

Differentiating through Menus of Challenging Mathematical Tasks

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“How are you challenging _____ in math?”

“An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.”

NCTM, *Principles to Actions*, p. 11

Why is challenging students so challenging?

- Content
 - Management
- Student Priorities
- TIME

Goals for Mathematics Differentiation

- Students have curriculum-aligned opportunities for students to enrich/deepen/extend their understanding of content.
- Students are provided with problem solving experiences - engaging in a task for which the solution method is not known in advance.
- Students have some opportunity for choice and self-regulation.
- Students have an opportunity to work independently or collaboratively.
- Students have opportunities to develop the Standards for Mathematical Practice.

Math Menus

A math menu is “a collection of activities for students to do [that] may provide classwork for several days, a week, or for a longer period of time.”

Marilyn Burns, 1992, p. 37

Math Menus

- Curriculum-Aligned
- Problem Solving Tasks
- Challenging Puzzles
- Choice: Tasks & Pacing
- Choice: Independent or Collaborative
- Engagement: 8 Standards for Mathematical Practice
- Engagement: Productive Struggle

Not to be confused with...

- Actual Menus
- Varieties of options found on Pinterest and TPT

Math Menus

- Main Course Tasks
 - apply and deepen students' understanding of essential concepts
 - ask students to apply the mathematics they are currently working on in novel ways and/or authentic contexts
- Dessert Tasks
 - extend students' thinking and provide opportunities for them to make important mathematical connections
 - are significantly more challenging

Math Menus - Agenda

- Setting Up
- Selecting and Developing Tasks
 - Digging In
- More Differentiation Opportunities
- Questions

Math Menus - Setting Up

- Construction
 - Student Record Sheet
 - 6 Main Course Tasks
 - 6 Dessert Tasks
- Formatting
 - 1 page per task
 - NOT worksheets
 - NOT packets

Math Menu Grade 6, Comparing Bits and Pieces

Main Courses

	Task	Task Complete	Teacher Initials
A	Up and Down the Ladder	<input type="checkbox"/>	
B	Mystery Fraction	<input type="checkbox"/>	
C	Bellingham Police Force	<input type="checkbox"/>	
D	Equivalent Temperatures		
E	Chocolate Milk		
F	Everybody Dance Now!		

Desserts

	Task
A	Spilling Marbles
B	Fit the Ratio
C	Fraction Fascination
D	Keep It Simple
E	Mixing Paints
F	Royal Wedding

Buying Organic

G6 DOPS EXC A

Isabelle needs to purchase produce at Fred Meyer's. She knows that her family likes to buy organic produce, but she also notices that it is more expensive. How much more will Isabelle spend in the produce section if she buys all organic produce?

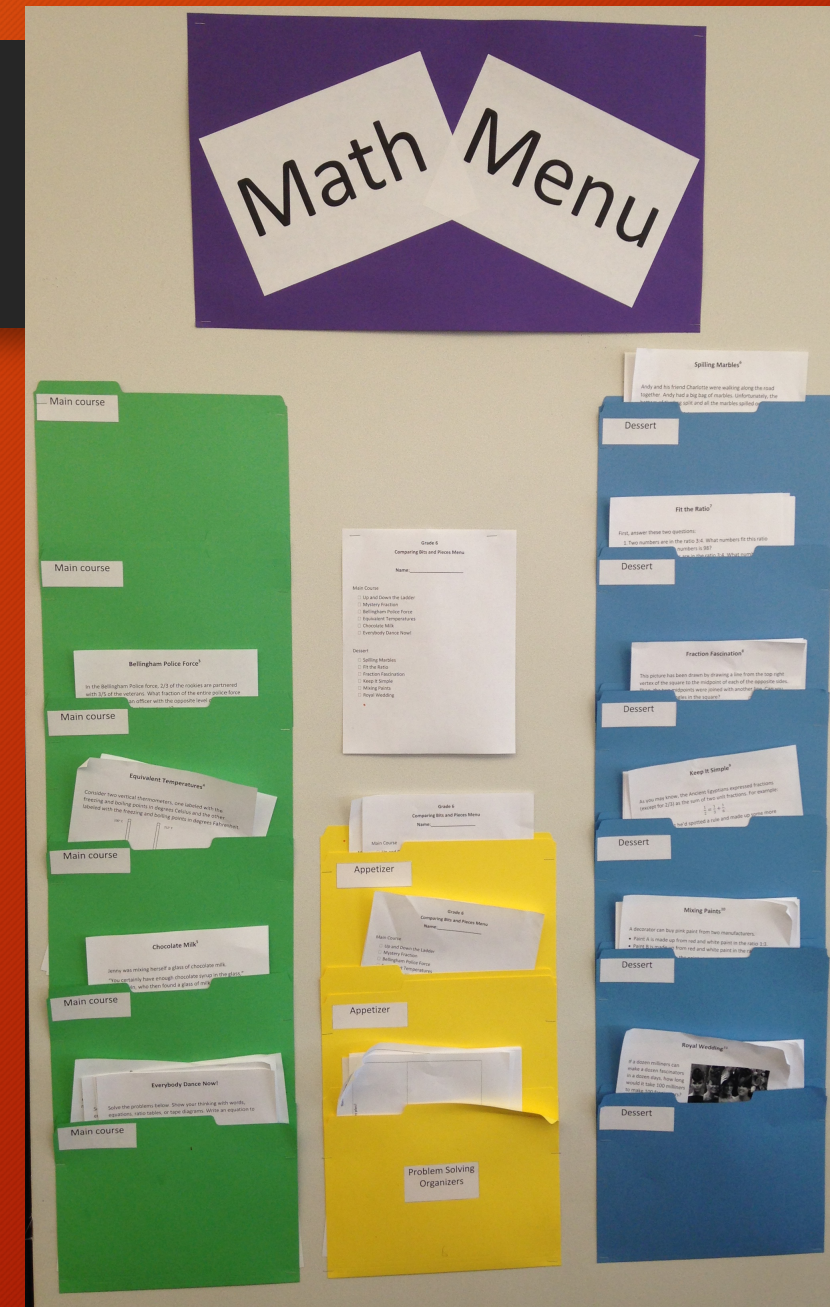
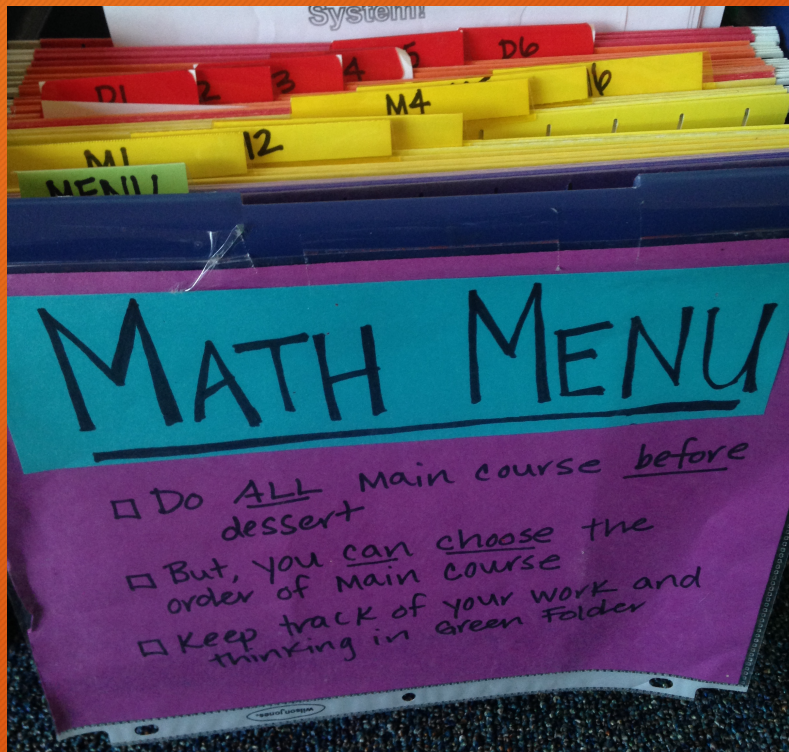
Produce	Regular Price	Organic Price	What Isabelle Needs
Gala Apples	\$1.79 / lb.	\$2.29 / lb.	3.2 lbs.
Pineapple	\$3.49 each	\$4.49 each	2 pineapples
Navel Oranges	\$1.29 / lb.	\$1.89 / lb.	2.5 lbs.
Honeycrisp Apples	\$3.49 / lb.	\$3.99 / lb.	5.1 lbs.
Tomatoes on the Vine	\$1.99 / lb.	\$2.49 / lb.	3.75 lbs.
Russet Potatoes	59¢ / lb.	89¢ / lb.	10 lbs.
Raspberries	\$2.99 / pkg.	\$3.99 / pkg.	2 packages
Strawberries	\$3.99 / pkg.	\$5.99 / pkg.	1 package

Math Menus - Setting Up

- *Main Courses* can be completed in any order.
- All *Main Courses* are successfully completed before moving on to *Desserts*.*
- *Desserts* can be completed in any order.
- There are no time limits or initial expectations for efficiency.
- Students choose to work independently, collaboratively, or both.

Math Menus - Setting Up

ALL students should have independent, physical access to the tasks:



Math Menus - Selecting and Developing Tasks

Cognitive Demand Framework

- Low Cognitive Demand Tasks
 - Memorization
 - Procedures without connections (to understanding, meaning, or concepts)
- High Cognitive Demand Tasks
 - Procedures with connections (to understanding, meaning, or concepts)
 - Doing mathematics

Math Menus - Selecting and Developing Tasks

Buying Organic

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Bang for the Buck

G7 CAS EXC B

Ella is joining the Bellingham Figure Skating Club at the Sportsplex, and she and her mom are shopping for a new pair of figure skates. They have a 20% off coupon for the skates which will lower the price, but they also know that they will be charged sales tax of 8.7%.

Ella is wondering if it is more advantageous to add the sales tax before applying the discount, or to apply the discount before adding the sales tax. Investigate, and help Ella which order (tax then discount *OR* discount then tax) will result in a better buy. Explain your reasoning.

Would your answer change if instead of having a 20% off coupon, Ella had a \$20 off coupon? Explain your reasoning.

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Teachers and Administrators

G8 TWMM EXC E

A group of teachers and administrators are meeting for a retreat to plan for the next school year.

- The average age of the teachers is 35.
- The average age of the administrators is 50.
- The average age of both teachers and administrators is 38.

What is the ratio of the number of teachers to the number of administrators?

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Math Menus – Selecting and Developing Tasks

Sums and Differences

G6 LBR EXC F

Examine the following equations involving sums of unit fractions.

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

$$\frac{1}{5} + \frac{1}{7} = \frac{12}{35}$$

Are these sums correct? What pattern do you see?

What generalization can you make about the sums of unit fractions? How could you complete the following equation:

$$\frac{1}{m} + \frac{1}{n} =$$

Use your generalization to find the sum of $\frac{1}{9} + \frac{1}{13}$.

Investigate what happens when you change the equations above to subtraction equations: $\frac{1}{2} - \frac{1}{3}$, $\frac{1}{3} - \frac{1}{4}$, and $\frac{1}{5} - \frac{1}{7}$. Find and apply a generalization for $\frac{1}{m} - \frac{1}{n}$.

Consecutive Negatives

G7 ATN ADV E

Take four consecutive numbers, for example, -7, -6, -5, and -4. Place + and/or – signs between them.

$$-7 + -6 + -5 + -4 = -22$$

$$-7 + -6 - (-5) - (-4) = -2$$

There are many more possibilities. Try to list all of them, and work out the solutions to the various calculations.

Choose a different set of four consecutive negative numbers and repeat the process. Take a look at both sets of solutions. Explain any similarities.

Predict some of the solutions you will get when you start with a different set of four consecutive numbers. Test out any conjectures you may have. Explain and justify your findings.

Flippant Factoring

G8 SIW5 EXC C

The following trinomial expressions can be factored into two binomial expressions in the form $(x \pm j)(x \pm k)$ where j and k are integers. Find all possible values missing from the quadratic expressions below.

$$x^2 + \underline{\hspace{1cm}}x + 12$$

$$x^2 - 8x + \underline{\hspace{1cm}}$$

$$x^2 - \underline{\hspace{1cm}}x - 10$$

Extra Challenge: Given the same criteria as explained above, find four possible values for the quadratic expression below. Generalize a pattern for finding all possible values.

$$x^2 + 7x - \underline{\hspace{1cm}}$$

Math Menus - Let's dig in!

Math Menus - Selecting and Developing Tasks

Our Favorite Resources

- *Teaching Children Mathematics* - Problem Solvers, Math by the Month
- *Mathematics Teaching in the Middle School* - Palette of Problems
- The Curriculum
 - Turning around or opening up problems
 - Problems in a new context
 - Working backwards

Math Menus - Selecting and Developing Tasks

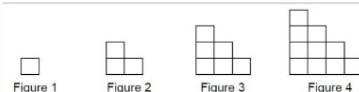
Our Favorite Resources

- www.youcubed.org
 - 7th grade
 - Functions



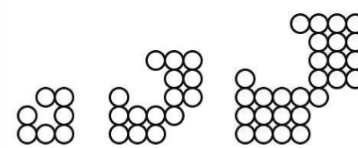
Penny Collection

3 4 5 6 7 8
9 10 Number Sense
Pattern Recognition Division
Factors Multiplication



Squares To Stairs

7 8 9 Algebra
Quadratic Equation



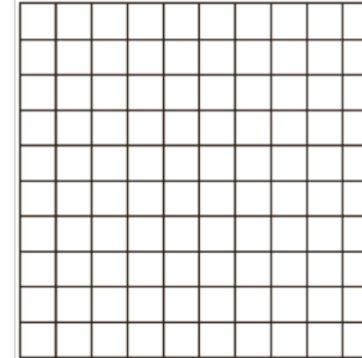
Circle Fever

7 8 9 10 11 12
Generalization
Pattern Recognition
Shape & Space Algebra



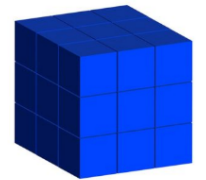
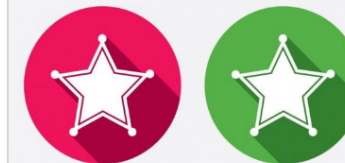
Patterns and Products

4 5 6 7
Number Sense Numbers



How Many Rows? How Many in Each Row?

3 4 5 6 7 8
9 Number Sense Area
Multiplication



Painting Youcubed

5 6 7 8 9 10
Generalization Number Sense
Pattern Recognition
Shape & Space 3D Shapes
Geometry Modeling
Patterns Surface Area



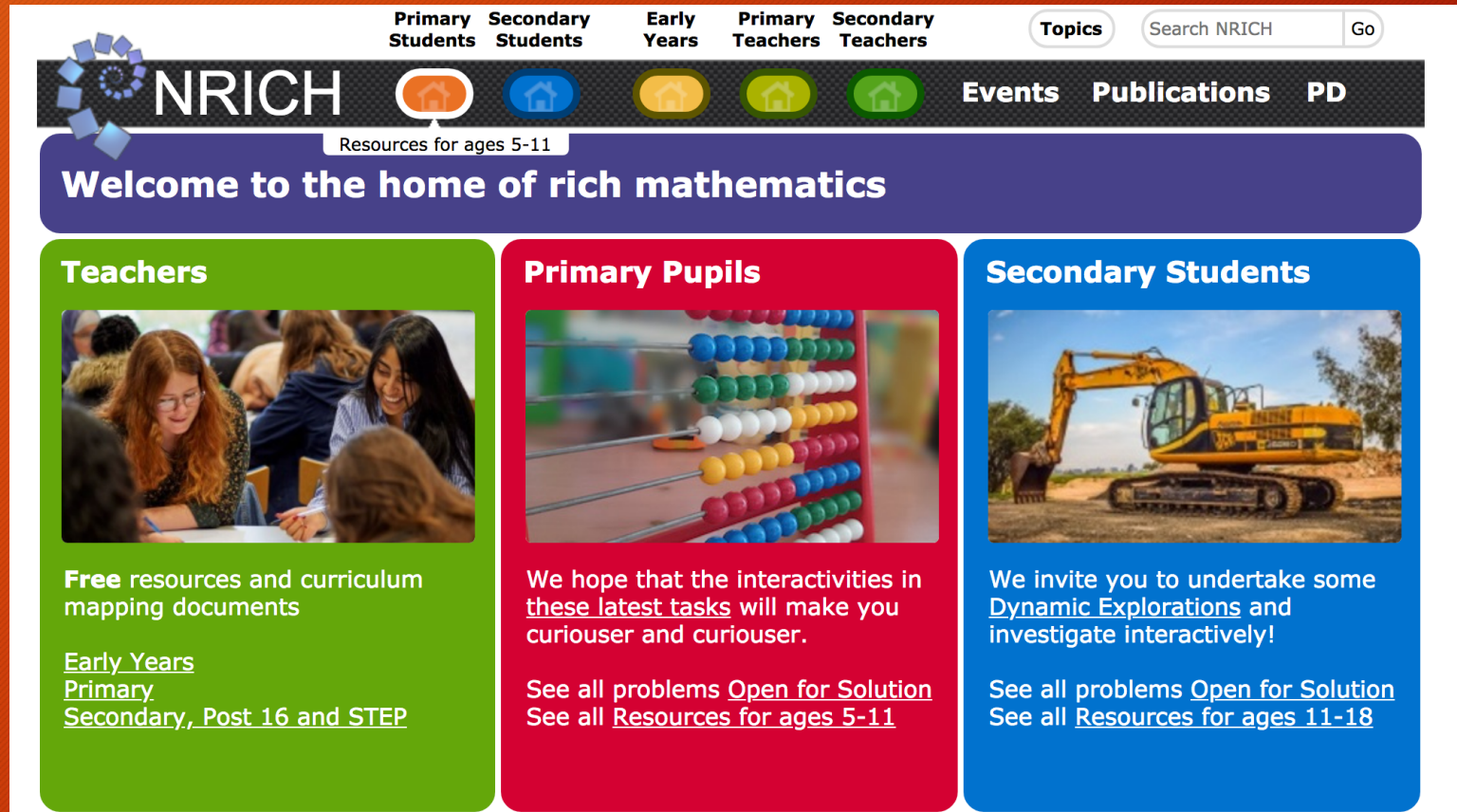
Squares Upon Squares

3 4 5 6 7 8
9 10 Generalization

Math Menus - Selecting and Developing Tasks

Our Favorite Resources

- <https://nrich.maths.org/>
 - Functions
 - Ages 11-14
 - Ages 14-16




The screenshot shows the NRICH website homepage. At the top, there is a navigation bar with links for Primary Students, Secondary Students, Early Years, Primary Teachers, and Secondary Teachers. To the right of these links are buttons for Topics, Search NRICH, and Go. Below the navigation bar is a large purple banner with the NRICH logo and the text "Welcome to the home of rich mathematics". Below the banner are three main content areas: Teachers (green background), Primary Pupils (pink background), and Secondary Students (blue background). Each area features a representative image, a brief description, and links to resources.

Primary Students **Secondary Students** **Early Years** **Primary Teachers** **Secondary Teachers** **Topics** **Search NRICH** **Go**

NRICH Resources for ages 5-11

Welcome to the home of rich mathematics


Teachers



Free resources and curriculum mapping documents

[Early Years](#)
[Primary](#)
[Secondary](#), [Post 16](#) and [STEP](#)


Primary Pupils



We hope that the interactivities in [these latest tasks](#) will make you curiouser and curiouser.

See all [problems Open for Solution](#)
See all [Resources for ages 5-11](#)

Secondary Students



We invite you to undertake some [Dynamic Explorations](#) and investigate interactively!

See all [problems Open for Solution](#)
See all [Resources for ages 11-18](#)

Math Menus - Selecting and

Our Favorite Resources

- Puzzles!
 - KenKen Puzzles
 - <https://www.kenkenpuzzle.com>

KenKen Puzzles

G7 SAP ADV F

A KenKen is an arithmetic puzzle invented in 2004 by a Japanese math teacher, Tetsuya Miyamoto. In the KenKen puzzle below, the numbers 1-5 only appear once in each row and once in each column. The bold, outlined sections indicate what numbers add, subtract, multiply, or divide to the number provided.

The 30x here means that the product of the numbers in these boxes is 30. This could be 1, 5, and 6, or 2, 3, and 5.

4	30x			5+
2÷	5+		5	
	3-	6+	2÷	8+
2-				
	12x		2÷	

Solve the four KenKen puzzles (including the one above) on the sheet provided.

Math Menus - Selecting and

Our Favorite Resources

- Puzzles!
 - KenKen Puzzles
 - <https://www.kenkenpuzzle.com>
 - Sudokus on NRICH

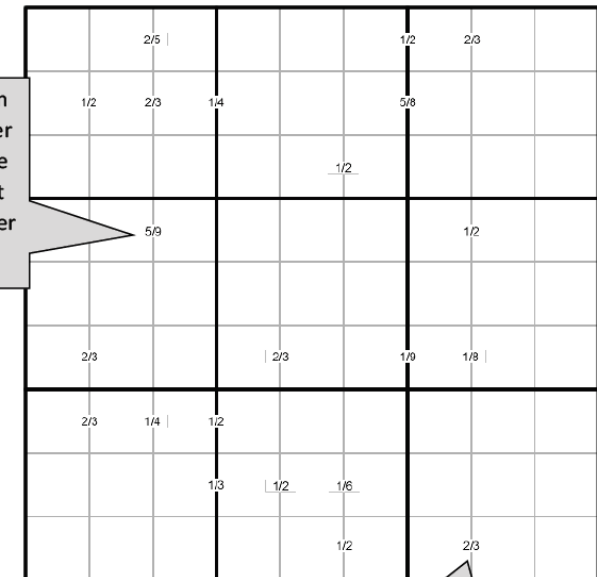
Ratio Sudoku

G6 LBR ADV F

In a regular Sudoku puzzle, each digit from 1-9 is exactly once in each row, each column, and each 3 x 3 highlighted square. In this Ratio Sudoku puzzle, the numbers in some adjacent squares are marked by clues: the two numbers in those squares form the ratio provided in simplest form.

For example, the numbers in these two squares may either be 5 and 9, or 9 and 5. These are the only two values that give this ratio. Note that order doesn't matter!

Solve the Ratio Sudoku on the sheet provided!



However, the numbers in these two boxes could be 2 and 3, 3 and 2, 4 and 6, 6 and 4, 6 and 9, or 9 and 6.

Math Menus - Selecting and

Our Favorite Resources

- Puzzles!
 - KenKen Puzzles
 - <https://www.kenkenpuzzle.com>
 - Sudokus on NRICH
 - Other Puzzle-y Tasks
 - Futility Closet Blog

E Pluribus Unum

Replace each * with a different digit 1-9 to make this equation true:

$$\frac{*}{**} + \frac{*}{**} + \frac{*}{**} = 1$$

Prime Puzzle

G6 PT ADV F

Arrange the numbers 1 through 25 in the squares below so that the sum of every pair of numbers, either horizontally or vertically, is a prime number.

Math Menus - Let's dig in!

Math Menus - Selecting and Developing Tasks

What to Avoid:

- Games that require more than one person
- Skills practice (that isn't more than that)
- Regurgitation tasks
 - Write a letter....
 - Make a video....
- Giving instructions on how to solve

Math Menus - Selecting and Developing Tasks

What to Aim for:

- Balance
- Relevant Contexts
- New Contexts
- Variety
- Novelty
- Representation
- Assessment

Math Menus - More Differentiation Opportunities

- Appetizers
- Working with Small Groups
- Using Tasks to Pre-teach Concepts
- Task Experts
- Writing About Mathematics

Questions?

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Thank you!

References

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National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM: Reston, VA.