

Authentic more than Real-World Mathematics Tasks for Middle School

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**Pseudo contexts can give
the impression to students
that mathematics is
other-worldly and unreal ...**

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Classic – Ages of Children

- **Johnny is twice as old as Jamie**
- **Jamie is four years older than Jules**
- **Their mother, Jill, was 32 years old when Jules was born.**
- **Jill is now 35 years old.**
- **How old are Johnny, Jamie and Jules?**
- **Answer – NEXT SLIDE PLEASE**

Answer

Why use mathematics? Just ask them their ages?

How do you know all of the facts if you didn't know their ages?

Use mathematics to find the ages of siblings? → UGH!

This problem not salvageable, but others are.

Authentic is more than Real-world

Real-world → anything visible or touchable

- **Children and their ages**
- **10 ft baseball glove in centimeters**
- **\$100 in bank account – interest compounded annually**
- **Shadow of flagpole**

Authentic is more than Real-world

Authentic

- 1. Problems have real purpose**
- 2. Relevant to the learner**
- 3. Mathematics adds advantage**
- 4. Problems foster discussion beyond the mathematics**
- 5. Answers are NOT always integers**

I. Authentic problems have real purpose

- **There is a real purpose for using mathematics to find an answer.**
- **Often place mathematics in a decision context**
- **“Why would one want to know the answer to this question?”** 
- **Not just descriptive**

Ratio Classic: Boy-Girl Ratio Problem

“In language class, the girl to boy ratio is 5 to 8. If there is a total of 65 students, how many girls are there?”

What are some shortcomings of this question?

“In language class, the girl to boy ratio is 5 to 8. If there is a total of 65 students, how many girls are there?”

What are some shortcomings of this question?

- **Why do I want to know the answer?**
- **The answer had to be known in order to create problem**
- **There is nothing to discuss**
- **The answer simply describes what exists**

Ratio Proposed: Field Day Problem

Twenty-four fifth graders and 16 sixth graders come to play softball on Field Day. Coach Cabrera wants the same ratio of fifth graders and sixth graders on each team. Create 4 softball teams of 10 players each.

- **How many fifth graders and sixth graders should she put on each team?**
- **Why do you think the coach wanted to keep the ratio of fifth graders and sixth graders on each team the same?**
- **How would you form six-person volleyball teams?**

How *would* you form six-person volleyball teams?

We don't want to give students the message that if the numbers don't work out nicely, you can't use mathematics!

2. Relevant to the learner

- **Context or analogous context could relate to children**
- **Issues raised would be relevant to child**
- **Answers of interest to child**

Cleaning Pools Rate Problem

Sarah can clean pools at a constant rate of $\frac{3}{5}$ pools per hour. How many pools can she clean in 10 hours?

What shortcomings does this one have?

Sarah can clean pools at a constant rate of $\frac{3}{5}$ pools per hour. How many pools can she clean in 10 hours?

- **Question posed lacks motivating context.**
- **Why do we want to know how many pools Sarah can clean in 10 hours?**
- **How many children would be interested in the rate of cleaning pools?**
- **Rate is given and not to be calculated.**
- **Is the rate likely to be constant?**

Cleaning Pools In Context

Sarah's mother, Martina runs a pool cleaning service from late spring until the end of the summer. During the first week of cleanups, Martina completed 24 pools in 40 hours. Sarah helped her mother on Saturday clean four pools in five hours. How much faster are the two together when compared to Martina working alone? Sarah hopes to work for 10 hours next weekend. How many more pools can they clean together compared to Martina working alone for 10 hours? Martina charges \$50 per cleanup. She plans to pay Sarah \$8 for each pool they clean together. Is it worthwhile for Martina to have Sarah's help for 10 hours next weekend? How much does Sarah earn per hour of work?

Cleaning Pools In Context

- **What are some features of this problem that are different from the previous pool-cleaning rate problem?**

Cleaning Pools In Context

- The problem asks to calculate an *average* rate, which is not a given.
- There is a reason to calculate the 2nd average rate and compare the two.
- A decision is embedded in the problem.
- Pool cleaning might not interest a child, but earning money might.
- There are many opportunities for discussion.

3. Mathematics adds advantage

Some standard textbook problems have no advantage:

- **Lemonade comparison → taste**
- **Ages → Ask**
- **Nickels and dimes in pocket → just count**
- **Answers had to be known to create problem**

Ratio: Which one is more lemony?

The first glass had 200 ml of lemon juice and 300ml of water. The second glass had 100 ml of lemon juice and 200ml of water. Which mixture has the stronger lemon taste? How do you know?

- **What shortcomings do you see here?**

Penny's Lemonade Stand

Penny Profit runs a lemonade stand on her street all summer

- **Weekdays open for 4 hours and averages 52 customers per day.**
- **Saturdays, open for 6 hours and averages 72 customers.**
- **Younger brother, Ben, wants to help her, but only one day per week.**
- **Jenny wants Ben to help her when she is the busiest. Should she have Ben help her on a weekday or on a Saturday?**

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- **Jenny wants Ben to help her when she is the busiest. Should she have Ben help her on a weekday or on a Saturday?**
- **What does this problem have that the other lacked?**

4. Problems foster discussion

- **Context leads to discussion beyond the math.**
- **Problem is not finished when answer is found.**
- **Solution involves not only the answer, but also mathematical reasoning to support it.**
- **Need to first understand the problem enough to generate appropriate questions.**
- **Questioning is part of the process of exploring the context.**
 - **The 5th and 6th grade teams: discuss designing balanced teams.**
 - **Organic food : general science discussion of the benefits, popularity, and the cost of organic foods.**

Event Hall Customers: Compare Ratios or Rates

Last year, Douglas County's event hall was used for 18 wedding receptions and 20 anniversary parties. This year it was used for 15 wedding receptions and 19 anniversary parties. During which year was the event hall used for a higher ratio of wedding receptions to anniversary parties?

- **Do you see any shortcomings?**

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Last year, Douglas County's event hall was used for 18 wedding receptions and 20 anniversary parties. This year it was used for 15 wedding receptions and 19 anniversary parties. During which year was the event hall used for a higher ratio of wedding receptions to anniversary parties?

- No reason for wanting to know the answer.
- Why is the ratio of one event to the other meaningful?
- Topic unlikely to be of interest to students.

Organic Grapes

In 2014, 12% of all produce sold was organic. One in eight customers surveyed is willing to pay up to 35% more for organically grown fruits and vegetables. Farmer John's Market has 40 square feet of space for all grapes. The manager wants to divide available space between organic and regular grapes in direct proportion to the responses in the survey.

- **What sorts of good questions could be asked?**

In 2014, 12% of all produce sold was organic. One in eight customers surveyed is willing to pay up to 35% more for organically grown fruits and vegetables. Farmer John's Market has 40 square feet of space for all grapes. The manager wants to divide available space between organic and regular grapes in direct proportion to the responses in the survey.

- **How many sq. ft. should be used for organic grapes?**
- **How many sq. ft. should be used for regular grapes?**
- **Can you suggest a reason why the manager might want to allocate more space than its proportion to organic grapes?**

Practice – Concerns and Revise

PARKING: 15% of the school parking spaces are handicap spaces. The school has 18 handicap spaces. How many parking spaces are there?

- **Shortcomings?**
- **How would you revise?**
- **Share**
 - **With a neighbor**
 - **With the group**

Practice – Concerns and Revise

1. Mr. Yoshi has **75** papers. He graded **60** papers, and he had a student teacher grade the rest. What percent of the papers did each person grade?
 2. Mrs. Bennett has graded **20%** of her **150** students' papers. How many papers does she still need to finish grading?
- Shortcomings?
 - Salvageable?

Need Answer – Forecast Instead of Decision

How long will it take:

- **to complete a task?**
- **to earn an amount of money?**
- **to reach your goal?**

How much work will there be:

- **at the local hospital, if 50% of the population has the flu?**

Identify a problem:

- **Which neighborhoods have highest crime rates?**

21st Century – Additional Message

- **Organic foods**
- **Compound growth of downloads of app**
- **Percentages and gerrymandering congressional districts**
- **Time travel between cities? REALLY?!?**
 - **Routing in Google maps and Waze**
- **Environmental: fuel economy or energy use**
- **Smart phone battery life**

Make Problems Engaging

- **Meaningful → Why do I care about the result?**
 - **Decision-making**
 - **Planning and allocating time or money**
 - **Not just descriptive**
- **Expand to learn about the 21st Century**
- **Something to discuss in addition to the math**
- **Imagine himself or herself in business, government, or addressing societal concerns.**
- **Latest Relevance**
 - **apps, I-pads, Google maps, twitter**
- **Things that impact them**

Message! – Challenge!

- **Mathematical concepts should be developed and presented in meaningful modern contexts.**
- **Meaningful contexts are necessarily authentic.**
- **Contexts should be wide ranging and include the 21st Century.**
- **Mathematics in meaningful scenarios can develop critical thinking skills.**
- **The key to choosing appropriate contexts is whether the *students* find them interesting and engaging.**