Solutions to the Neanderthal Problem, logarithm version

1. $C(t) = (0.5)^{\frac{t}{5730}}$

2. $C(50{,}000) \approx 0.00236$. A 50,000-year-old fossil will contain only about 0.236 percent of its original carbon.

3. $(0.99)(0.00236) + (0.01)(1) \approx 0.012338$

4. We need to solve the following equation for t:

$$0.012338 = (0.5)^{\frac{t}{5730}}$$

Using logarithms:

$$\log(0.012338) = \left(\frac{t}{5730}\right)\log(0.5)$$

$$\vdots$$

$$t \approx 36{,}332$$

The scientist would think the fossil is only 36,332 years old instead of 50,000 years old.

5. The researcher's statement in the article actually *underestimates* the error in carbon dating!

6. The amount of original carbon 14 from an x-year old fossil would be:

$$(0.5)^{\frac{x}{5730}}$$

With one percent contamination of modern carbon, the total amount of carbon 14 in a sample would be:
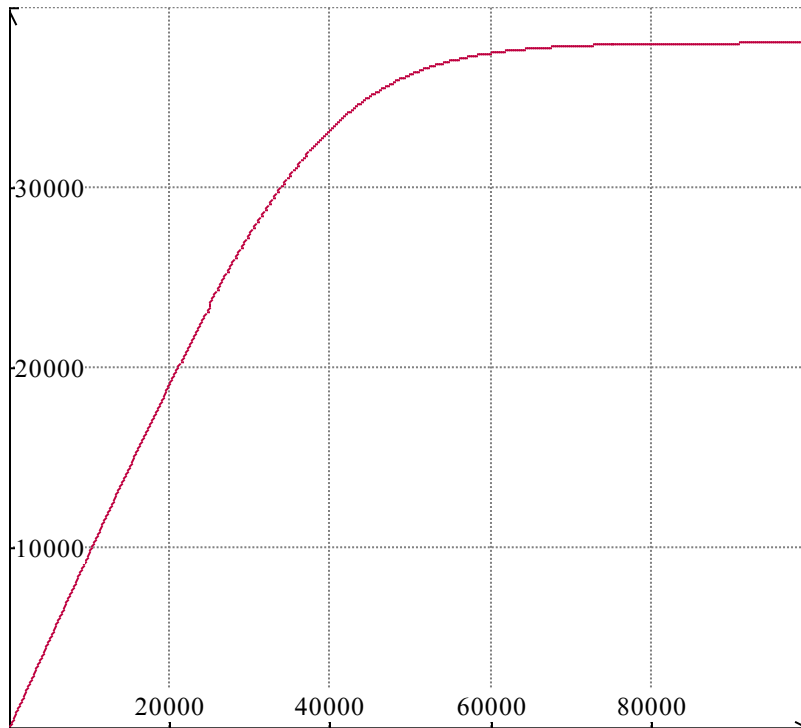
$$(0.99)(0.5)^{\frac{x}{5730}} + (0.01)(1)$$

The apparent age (A_1) of the fossil would be the solution to the equation:

$$(0.99)(0.5)^{\frac{x}{5730}} + (0.01)(1) = (0.5)^{\frac{A_1}{5730}}$$

Using logarithms, this gives:

$$A_1 = \frac{\log\left((0.99)(0.5)^{\frac{x}{5730}} + (0.01)(1)\right)}{\log(0.5)} \times 5730$$

A graph of this function is shown on the next page.

For relatively young fossils, the graph resembles the line $y = x$. That's because for young fossils, not a lot of original carbon 14 is gone yet, so most of the carbon 14 we see is original, and the age estimate is pretty accurate.

For relatively old fossils, this graph has an asymptote approaching about 37,000 years. Once a fossil is about 50,000 years old, nearly all of the original carbon is gone and most of what you see is the contamination. So the asymptote is at the age that a fossil would appear if it had one percent carbon 14.

7. This time the apparent age (A_2) of the fossil will be the solution to this equation:

$$A_1 = \frac{\log\left(\left(\frac{1-x}{100}\right)(0.5)^{\frac{50,000}{5730}} + \left(\frac{x}{100}\right)(1)\right)}{\log(0.5)} \times 5730$$

A graph of this function is below:
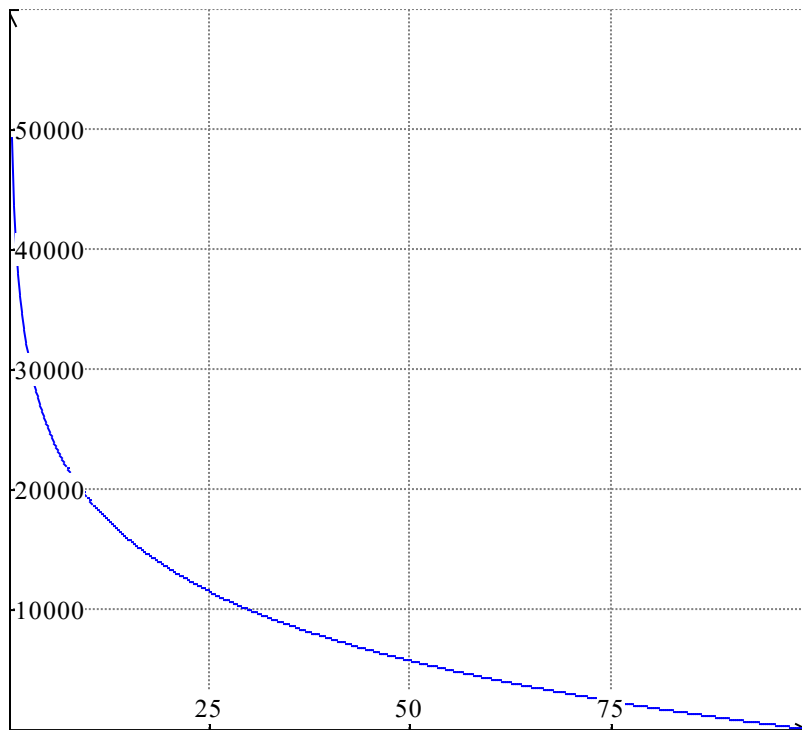
The points that are known are (0, 50000) and (100, 0). That's because with no contamination the age estimate would be completely accurate, and with 100% contamination, the fossil would look perfectly contemporary.

You can tell from the graph that purifying samples makes a big difference in estimating the age of a fossil because the graph is so steep for small amounts of contamination. Tiny changes in the amount of contamination will result in big changes in the age estimate—and the less contamination there is, the greater the improvement in the age estimate that will come with a purer sample.