# Math Revolution Needed in Geometry NCTM 2019

Jenny K. Tsankova, Ed.D., jtsankova@rwu.edu
Roger Williams University, RI
Jim Matthews, Siena College, Matthews@siena.edu
Madge Thombs, Ph.D., mthombs@rwu.edu
Roger Williams University

## Geometry – The Forgotten Strand

Glenda Lappan (President's Message, 1999)

- What do students think about Geometry?
- What do teachers think about Geometry?
- Catalyzing change in HS Math: Initiating Critical Conversations (NCTM, 2018)
  - Why do we teach Geometry?
  - When do we teach Geometry?
  - How do we teach Geometry?

## Key Ideas

- Geometry – The Forgotten Strand?
- The Why, the When, and the How
- The CCSS and Geometry
- The Nature of Proof
- Double-Column Proof
- Vocabulary
- Constructions
- Traditional topics in context
- Summary and First steps

## *Catalyzing Change in High School Mathematics: Initiating Critical Conversations, NCTM 2018*

One of the Key Ideas: Arguments, Reasoning and Proof:

1. To prove is a process:
   a. To Discover
   b. To Communicate
   c. To Certify a proof (Dimmel & Herbst, JRME, May 2018)
2. Variety of formats for writing proofs
3. Include proofs using coordinates and transformations.

Who is putting all the Geometry books in the Horror section?

## The Idea of Proof

**A Definition of a Proof:** An argument which convinces people that something is true. A good proof also helps people understand why something is true.

Proof statements are based on:

- Definitions
- Axioms (Postulates)
- Theorems (Combined Axioms and more complicated statements)

**AND**

Once we prove a theorem, we agree to use it from that point on as a fact, as a stepping stone for other proofs.

# Types of Proofs

- Direct
  - Using prior knowledge and previously proven theorems
  - Using Transformations
  - By Construction (including drawing auxiliary lines)
- Indirect (by contradiction; by assumption)
- By induction

## What details do teachers expect from students proofs? A study of proof checking in geometry (Dimmel & Herbst, JRME, May 2018)

- 44 HS teachers viewed and rated instructional practice

**Results**

- Students are not expected to draw or modify a diagram;
- Writing a double-column proof was considered as the activity of learning to do mathematical proof
- Teachers spend a "tremendous amount of time" teaching students how to write a proof and check proofs "line by line" to make sure it is acceptable, focusing mainly on students identifying correctly reasons
- Teachers insisted on the first line being the given information
- Teachers had different expectations for details when checking double-column proof

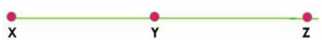
**Example:** A problem stated that two angles (drawn) were a linear pair. The teachers insisted the students should provide:

a) a step for given information,
b) a step to define what linear pair is (prior knowledge),
c) and a step to restate that the angles given constitute a linear pair.

# The Struggle to Prove

**Consider the following task and the solutions:**

Given: Y is the midpoint of XZ
Prove: XZ = 2XY




**Solution A:**

| | |
|---|---|
| Y is the midpoint XZ | Given |
| XY = YZ | Definition of midpoint |
| XY + YZ = XZ | Segment addition postulate |
| XY + XY = XZ | Substitution property |
| 2XY = XZ | Simplify (like terms) |
| XZ = 2XY | Symmetric property |
| XZ = 2XY | QED |

**Solution B:**

There is nothing to prove, Y is the midpoint.

## Why Do We Teach Double-Column Proof?

- Helps with Organization
- Preparing for College
- Needed for SAT
- Infuses Rigor

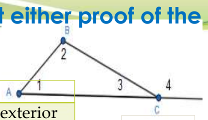
# Questions to consider:

- What is the big idea involved in this problem?
- Why is a double-column proof required for this problem?
- What are the students discovering, communicating, certifying?
- Where is the rigor?

# What else can students do to show us they understand the problem?

- Write a narrative; bullet list
- Explain in your own words
- The question is – what are the students learning? What do I need to see as a teacher as evidence that a student understands?

**Example:** **Would you accept either proof of the Exterior Angle Theorem?**




**A**

| Statements | Reasons |
|---|---|
| **1.** ∠4 and ∠3 are supplementary angles. | 1. Definition of exterior angles |
| **2.** m∠4+m∠3 = 180° | 2. Definition of supplementary angles |
| **3.** m∠1+m∠2 +m∠3 = 180° | 3. Triangle Sum Theorem |
| **4.** m∠4+m∠3 = m∠1+m∠2 +m∠3 | 4. Substitution Property of Equality |
| **5.** m∠4 = m∠1+m∠2 | 5. Subtraction Property of Equality |

**B**

a) ∠1+ ∠2+ ∠3 = 180°

From line $\overleftrightarrow{AC}$ we have
b) ∠3 + ∠4= 180°

a – b => ∠1+ ∠2 - ∠4=0
∠1 + ∠2 = ∠4

## Question:

How many different angle measures are there when we have two parallel lines cut by a transversal?

**Answer: Two**

## Now we focus on rigor!

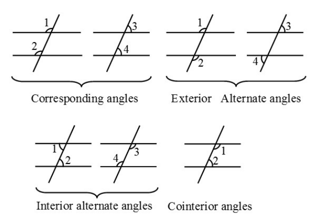
**Prior knowledge: Isosceles Triangles Theorem; 360° around a point**

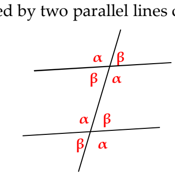
**Example:** You are given equilateral triangle ABC. On sides BC and AC, outside of triangle ABC, are constructed two squares, BDEC and ACFG. Find the measures of the angles of triangle EFB.

**Note: Do not provide the picture, let students draw it.**

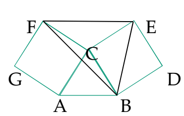
***Focus on ideas, vocabulary will come!***

**Example:** Congruent angle pairs formed by two parallel lines cut by a transversal.




**Supplementary; (Vertical)**




1. Around p. C is 360°. 360 = 90°(square + 90° (square) + 60° (given equilateral) + 120° (∠FCE).
2. FC = CE (squares with same side), => ΔFCE is isosceles, => ∠EFC = ∠FEC = (180 – 120)/ 2= 30°
3. Similarly, ΔFCP is isosceles, ∠CFB = ∠FBC = {180 – (90 + 60)}/ 2= 15°
4. Similarly, ΔECB is isosceles, ∠CEB = ∠EBC = (180 – 90)/ 2= 45°

**Answer: 45°, 60°, 75°**

## Traditional but Making Sense

- Why do we teach how to construct a perpendicular bisector?
- Do the kids understand the steps?

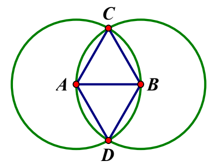
# Suggested Sequence

**Prior Knowledge:** Properties of quadrilaterals including diagonals

**Vesica Piscis**

The Vesica Piscis is a symbol made from two circles of the same radius, intersecting in such a way that the center of each circle lies on the circumference of the other circle. The term "vesica piscis" is first recorded in literature in 1809, but is no doubt much older. The almond-shaped center of the image is called a mandorla (Latin for almond). The mandorla can easily be seen as a grail or chalice, or womb.

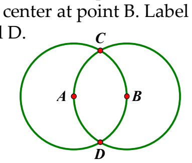
2. What can you tell about quadrilateral ACBD. Prove your hypothesis.




# Construct Vesica Piscis

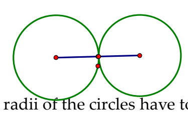
**Note: A circle is defined by a center and a radius!**

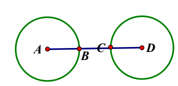
Draw circle with radius 2in and center A. Make a point B on the circle. Construct a circle with radius 2in and center at point B. Label the intersection points of the circles C and D.




# Questions:

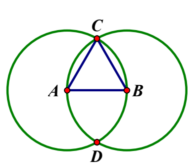
- What do we know about the rhombus? What about its diagonals?
  - Quadrilateral, parallelogram, 4 congruent sides, bisecting, perpendicular diagonals
- So, if I have a segment and I would like to bisect it with a perpendicular, what would be one way to do it?
  - Construct circles with same radii with centers the endpoints of the segment.





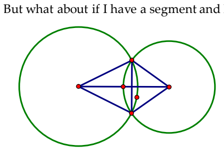

  - The radii of the circles have to be a little longer than halfway the length of the segment

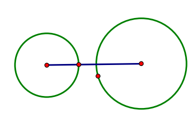
1. Draw segments AB, AC, and CB. What kind of a triangle do you think this is? Prove your observation.

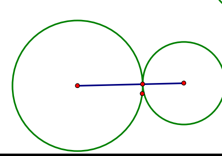



Questions continued:

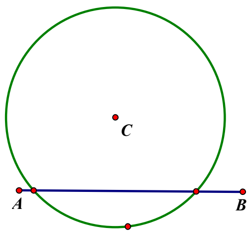
But what about if I have a segment and I just want a perpendicular not necessarily a bisector?





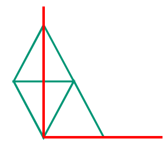


Lastly, What if I have a segment and point not on the segment. I would like to construct a line, through the point, perpendicular to the segment. How can I do that?




**Exploring History**

How Mayans Created right angles, reported by Christopher Powell in a 2011 lecture at MAA.



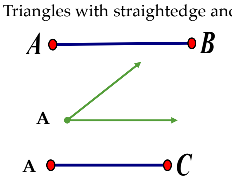
- https://youtu.be/n8G0knuX3mE

# Traditional but Making Sense, Part II

Q: Why do we teach students to copy a segment or copy an angle?

**Task:** Construct Triangles with straightedge and compass by given sides and angles to copy.

**Example:**



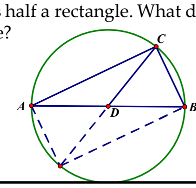

https://www.mathopenref.com/consttrianglesas.html

**Exploring History 2**

**Prior Knowledge:** Properties of diagonals of quadrilaterals; definition of median

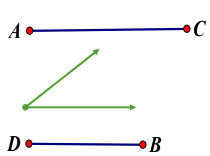
a) Prove that you can always circumscribe (draw around a triangle passing through the 3 vertices) a circle about a right triangle.

**Hint:** A right triangle is half a rectangle. What do we know about the diagonals of a rectangle?




- **Example 2:** Construct a parallelogram by given two diagonals and an angle between them.

Prior knowledge: Properties of quadrilaterals (including diagonals); know how to copy a segment and an angle; know how to construct a perpendicular bisector.




**Summary and First Steps**

- Students learn by solving problems, variety of problems.
- Students should be able to use various proof formats
- Focus on reasoning and not memorizing.
- Accept general justifications and not exact wording of postulates and theorems.
- Have students sketch pictures to understand a problem.
- Teach traditional ideas in context
- Ask, "Why am I teaching this concept? What does it connect to? What is the big picture? Is this problem/ task making students think?"
- Grab an accomplice and create a revolution in your Geometry teaching!