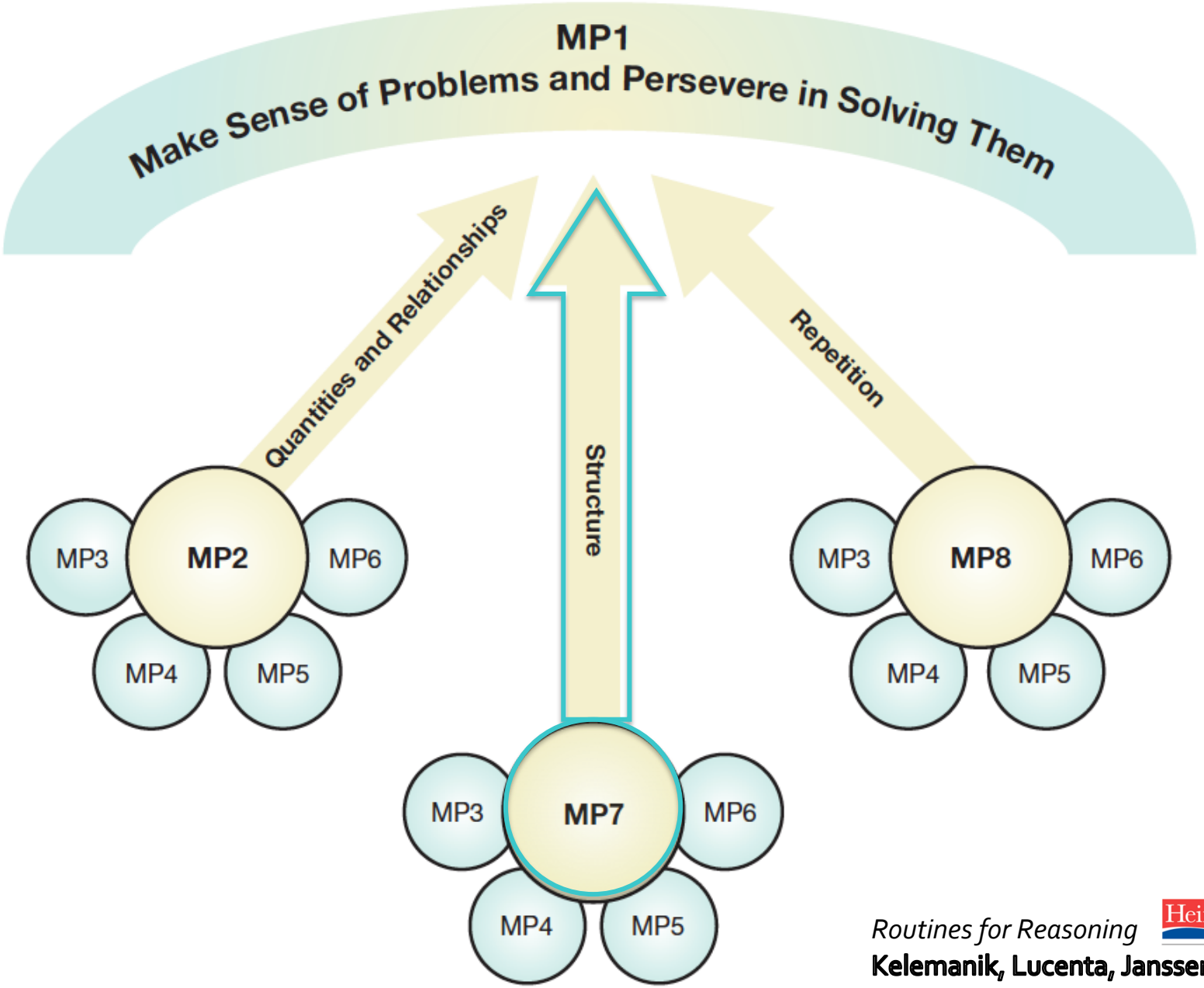
How many dots?

Solve for *x*

$|100 - x| = 3$

We are currently preparing students for jobs that don't yet exist . . . using technologies that haven't yet been invented . . . in order to solve problems we don't even know are problems yet.

—*The Jobs Revolution* Richard Riley



Routines for Reasoning Heinemann
Kelemanik, Lucenta, Janssen Creighton

Structural Thinking (MP7)

Attend to…

Organization & Properties of Number & Space

Type and Composition of Mathematical Objects

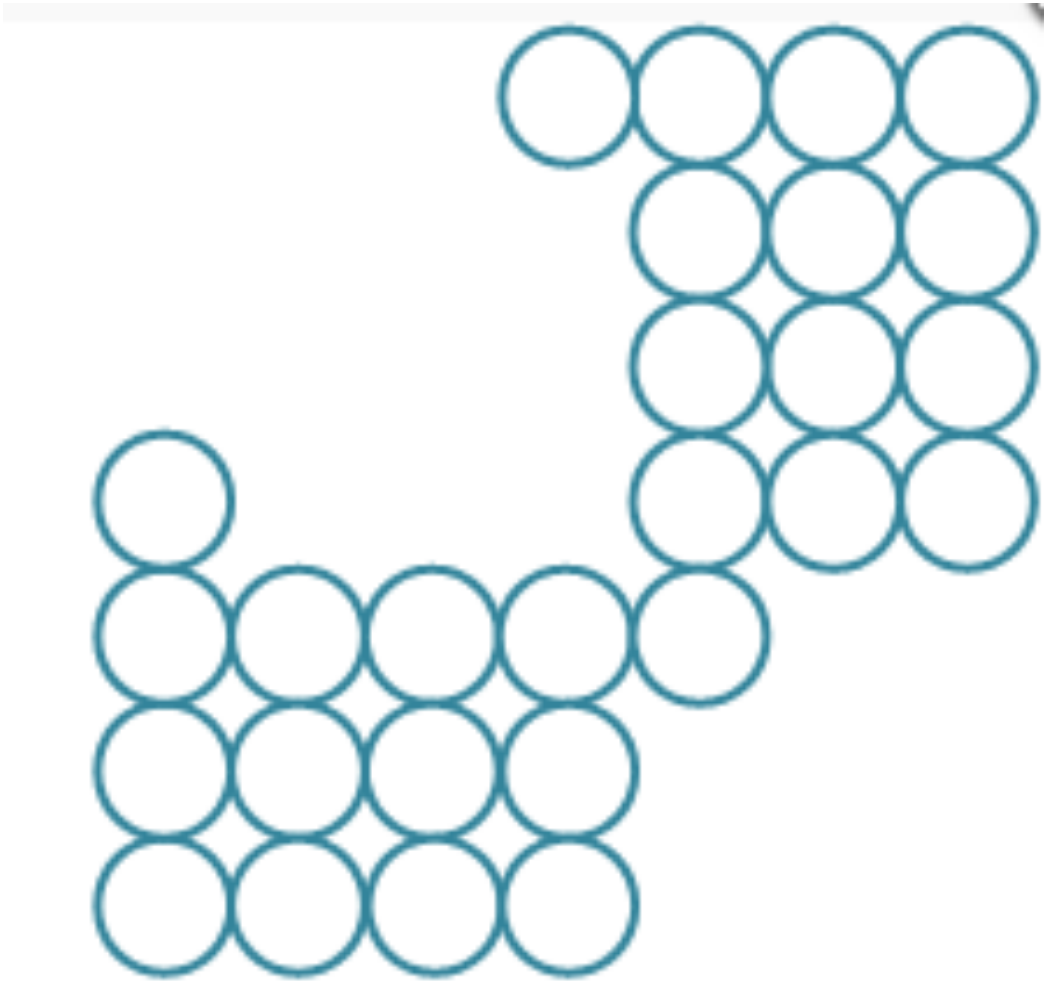
Ask Yourself…

- Is there another way I can think about 15% off? Can I change the form of it?
- Do 16 and 25 have something in common?
- Can I chunk this visual into parts that are easier to work with?

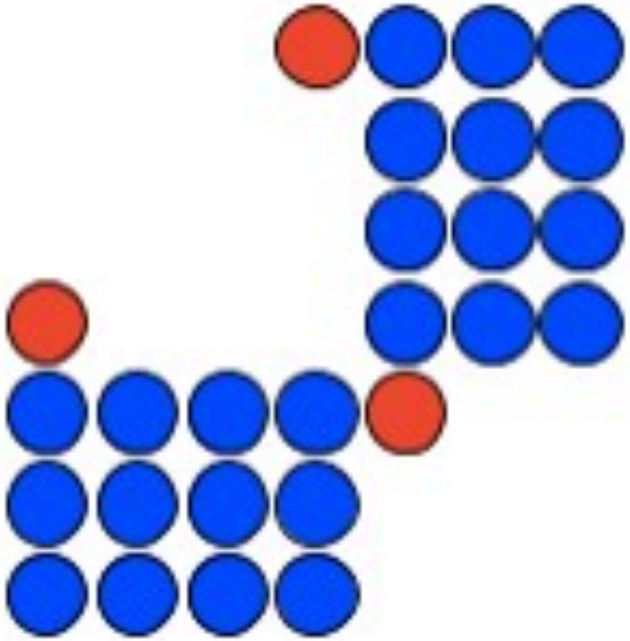
Structural Thinking Actions

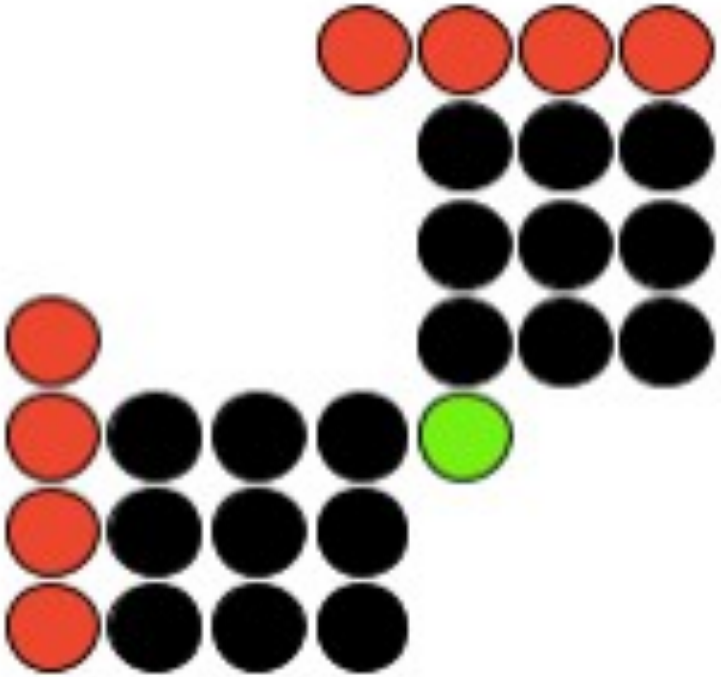
- *Chunk* complicated objects
- *Connect* math ideas and representations
- *Change* the form of objects
- Recall and use properties, rules of operations, and geometric relationships

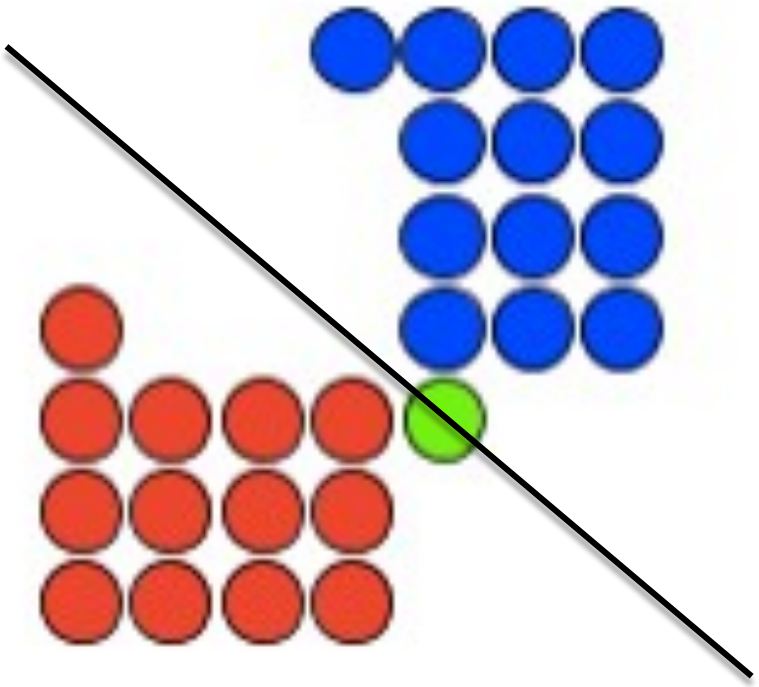
Find the total number of circles quickly "in your head" (i.e. without counting each circle individually)

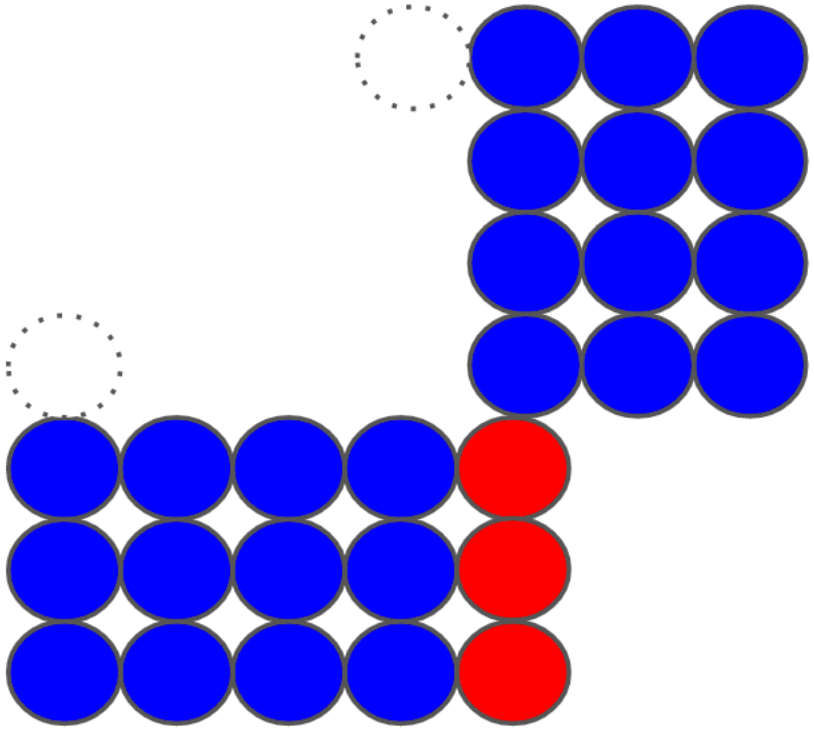


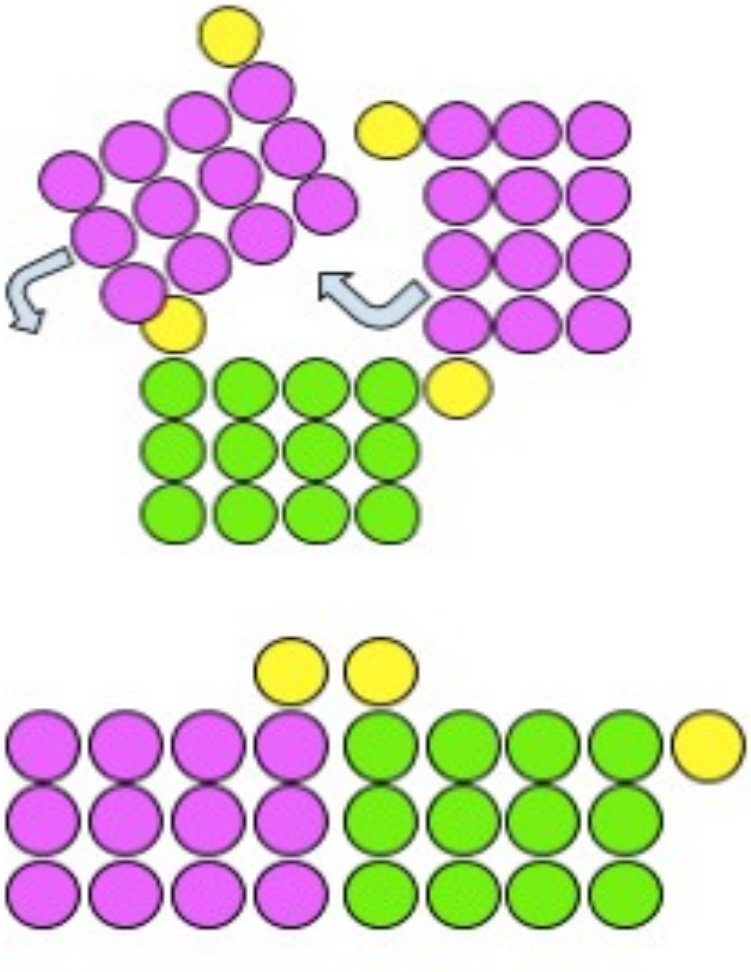
http://www.visualpatterns.org/

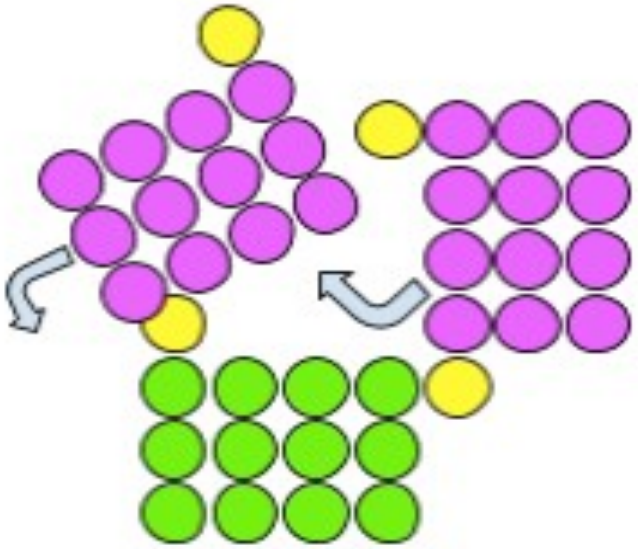


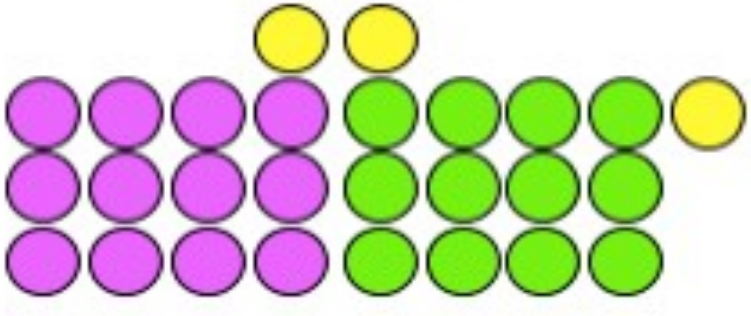


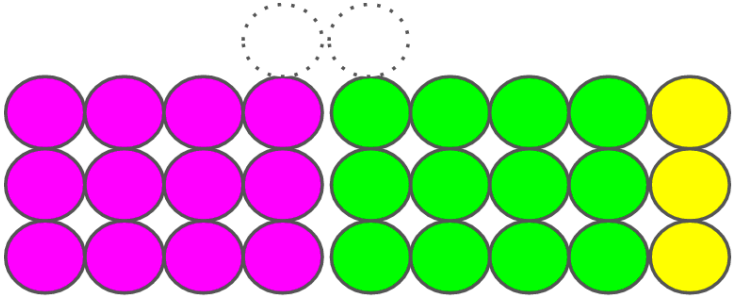












Consider the structure in this equation,

then solve for *x* by *chunking, changing the form, & connecting mathematical ideas*

$$5(x + 3.5) - 3(\frac{7}{2} + x) + 1 = 17$$

Structural Thinking Shifts

A collection of unrelated results and procedures to know

A set of interconnected ideas that build on each other and make sense

Structural Thinking supports ALL students….especially

- Students who lose track of their work and/or calculations
- Students who see the ‘big picture’
- Students who benefit from multiple representations

An example of a structural trajectory

A little ditty

Think like a 2nd grader…

Add 47 and 12

Think like a 4^{th} grader…

Add 4.75 and 12

How might a 5th grader approach

$\frac{1}{4} + \frac{1}{2}$

Add same to same, like units to like units, like terms to like terms.

When students learn through structure, addition works the same way every year.

In Algebraic Expressions, addition continues to behave the same way.

$3x + 7x$

$3(x+2) + 7(x+2)$

Even when considering trigonometric functions…

$3\sin^2 x + 7\sin^2 x$

As students think structurally, a major shift will take place…

Instead of asking,
"What's the topic for the next lesson/unit/grade?"

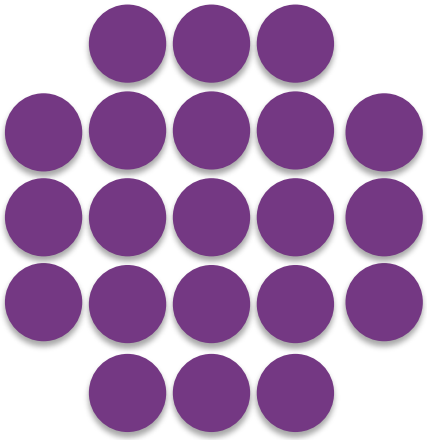
Students will ask themselves,
"How can this math be extended and applied in a new domain?" and
"How is this new content related to what I've learned in the past?"

Pedagogical Content Mindset necessary to teach structural thinking

- See structure in mathematical objects
- View content as a smaller slice of a larger trajectory
- Look for the glue

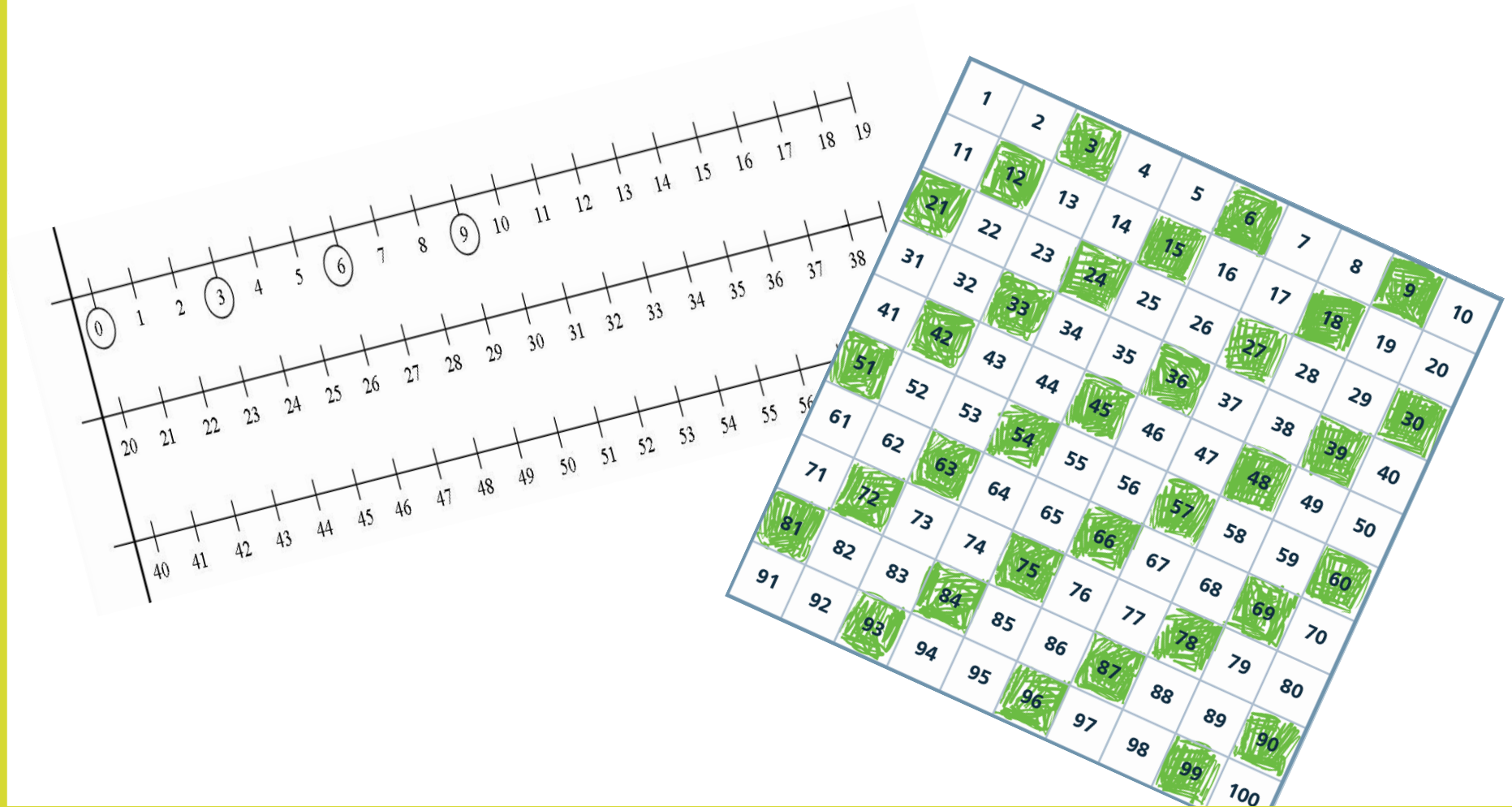
Task Analysis

In what grade might this task sit?
What content might you teach with this task?

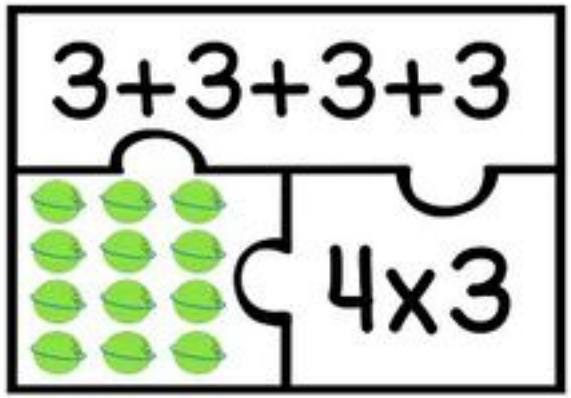


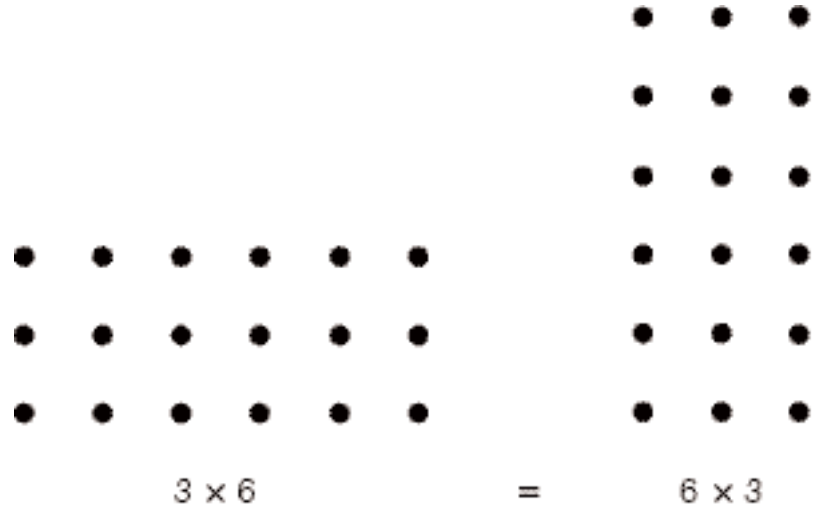
How many dots?

Subitizing to skip counting

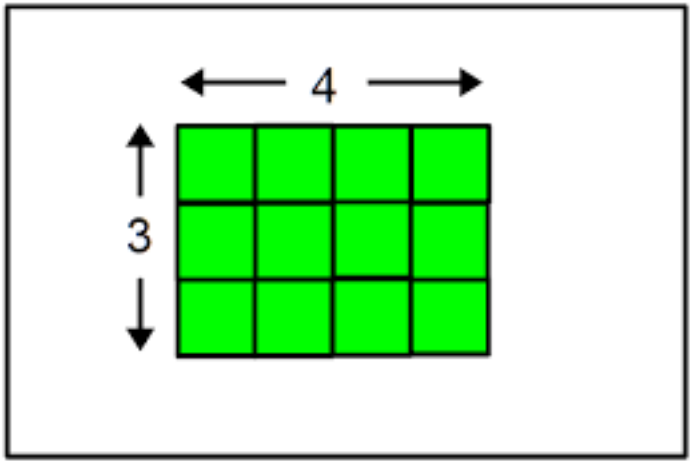


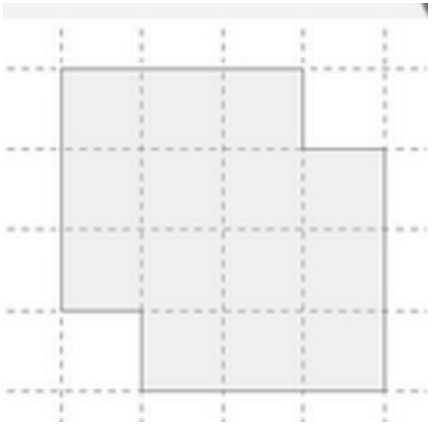
Skip Counting to Arrays & Multiplication



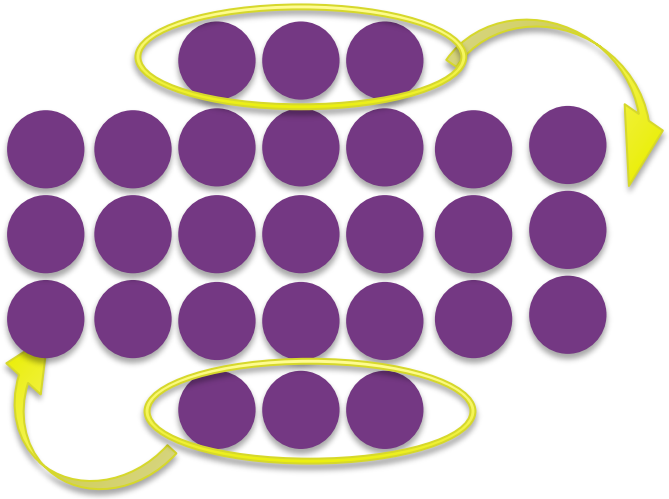


Arrays to Area and Composite Area

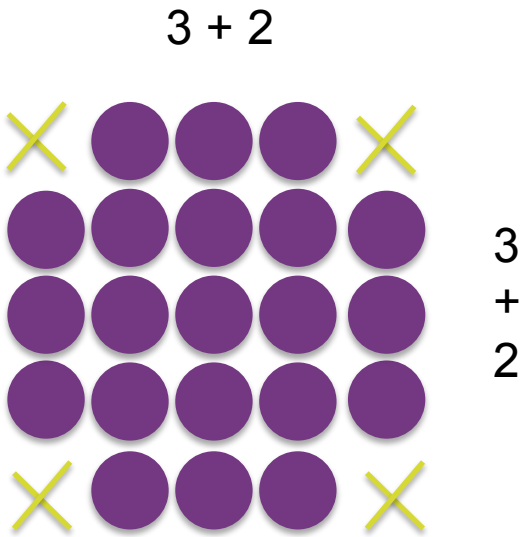




Arrays & Transformations



An ‘Almost’ Square



$$(3 + 2)^2 - 4$$

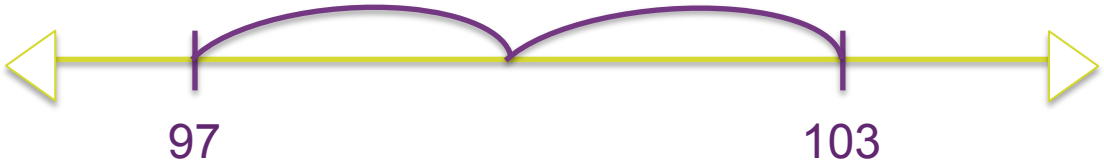
$$n^2 + 4n = (n+2)^2 - 4$$

Task Analysis

In what grade might this task sit?
What content might you teach with this task?

103 – 97

103 – 97



Subtraction

- Constant difference
- Compensation
- Using Friendly Numbers (100)

Symmetry on the number line

Properties

(100 + 3) - (100 – 3)

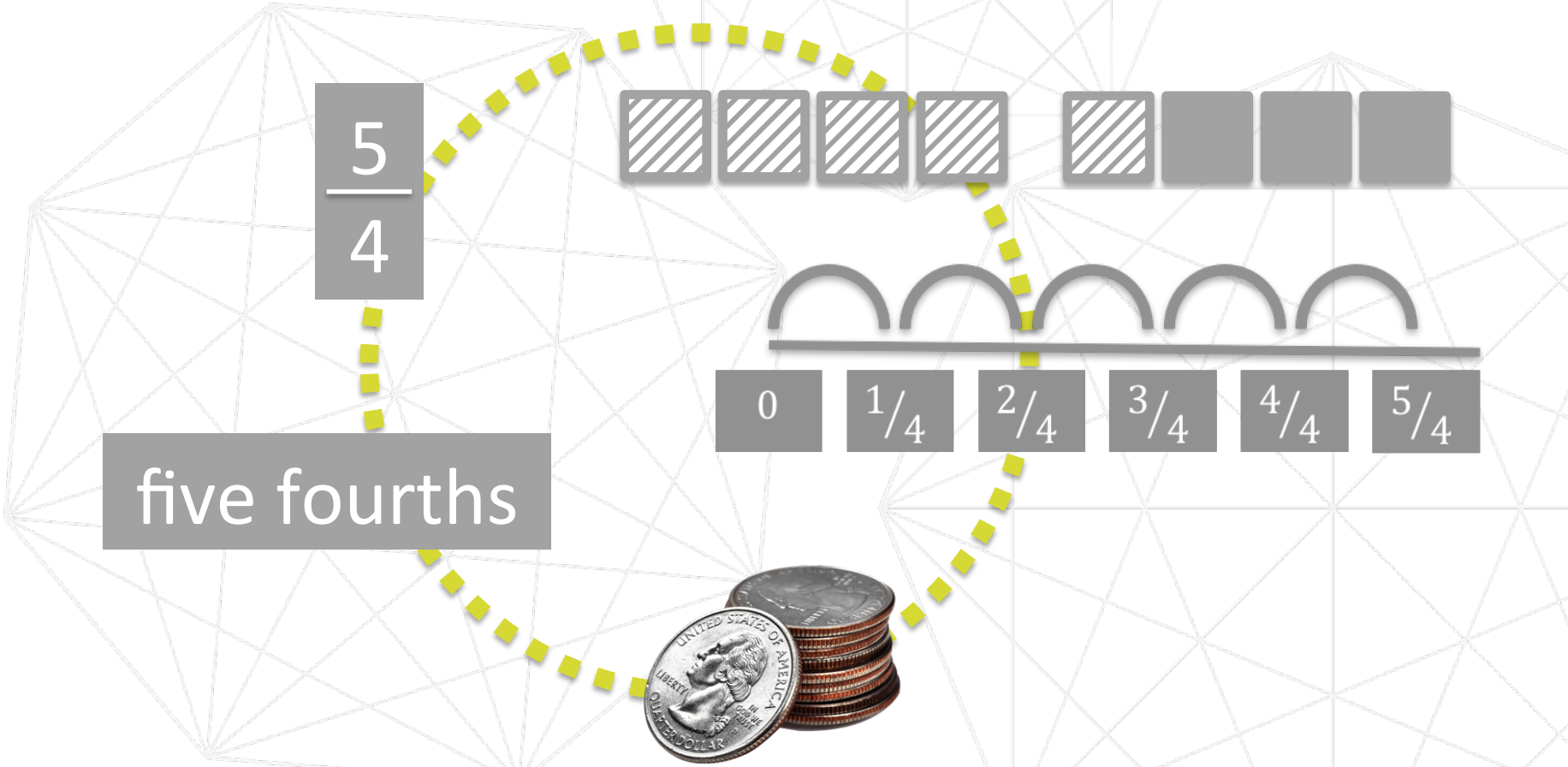
Absolute value

$|100 - x| = 3$

Representations:
A Cornerstone of Mathematical Structure

Write a number or expression that is represented by this visual

Connecting Representations: A Hallmark of Structural Thinking



Connecting Representations: A Hallmark of Structural Thinking

Why else should we focus on structural thinking?

Why focus on structural thinking?

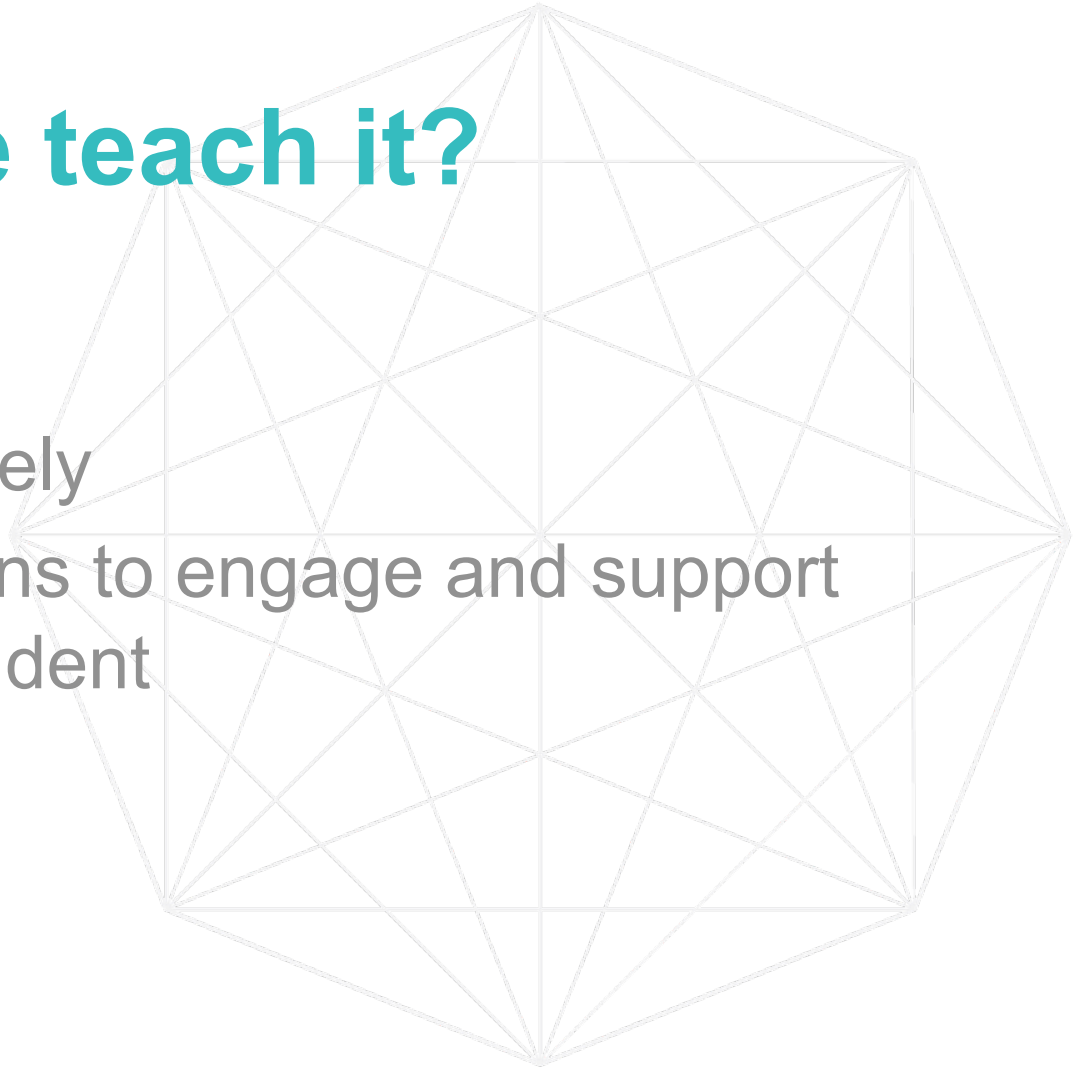
- Apply and extend…
  - …previous understandings of multiplication and division to divide fractions by fractions.
  - …previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
  - …previous understandings of arithmetic to algebraic expressions.

Mathematical ‘glue’ fosters a growth mindset

- Helps students see the interconnectedness of mathematics
- Helps students understand that mathematics makes sense
- …and therefore mathematics is doable with effort!

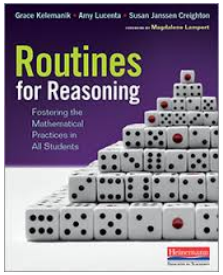
So how do we teach it?

- Explicitly
- Regularly & Routinely
- With specific designs to engage and support each and every student



Connecting Representations

An Instructional Routine to Develop
ALL Students' Structural Thinking



Routines for Reasoning
Kelemanik, Lucenta, & Creighton

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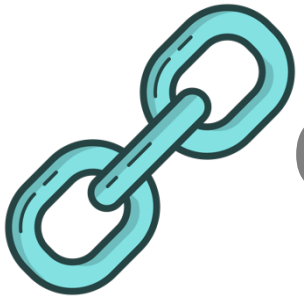


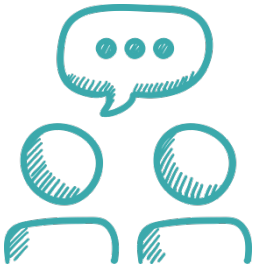
Connecting Representations

WHAT: Match visuals to expressions by **chunking** and **connecting** to math you know

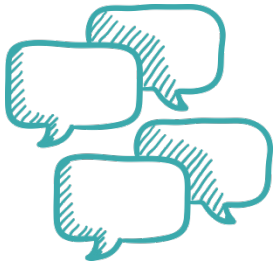
WHY: To "think like mathematicians", to use mathematical *structure* to match two different representations.

Connecting Representations

Think



Make connections



Share & study connections

Create representation

Reflect on learning

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Think

Ask yourself…

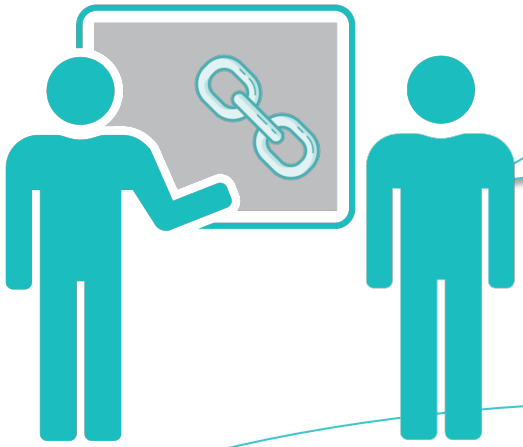
- What part of the *visual* will help me connect to a chunk of the *expression*?
- What about the *expressions* will help me connect to a *visual*?

Make Connections

“I noticed… so I looked for...”

“I saw... so I knew...”

Share & Study Connections



We noticed… so we ...
We knew... so we...

They noticed… so they ...
They knew... so they...

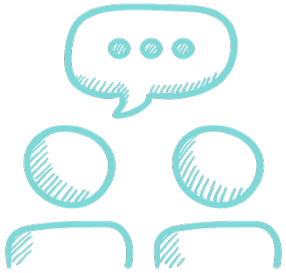
Create a Representation

THINK

Ask yourself…

- "What do you notice about this *expression*?"
- "How can you chunk this *expression* into pieces you can represent?"

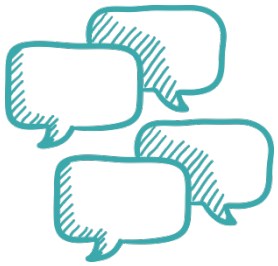
Create a Representation



Pair

- Share your interpretations of the *expression*.
- Together create a matching *visual*.

Create a Representation



Share

They noticed… so they…

When they saw…it made them think of… so they…

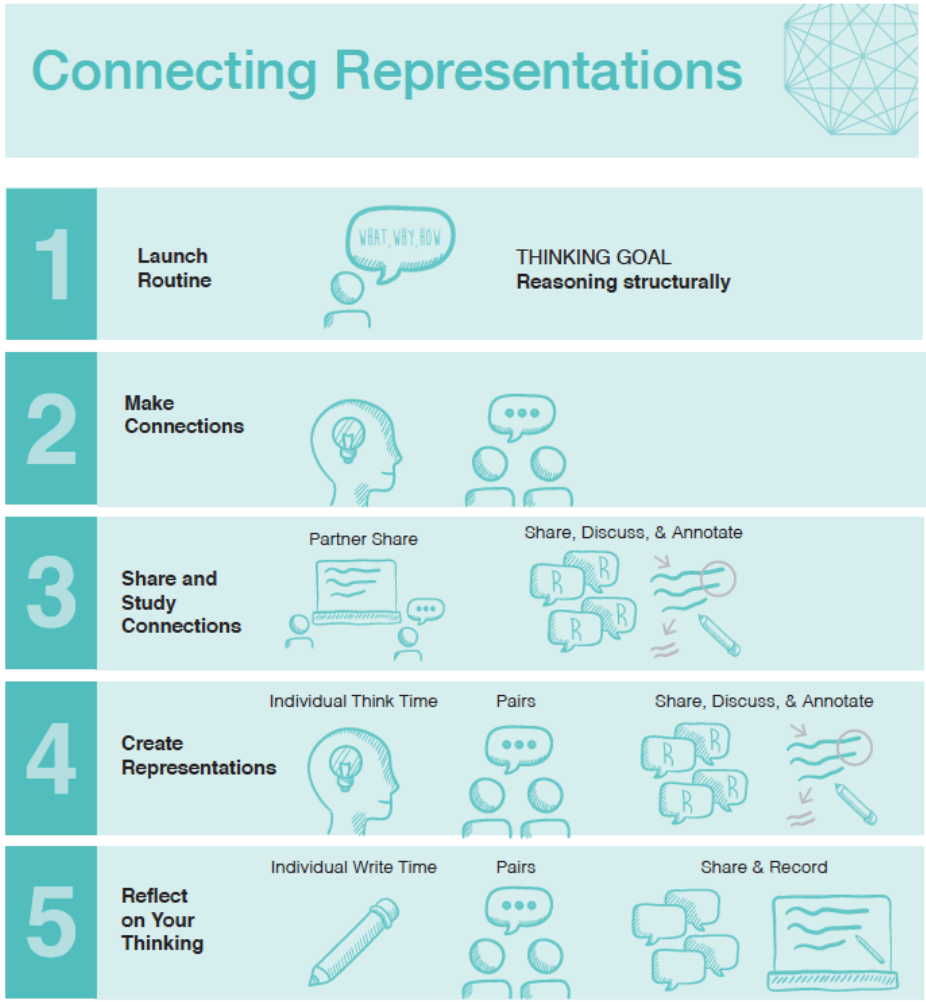
Meta-Reflection

A. When interpreting an *expression/visual*, I learned to pay attention to…

B. When connecting representations, I learned to ask myself…

C. A new mathematical connection I made is …

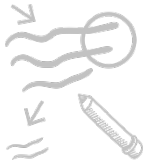
Reflect on CR Instructional Routine



How does/can the Connecting Representations instructional routine provide access and support for SWLDs and ELLs to develop the math practices?

4 Essential Instructional Strategies that keep the focus on mathematical thinking while providing access for ALL learners

Ask-yourself questions



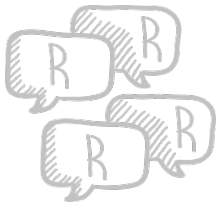
Annotation

I noticed…so I knew...

I saw...so I looked for...

... Connects to ... because

Sentence frames and starters



The Four Rs – repeat, rephrase, reword, record

How does Connecting Representations Support Special Populations?

- It's predictable!
- Combats learned helplessness by inviting multiple sensing modalities and habitualizing "ask yourself" questions
- Articulates features of structure so that they are not "magic" or disconnected
- Places a premium on processing time and modalities
- Provides opportunities to develop and practice language

Connecting Representations: A Hallmark of Structural Thinking

3 Numeric Expressions

2 Visuals

3(10+2)

30+2

3(10+6)

?

3 Verbal Descriptions

2 Visuals

Three groups of
two more than a number

Two more than three
groups of a number

Three groups of six
more than a number

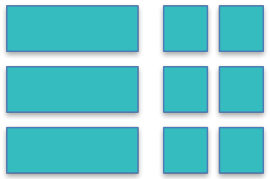
?

Numeric Expressions	Verbal Descriptions
$3(10+2)$	Three groups of two more than a number
$30+2$	Two more than three groups of a number
$3(10+6)$	?

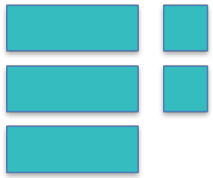
Connecting Representations: A Hallmark of Structural Thinking

3 Algebraic Expressions 2 Visuals

3(x+2)



3x+2



3(x+6)

?

Algebraic Expressions Verbal Descriptions

3(x+2) Three groups of two more than a number

3x+2 Two more than three groups of a number

3(x+6) ?

3 Verbal Descriptions 2 Visuals

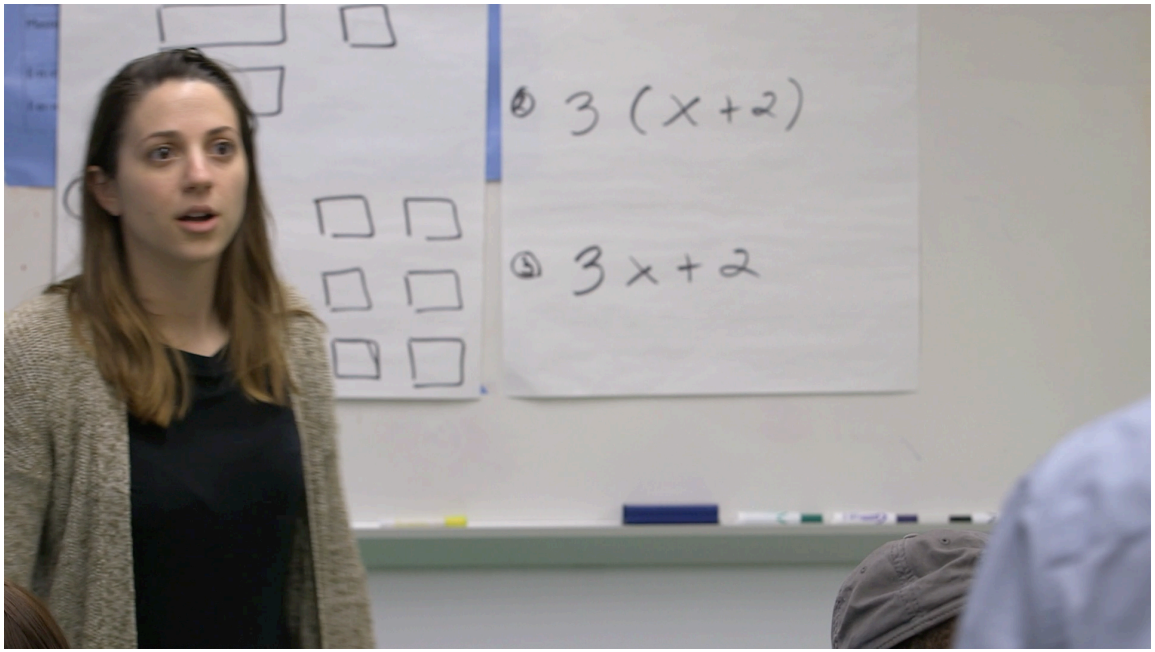
Three groups of two more than x

Two more than three groups of x

Three groups of six more than x

?

In action…Listen for MP7



Harris Mintz, Maria Ambroselli

Mather H.S., NYC

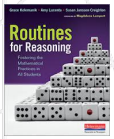
New Visions for Public Schools

This material is based upon work supported by the National Science Foundation under Grant No. 1238157

The 5th Essential Strategy: Make it Routine!

Where can I find more about…

- Instructional Routines?
  - Routines for Reasoning
  - TEDD.org
  - Fosteringmathpractices.com



- Tasks for Connecting Representations
  - Fosteringmathpractices.com
  - #ConnectingReps
  - YOUR OWN CURRICULUM

www.fosteringmathpractices.com

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Avenues of Thinking | Special Populations | Routines for Reasoning | Related Re

Free Resources (Site Registration/Login Required)

Go to Downloads

View Tasks

Classroom Planner

Classroom PPTX Template

Tasks & Discussion

Access and equitable practices for English learners and SWLDs don't happen by chance.

Routines for Reasoning bake in explicit supports for learners to develop critical mathematical thinking & reasoning…and make them routine

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INSPIRED BY A PUBLIC SCHOOL STUDENT WITH DISABILITIES

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Join the Conversation

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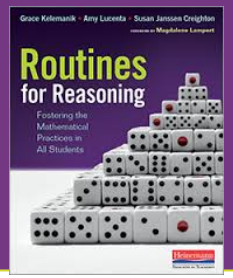
Read the Book

www.heinemann.com



See you tomorrow….

- Friday 8 am: Leveraging the Predictable Design of Instructional Routines to Elicit and Use Student Thinking
  Hilton Indigo BF
- Friday 8 am: Teaching in the Moment: Making Decisions that Elicit and Use Student Thinking Hilton Aqua 310
- Friday 9:30: Interpret and Communicate Math Ideas with Increasing Precision through the Decide and Defend Routine. Hilton Indigo BF
- Friday 10:30 -10:50 Representing Student Thinking, Networking Lounge
- Friday 6 pm IGNITE Our Greatest Challenge

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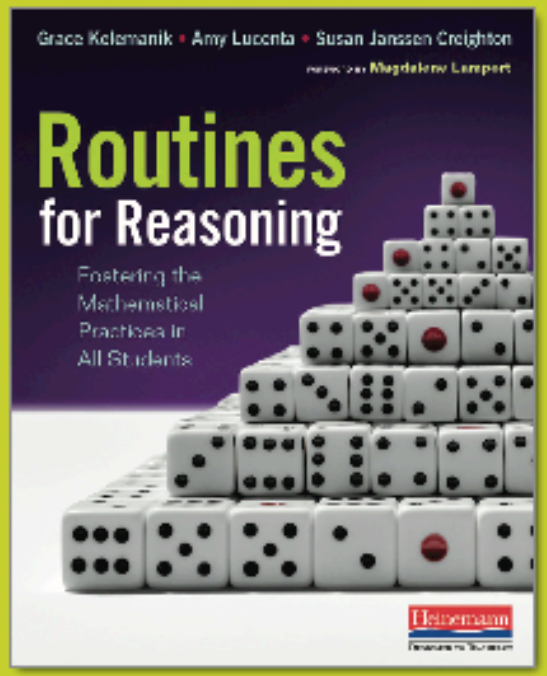

Courses at EDCO Collaborative

36 Middlesex Turnpike, Bedford, MA

Routines for Reasoning: Fostering Structural Thinking in ALL Students

June 24 - 27, 2019 • 8:30 AM to 3:00 PM

$599 before May 1st • $649 after May 1st

Register at:
https://edco.rocks/routines

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4 Essential Strategies for Teaching Students with Learning Disabilities to Think Mathematically

August 6-7, 2019 • 8:30 AM to 3:00 PM

$299 before May 1st • $349 after May 1st

Register at:
https://edco.rocks/strategies