Discrete Math Project Collaborative

Teaching the SMPs through "Continual" Meaning Making in Discrete Math

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https://dmpc.sdsu.edu

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Agenda

9:45-9:50	Welcome & Introduction
9:50-10:45	Do Math
10:45-10:55	Q & A
10:55-11:00	Reflection & Evaluation

Session Description

Empowering students begins with asking them to engage in mathematical thinking before telling them how to think and, consequently, honoring what learners bring. In this session, teachers will focus on Standards for Mathematical Practices as they solve holistic discrete math problems.

Students as partners in knowledge construction: Holistic Problems & Cognitive Demand

Author/ Term	Harel & Stevens (2011) Holistic Problems	Smith & Stein (1998) Cognitive demand (of tasks)	Schoenfeld et al (2014) Cognitive Demand
Concept/ Definition	A holistic problem is one where a person must figure out, from the problem statement, the elements needed for its solution. It does not contain hints or cues as to what is needed to solve it. In contrast, a non-holistic problem is broken down into small parts, each of which attends to one or two isolated elements.	Levels of Cog. Demand (of Tasks) 1. Memorization 2. Procedures without connections 3. Procedures with connections **4. Doing Math**	The extent to which students are supported in grappling with and making sense of mathematical concepts at an appropriate level (***productive struggle***)

Focus Questions

- What mathematical activities does each problem target? (e.g. defining, meaning-making, pattern finding, generalizing, conjecturing, justifying/proving, symbolic manipulation, interpreting, inferring, explaining, structuring, applying, predicting, classifying, searching, problem solving, etc.) Harel (2008)
- What SMPs did you engage in?

Nice To Meet You!

The Situation:
Jack and his wife, Jill, invited three other married couples to their house for dinner. Various handshakes took place. No one shook his/her own hand or his/her spouse's hand, and no one shook hands with the same person more than once.

The Question:
What mathematical questions might we raise about this situation?

Write questions on post-its provided and place them on posters.

Questions we anticipate

- How many handshakes took place in total? (What assumptions did you make to problematize the situation?)
- Is it possible to determine how many hands each person shook? If so, how many hands did Jack shake? How many hands did Jill shake?

Nice To Meet You!

Jack and his wife, Jill, invited three other married couples to their house for dinner. Various handshakes took place. No one shook his/her own hand or his/her spouse's hand, and no one shook hands with the same person more than once.

After all the handshakes took place, Jack asked each guest and his wife (but of course doesn't ask himself), how many hands he/she had shaken. Surprisingly, each person gave a different answer.

Dominoes

A domino is a rectangular tile with a line dividing its face into two square halves. Each half is either blank or imprinted with a number of dots starting at 1. No two dominoes have the same set of numbers and a complete set has every possible pair of numbers. The highest number varies from one set to another.

Here are a few sample dominoes.

Dominoes

a. How many dominoes make a complete game set? Explain the process which you used to count the number of dominoes. How do you know you did not overcount or undercount?

Dominoes

b. Jonathan found a different version of the domino game online. The number of dots on each square half can go up to 6. How many dominoes make a complete set for this version? (Remember: a complete set has no duplicates.)

Dominoes

c. Jonathan found a different version of the domino game online. The number of dots on each square half can go up to 9. How many dominoes make a complete set for this version? (Remember: a complete set has no duplicates.)

Dominoes

d. Jonathan found a different version of the domino game online. The number of dots on each square half can go up to 100. How many dominoes make a complete set for this version? (Remember: a complete set has no duplicates.)

Consider these Two Sequences

Discuss the ramifications of the following two sequences, in terms of **SMP's**.

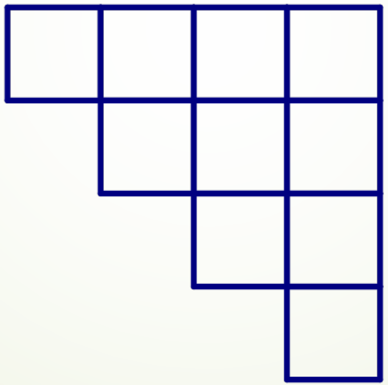
1. a, [b, c, d, a]

2. b, c, a

MAA's 10 problem-solving strategies!! [see #2 and 5]

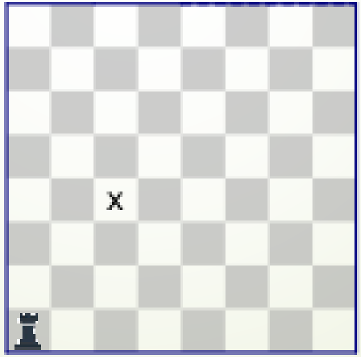
Stair-like Structure

Using exactly 400 squares, would you be able to build a structure like the one shown below. Why or why not?



Chessboard

In chess, a rook can move only in straight lines along rows or columns (not diagonally). How many different shortest paths are possible for the rook in the image below to get to the square marked ‘X’?



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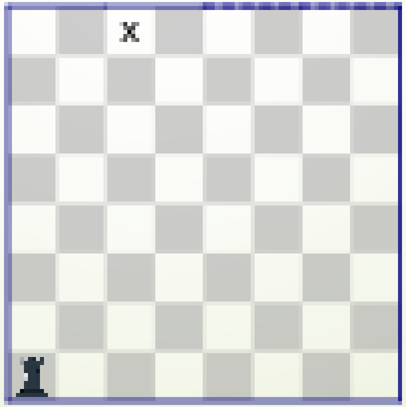
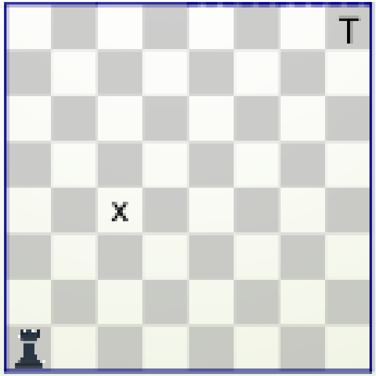


Chessboard

In chess, a rook can move only in straight lines along rows or columns (not diagonally). How many different shortest paths are possible for the rook in the image below to get to the square marked ‘T’ while avoiding the square marked ‘X’?



Turn and Talk: Revisit Focus Questions

- What mathematical activities does each problem target? (e.g. defining, meaning-making, pattern finding, generalizing, conjecturing, justifying/proving, symbolic manipulation, interpreting, inferring, explaining, structuring, applying, predicting, classifying, searching, problem solving, etc.) → Harel (2008)
- What SMPs did you engage in?

What Discrete Math Topics are Fertile Ground?

Games (Two-player Impartial Combinatorial)	Structuring, Pattern finding, Conjecturing, Generalizing, Justifying/Proving
Traceability	Modeling, Pattern finding, Generalizing
Crypto	Reasoning with and about function, modeling
Counting	Structuring, Symbol-Manipulating, Pattern finding, Conjecturing, Generalizing, Justifying/Proving
Iteration and Recursion	Structuring, Symbol-Manipulating, Reasoning with and about function, Predicting, Generalizing

For more info about DMPC…

Website: https://dmpc.sdsu.edu/

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References

Counting, Connectivity & Traceability, Sequences & Series Modules © 2018. Discrete Math Pre-Collegiate is licensed under CC BY-NC-SA 4.0. https://dmpc.sdsu.edu/

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Schoenfeld, A. H. (2014, November). What makes for powerful classrooms, and how can we support teachers in creating them? Educational Researcher, 43(8), 404-412. DOI: 10.3102/0013189X1455 (map.mathshell.org)

Smith,M.S. & Stein, M.K. (1998). Selecting and creating mathematical tasks. Mathematics Teaching in the Middle School,3, 344-350. (See Task Analysis Guide for synopsis)

Questions?

DMPC Course Overview

Major goals:

- To help students acquire knowledge of fundamental mathematics (Statement of Competencies in Mathematics, ICAS)
- To advance students' ways of thinking (CCSS Standards for Mathematical Practice)
- To foster students' mathematical curiosity and to demonstrate how mathematics can solve authentic mathematical problems.
- To facilitate students' development of problem-solving skills, while fostering critical thinking, within an interesting setting.

Beliefs About Learning

- Problem-solving and productive struggle are keys to knowledge construction
- Students need MANY opportunities to define, model, generalize, conjecture and justify
- Teachers should view students as partners in knowledge construction
- Learners should experience phenomena before labels are introduced
- Learning is both social and cognitive in nature
- Group tasks should be group worthy
- Teachers are orchestrators of discourse

5 Practices for Orchestrating Productive Mathematics Discussions

- **Anticipating**: Multiple solutions before teaching, student thinking
- **Monitoring:** Seating chart, tracking, anticipated solutions/ways of thinking
- **Selecting:** Using goals, handling errors (equity and agency)
- **Sequencing:** Long-term goals (chapter & year), progression of ideas, address misconceptions (equity and agency), from concrete to abstract
- **Connecting:** Focus on meaning & linking approaches, make ways of thinking explicit, connecting representations

Reflection & Evaluation

http://tinyurl.com/dmpc-nctm2019

1. How did you feel about the mathematics we did today?
2. What do you view as the benefits of today's presentation?
3. What would you like to know more about?
4. What suggestions do you have for us to improve our professional development activities?