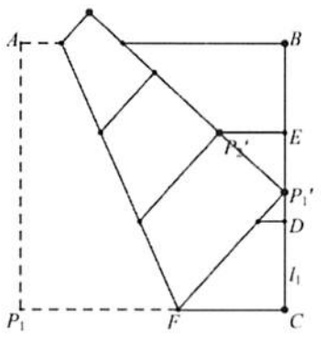

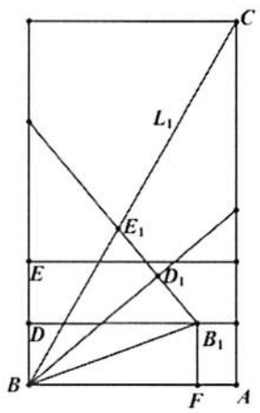

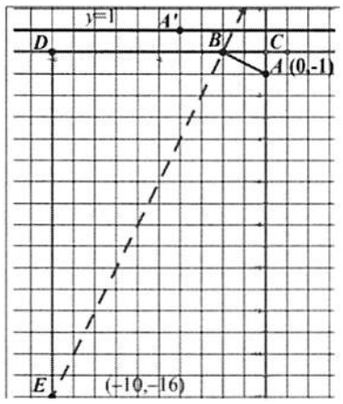

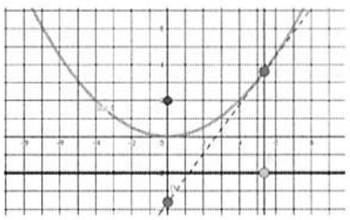

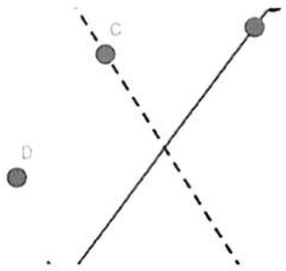

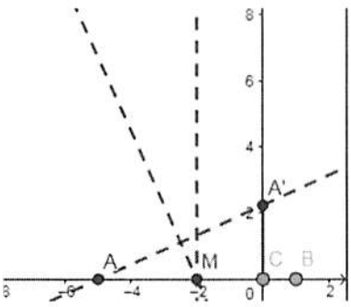

Paper Folding in Class? Not Just For Paper Airplanes Anymore!

Sites.google.com/view/paperfoldingmath/

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Stuyvesant High School

Math For America

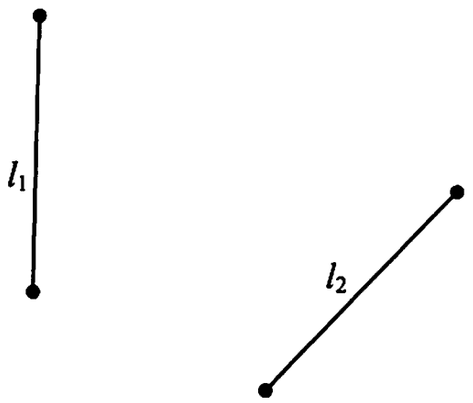
MfA

The first five Huzita-Hatori paper folding construction axioms.

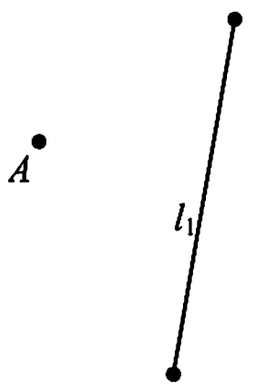
1. Make a fold that passes through both points A and B. How would you describe that fold geometrically?

2. Make a fold so that point A coincides with point B. How would you describe that fold geometrically?

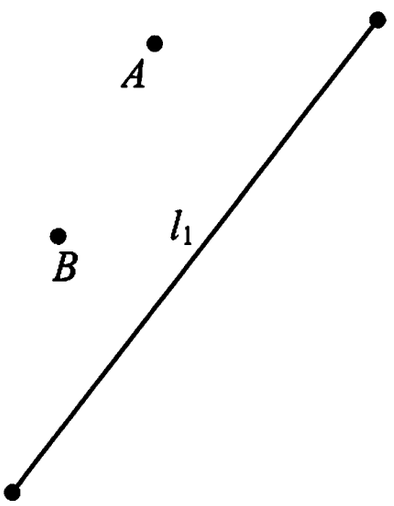
3. Make a fold so that line l1 coincides with line l2. How would you describe that fold geometrically?



4. Make a fold perpendicular to line l1 that passes through point A. How would you describe that fold geometrically?



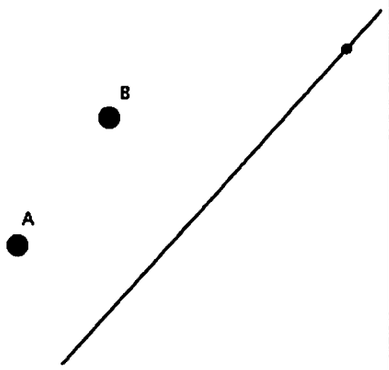
5. Make a fold that puts point A onto line l1 and that passes through point B.



For axiom 5, depending on the location of the points and the line, there are configurations where this is impossible. there are also configurations where there are multiple folds that make it work. Why is this? What is this fold accomplishing, geometrically?

For each question, find as many folds as you can that would place A onto the line so that the fold passes through B. Is it always possible to find a fold that does this? Are there ever two folds that accomplish this? Are there ever three folds that accomplish this?

1.



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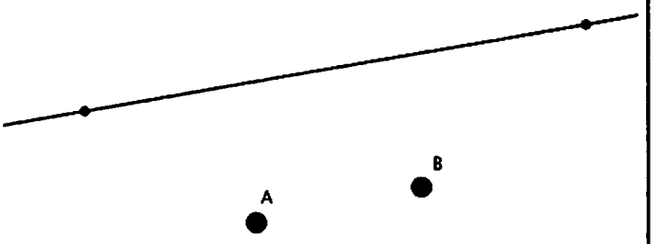

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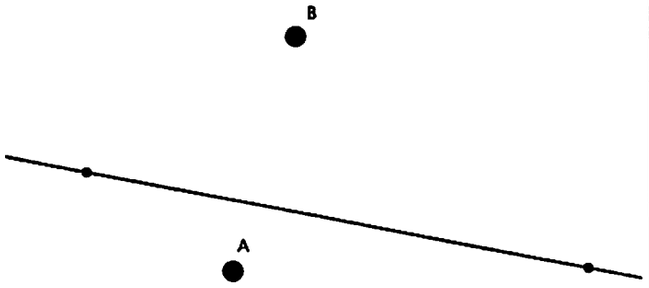

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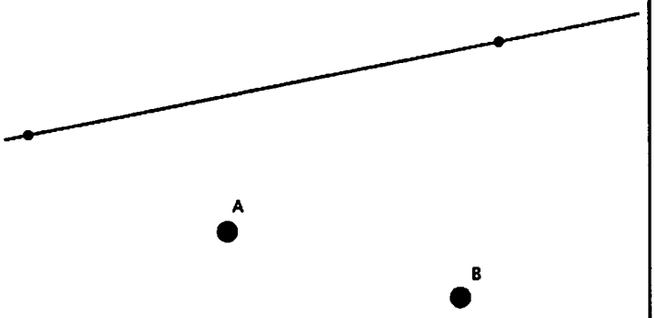

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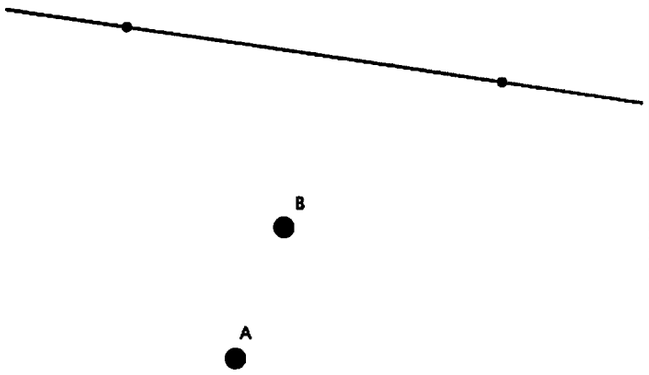

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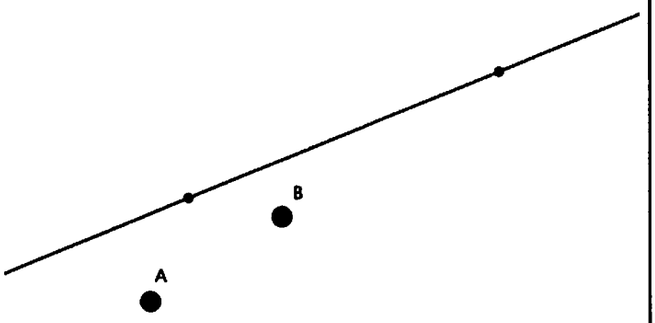
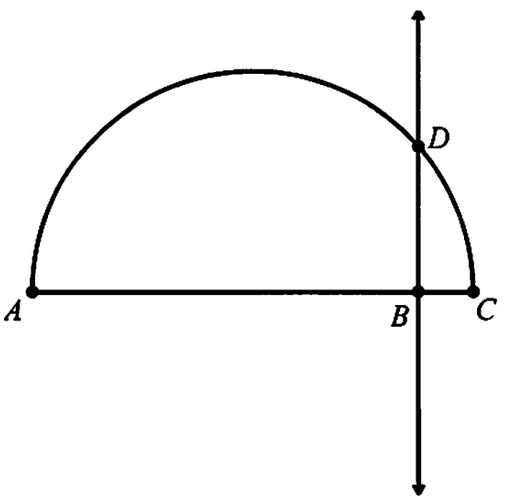


Finding the square root of 7 with a compass and a ruler

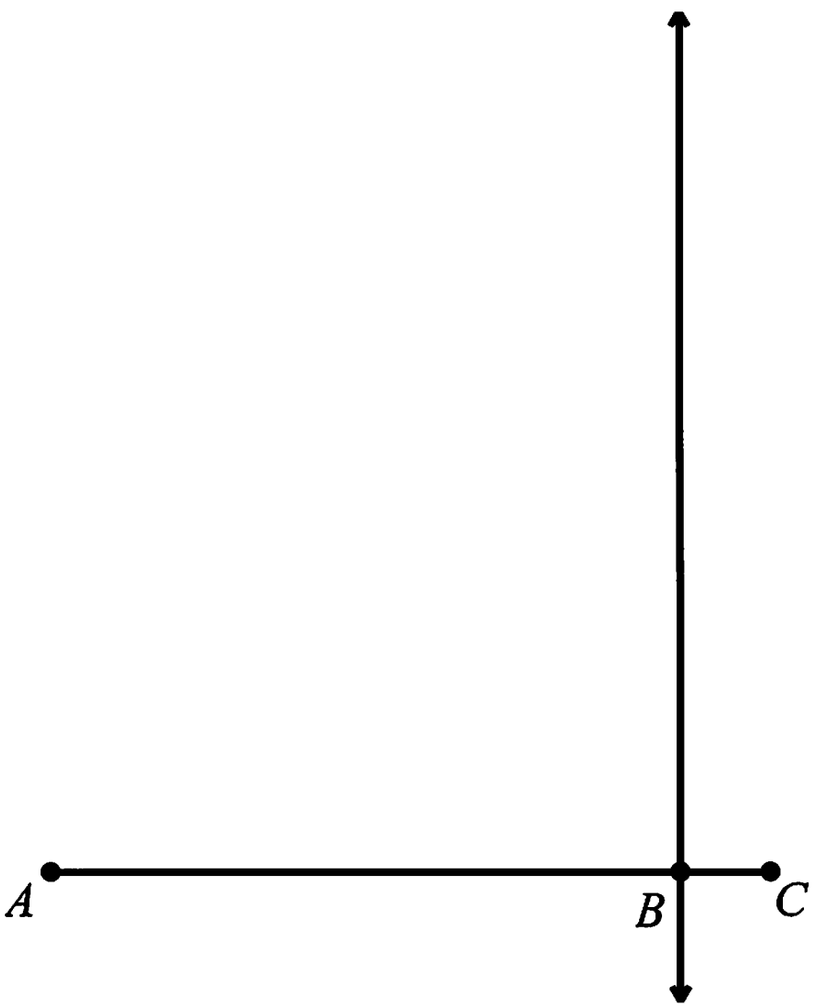
1. Make ABC so that AB=7 units and BC=1 unit

2. Make the semicircle with diameter AC and make a line through B perpendicular to AC. Call the intersection of the perpendicular line with the circle D. BD has length square root of 7. Why?

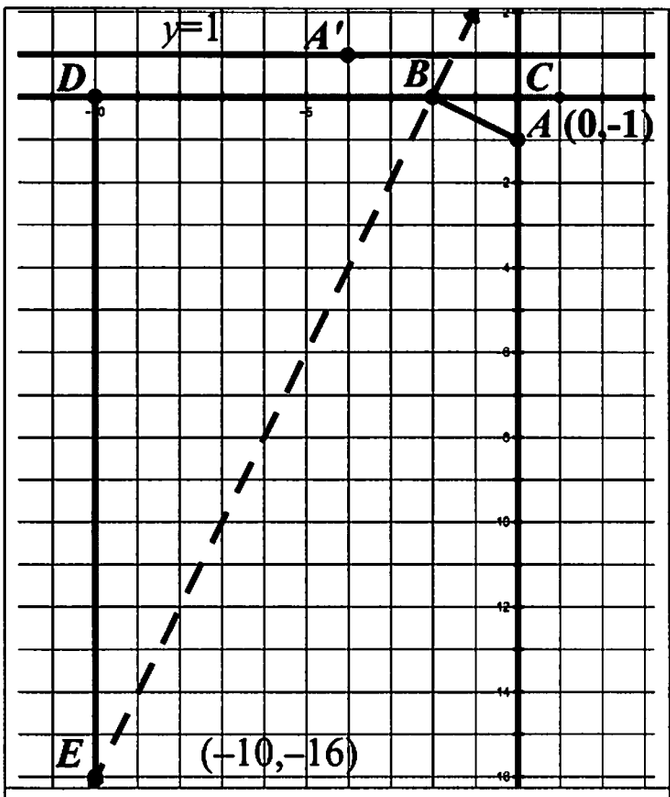


3. Paper folding axiom 5 can be used in a step for constructing the square root of 7.

Start again with AB=7 and BC=1 and the line perpendicular to AC through B (which you could create with Axiom 4 if you don't already have that perpendicular line and you just have points A, B, and C) Figure out how to use paper folding (it will require 3 folds, 4 if the perpendicular line is not given) to make a segment that is length square root of 7.



Solving quadratic equations of the form $x^2 - bx + c = 0$ with paper folding



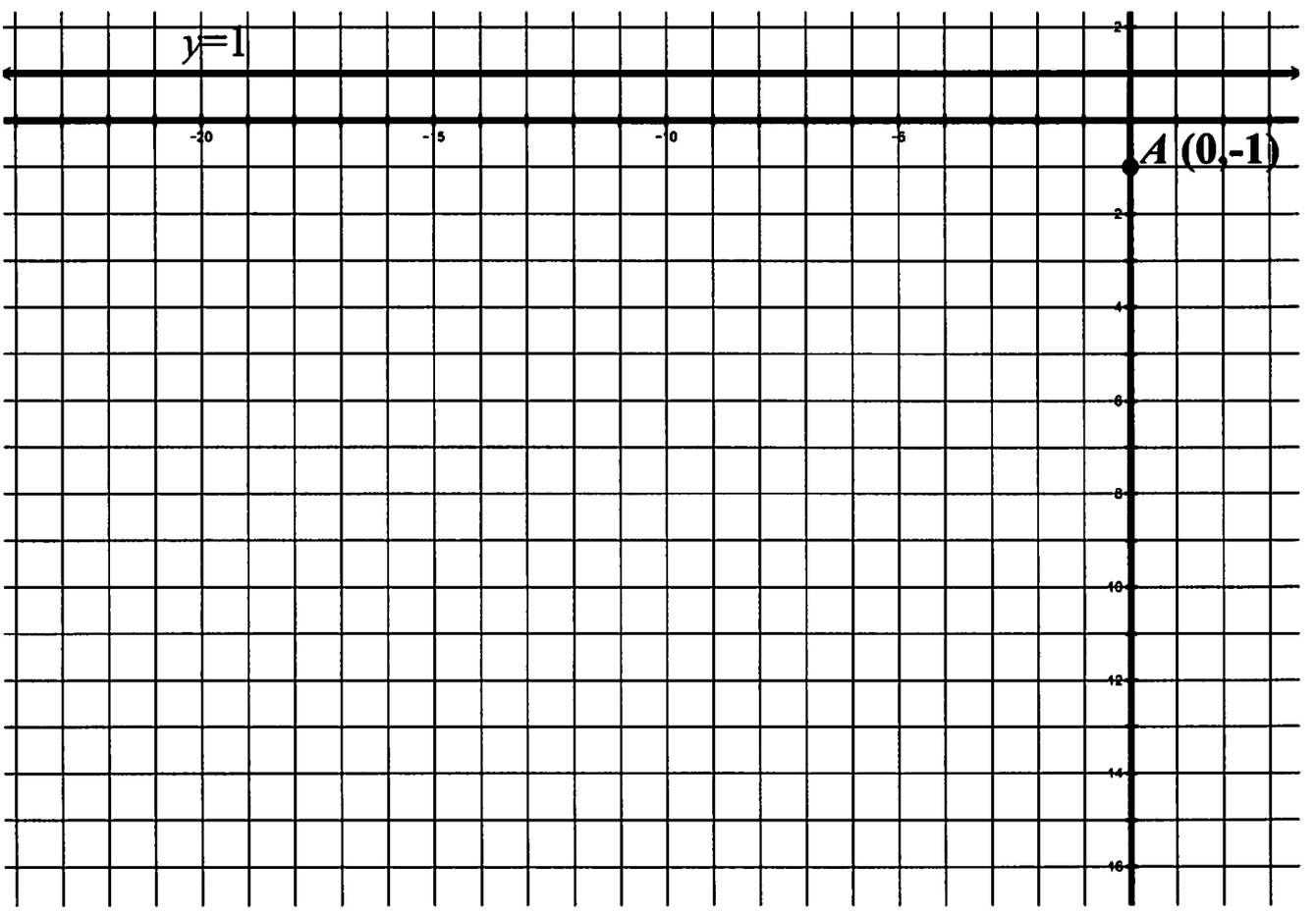
To solve $x^2 - 10x + 16 = 0$

1) Plot the point $E = (b, -c) = (-10,16)$
2) Fold A at $(0,-1)$ to line $y = 1$ so that the fold passes through E. This can be done 0, 1, or 2 ways.
3) Label where the fold intersects the x-axis B.
4) The length of BC will be one solution to the equation.

Why does this work? (Without using circles or parabolas)

Hint: What can be said about triangles ABC and triangles BED? Call BC x and find the lengths of AC, DE, DC, and DB, in terms of x and of the numbers 10 and 16.

Find all solutions to $x^2 - 6x + 8 = 0$ with paper folding



Conic Sections Activity

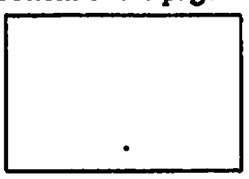
1. Draw ten points that are the same distance from both P and Q

P Q

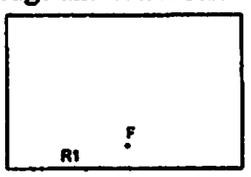
2. What is the relationship between the line segment PQ and the set of all points equidistant from both P and Q?

3. Fold this paper so that points P and Q from question 1 coincide. Make a conjecture about the significance of the crease formed by the fold.

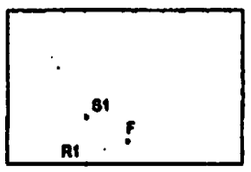
4. Take out a sheet of looseleaf paper, turn it horizontally, and put a point about an inch above the center of the bottom of the page.



Label the point F. Make a point on the bottom edge of the paper, where one of the looseleaf lines intersects the edge and call it R1.



Fold the paper so that R1 coincides with F. Make a point on the crease directly above point R1. Call that point S1..



5. Pick 9 other points on the edge of the bottom and repeat the last two steps, calling the points on the bottom R2 through R10 and the points on the crease directly above them, S2 through S10.

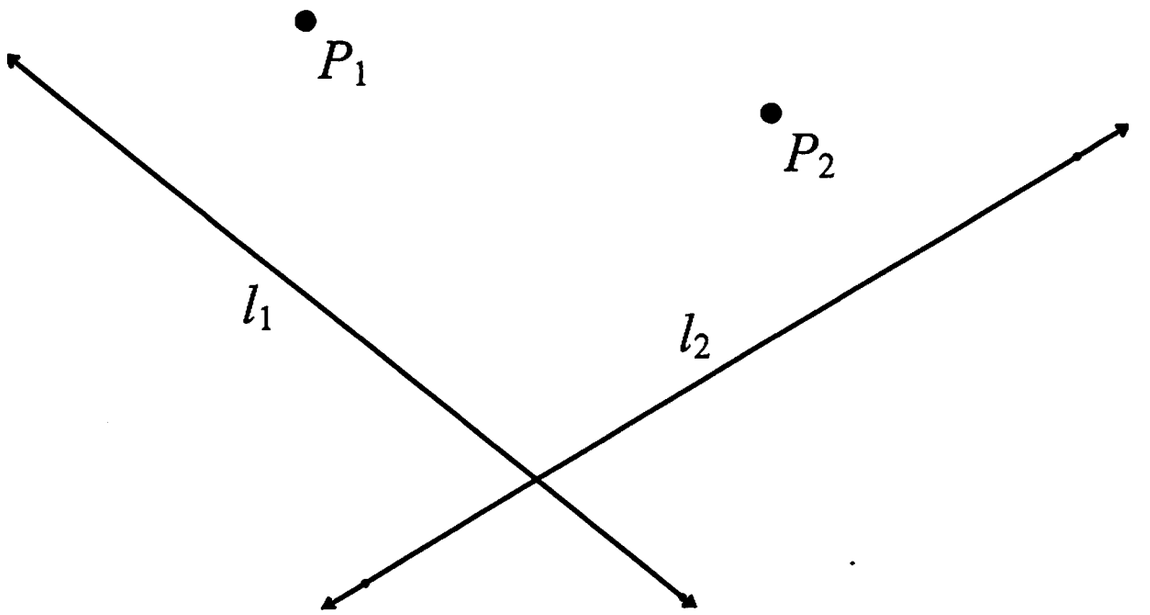
6. What curve appears to be formed by the points S1 through S10?

7. Which statement is most accurate:
a) Sn is closer to Rn than it is to F for all n.
b) Sn is closer to F than it is to Rn for all n.
c) Sn is the same distance from F as it is from Rn for all n.
d) It depends on the value of n

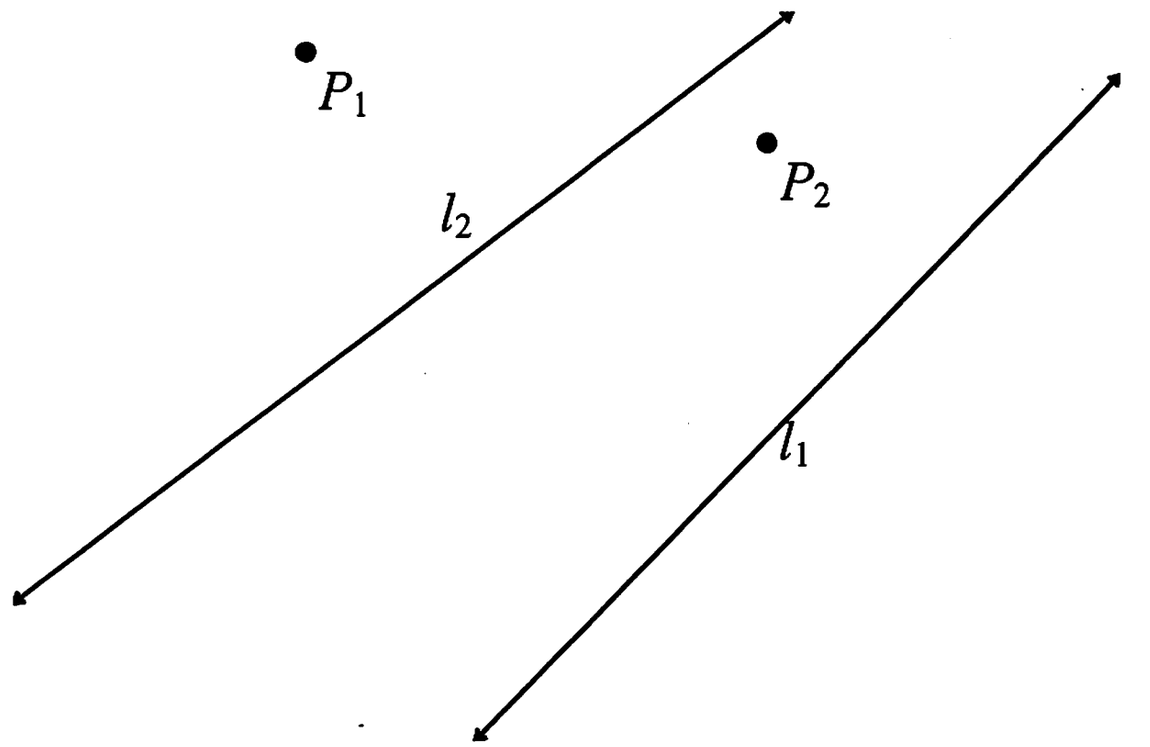
8. A circle can be described as the set of points a fixed distance from a given point. What is a similar definition of a parabola?

Paper folding axiom #6

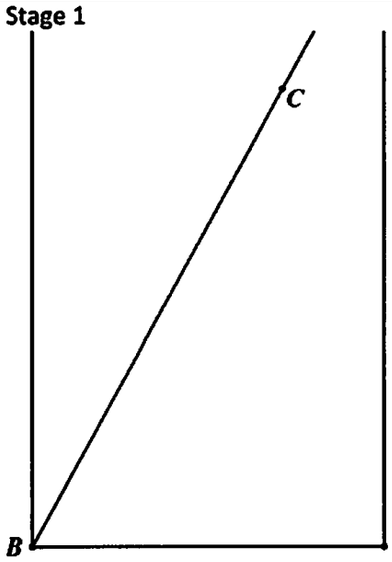
Make a single fold (if possible) that puts point P1 onto line l1 and point P2 onto line l2.



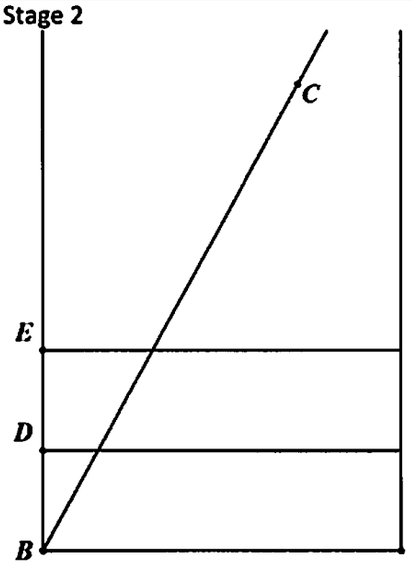
Try it for this one.



Stage 1

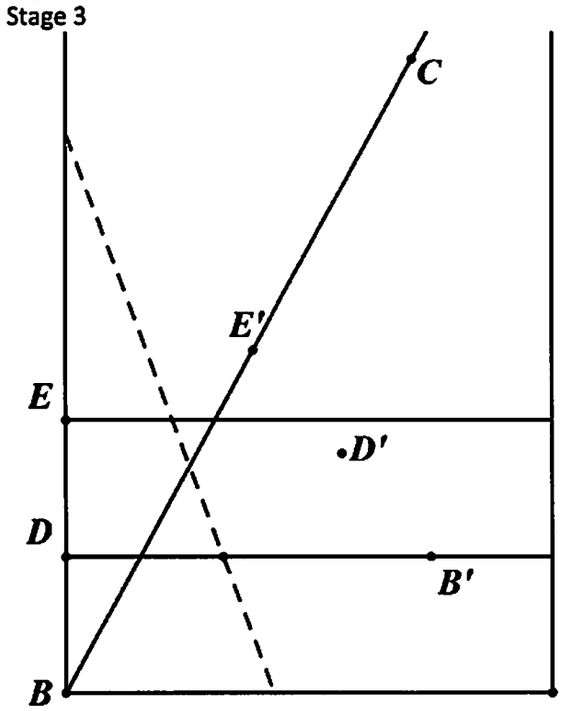


Stage 2



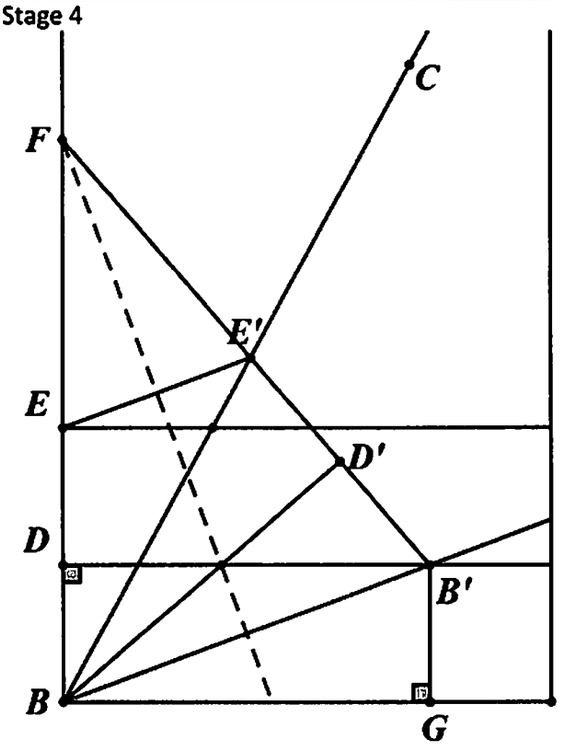
Make point E anywhere on the edge. Make the horizontal fold through E. Fold B to E to make D and the horizontal fold through D.

Stage 3



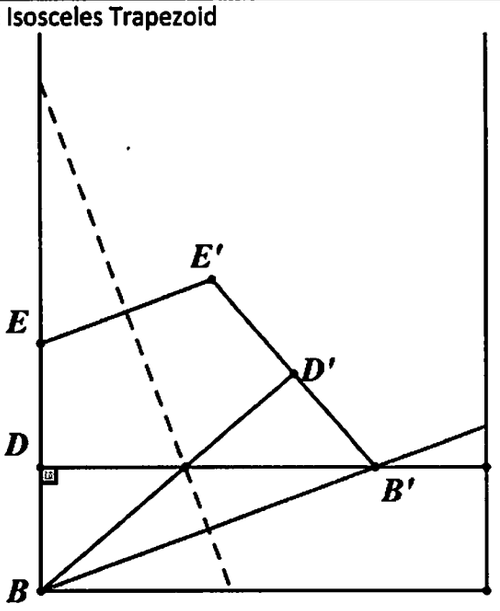
Use axiom 6 to make a fold that puts E onto BC and B onto the horizontal line through D. Label E' and B'. With this fold, D gets put at D' too.

Stage 4

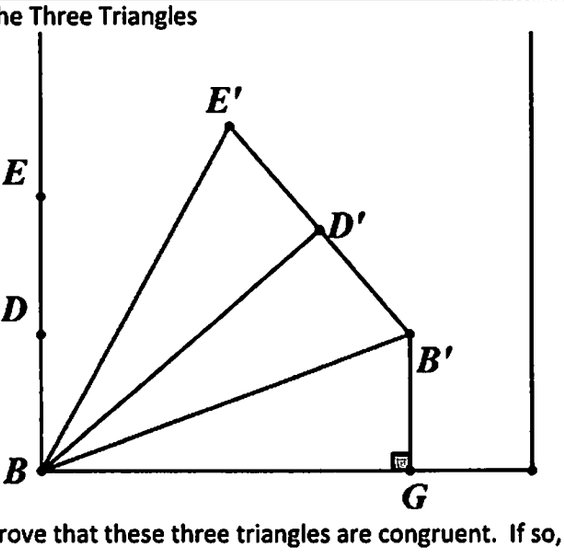


B'G is perpendicular to the bottom edge. Line segments can be made to form various triangles and also an isosceles trapezoid.

The Isosceles Trapezoid



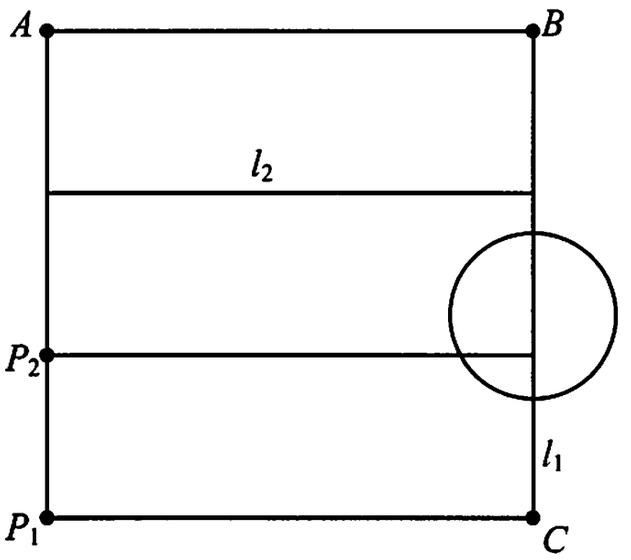
The Three Triangles



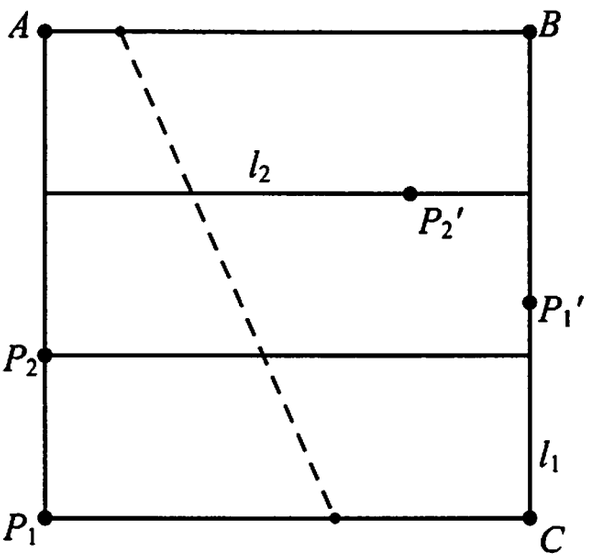
Prove that these three triangles are congruent. If so, then angle GBB' will be 1/3 of angle GBE' which is the original angle.

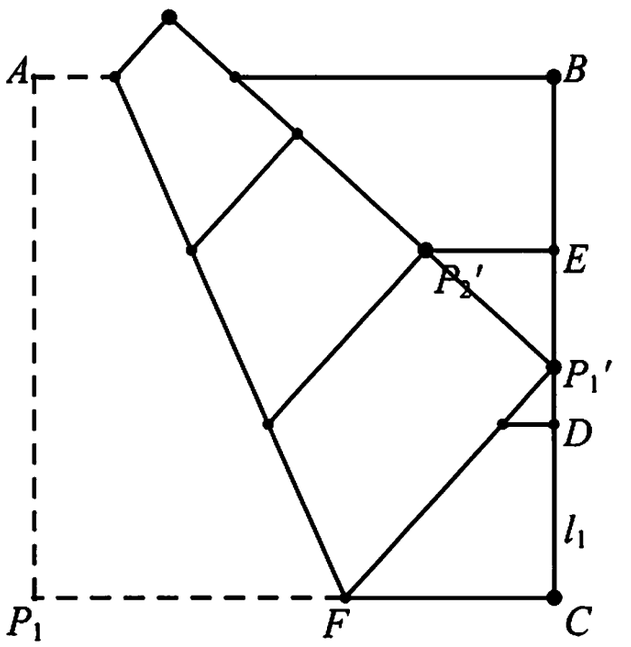
Step 1

Using Axiom 6, Fold P_1 to l_1 and P_2 to l_2



Step 2





Proof:

Label CP_1' as 1 'unit'. The goal is to show that BP_1' is $\sqrt[3]{2}$ 'units'

Call $BP_1' = x$.

What is an expression for the length of BC?

What is an expression for the lengths of CD, DE, and EB?

What is an expression for the length of line segment $P_1'P_2'$?

What is an expression for the length of line segment $P_1'E$? (simplify this as much as possible)

Prove that triangle $P_1'P_2'E$ is similar to triangle $FP_1'C$

Call the line segment CF d. What is an expression for the length of P_1F in terms of x and d?

Explain why P_1F is the same length as $P_1'F$.

Set up a proportion with the similar triangles $P_1'P_2'E$ and $FP_1'C$ involving d and x.

It should eventually become $\frac{2x-1}{x+1} = \frac{x^2+2x}{x^2+2x+2}$. Show that this simplifies to $x^3 = 2$.

There is a full show me video about this proof on the show me site.

Solving Cubic Equations with Origami

Cubic Equation

It is known that the straight-edge-and-compass construction is equivalent to solving quadratic equations. On the other hand, the origami construction is equivalent to solving cubic equations. So, any line or point can be constructed with origami if it can be constructed with a straight edge and a compass. In addition, we can solve with origami some problems which cannot be solved with a straight edge and a compass, such as doubling a cube or trisecting an angle.

Now, let's solve the cubic equation $x^3+ax^2+bx+c=0$ with origami. Let two points P1 and P2 have the coordinates (a,1) and (c,b), respectively. Also let two lines L1 and L2 have the equations $y=-1$ and $x=-c$, respectively. Fold a line placing P1 onto L1 and placing P2 onto L2, and the slope of the crease is the solution of $x^3+ax^2+bx+c=0$.

For example: $x^3+4x^2+5x+6=0$ has a solution $x=-3$.

Fold P1 (4,1) onto l1 ($y=-1$) and P2 (6,5) onto l2 ($x=-6$). After getting the fold, find the slope of the fold by finding two points on the fold that have x coordinates that differ by 1. The change in the y coordinate will be the slope of the line and also the solution to the original equation.