Catalyzing Change in High School Mathematics: Initiating Critical Conversations

NCTM

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Rethinking What Each and Every High School Student Needs Related to Algebra and Functions

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Achievement

- How can we improve mathematics achievement at grade 12? (NCES, 2013)
- How can we increase the number of high school students who are academically prepared for their next step, and increase the number of students whose next step is STEM? (ACT, 2016)

Achievement for Each and Every Student

- How can we decrease (or eliminate) the achievement gaps?
- How can we provide opportunities for students to pursue their own mathematical interests versus rushing to calculus?

Key Recommendation

- The purpose of learning mathematics and Essential Concepts
- Equitable structures
- Equitable instruction
- A common Essential Concepts Pathway

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Why Learn Mathematics?

This is a simple question, but worth considerable reflection. Because how you answer this question will strongly determine ***who* you think** should be doing mathematics, and ***how* you will teach** it.

Su, F. (2017). *Mathematics for Human Flourishing*. Presidential Address, AMS-MAA Joint Math Meetings, Atlanta, January 6, 2017.

Key Recommendation

Each and every student should learn the Essential Concepts in order to

- **expand professional opportunities,**
- **understand and critique the world, and**
- **experience the joy, wonder, and beauty of mathematics.**

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The role of technology: A shift in focus

- From learning to perform algebraic **manipulations "by hand**" to learning to recognize which techniques produce a desired outcome, to **interpret** the outcome mathematically, and to **use** the outcome to move forward in analyzing a situation or solving a problem …

- From learning many **individual procedures** for algebraic manipulations to considering multiple **equivalent forms** of expressions and equations, **interpreting** the results of manipulations, and **making strategic choices** about which forms of an expression or equation to use (p. 77)

Overview of Chapter

- Algebra
  - Modeling & Proof
- Connecting Algebra and Functions
- Flexible Understanding of Functions

Essential Concepts

- Number, **algebra and functions**, statistics and probability, geometry and measurement.
- Represent the most critical content from each domain – the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas.
- These are NOT another set of standards.

Algebra and Functions

- "… beyond specific techniques, **algebra** should be seen as a collection of unifying concepts that enable one to solve problems flexibly. " (p. 74)

- "… the notion of a **function** is broad and flexible, allowing the application of functions to many situations." (pp. 74-75)

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Essential Concepts in Algebra

- Expressions can be rewritten in equivalent forms … to make different characteristics or features visible.
- Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions ….
- The **structure of an equation or inequality** … can be purposefully analyzed …
- Expressions, equations, and inequalities can be used to analyze and make predictions, in mathematics and in contexts that arise in relation to linear, quadratic, and exponential situations.

E.C. The **structure of** an equation… can be purposefully analyzed

- Shifting the emphasis to pay attention to structure: “look before you leap”

- Solve:

  a) $3(2x-1)+10=37$

  b) $3(2x-1) = 6x$

  c) $4(x-2)=5(x-2)$

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- Solve:
  - 3(2x-1)+ 10 = 37
  - 3(2x-1) = 6x
  - 4(x-2)=5(x-2)

- $(x^2 - 5x + 5)^{(x-1)} = 1$

E.C. Equations …can be used to analyze …contexts that arise in relation to linear, quadratic, and exponential situations.

You're planning an event and find an online calculator that helps you predict sales revenue for different ticket prices. The online calculator asks you to enter information about how many people attended the event in the past (200 people) and what the ticket price was ($5.50).

The fine print on the website states they used this formula to predict the revenue R for different prices, p:

$R = (200 – 10(p – 5.50))(p)$

Ticket Price	Expected Revenue
$7	$1295
$10	$1550
$15	$1575
$18.50	$1295

E.C. Equations …can be used to analyze …contexts that arise in relation to linear, quadratic, and exponential situations.

You are curious what ticket price would give you the maximum revenue, and what the lowest ticket price is that would get you to your fundraising goal of $1400, but you aren't sure how to figure those things out from the formula on the website: $R = (200 - 10(p - 5.50))(p)$

You put the formula from the website into WolframAlpha (www.wolframalpha.com/) and it shows you these equivalent expressions:

- $R = -10p^2 + 255p + 0$
- $R = -5p(2p - 51)$
- $R = 1625.25 - 10(p - 12.75)^2$

Which formulas could help you answer your questions?

Let’s reflect on the task

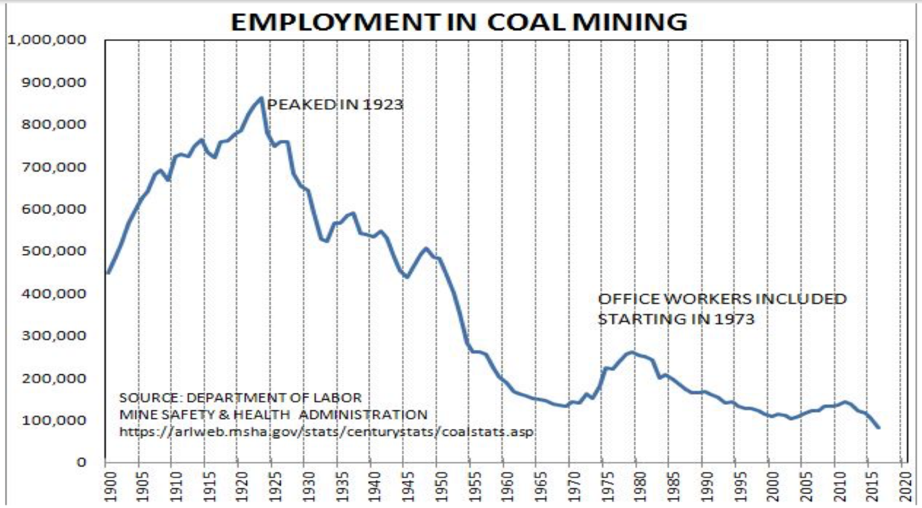
- What did you have to know to start the problem?
- Who has access to doing the mathematics?
- How could this activity affirm students’ mathematical identities?

Essential Concepts in Connecting Algebra to Functions

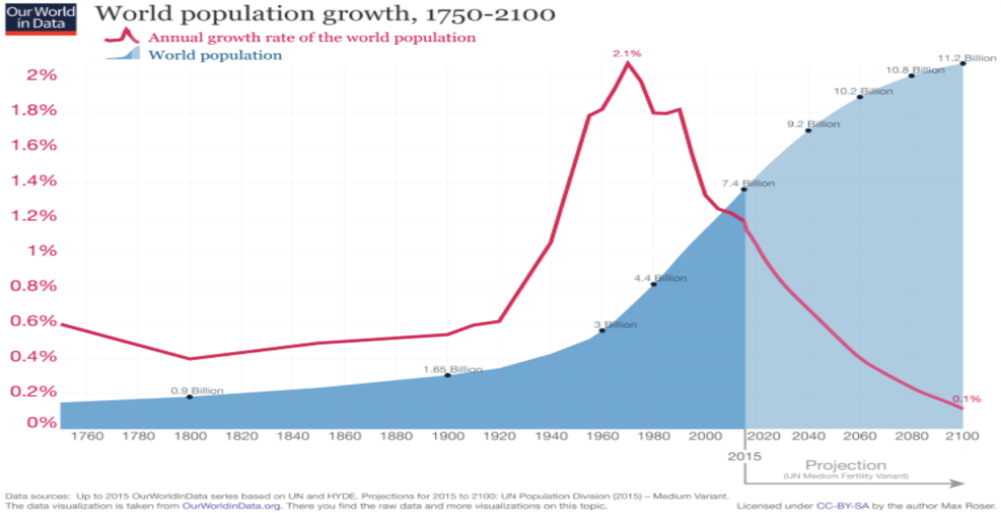
- Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.

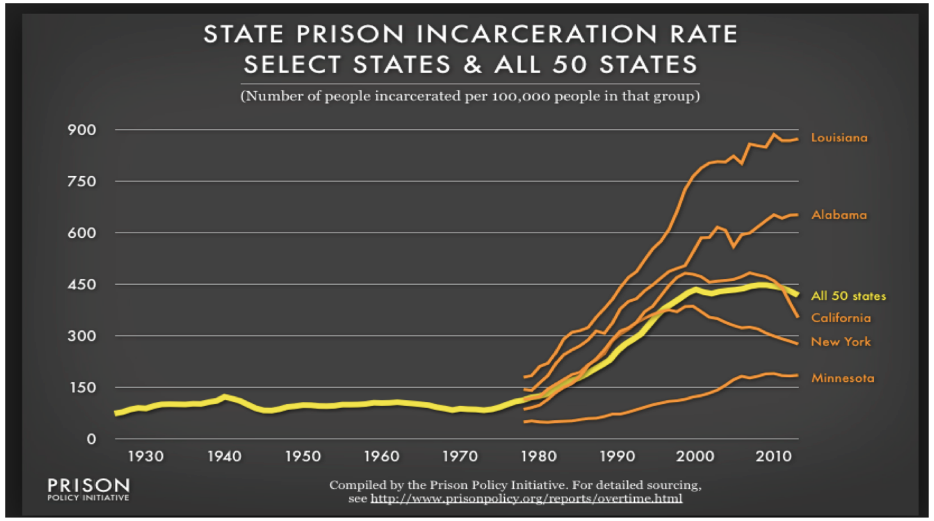
- Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).

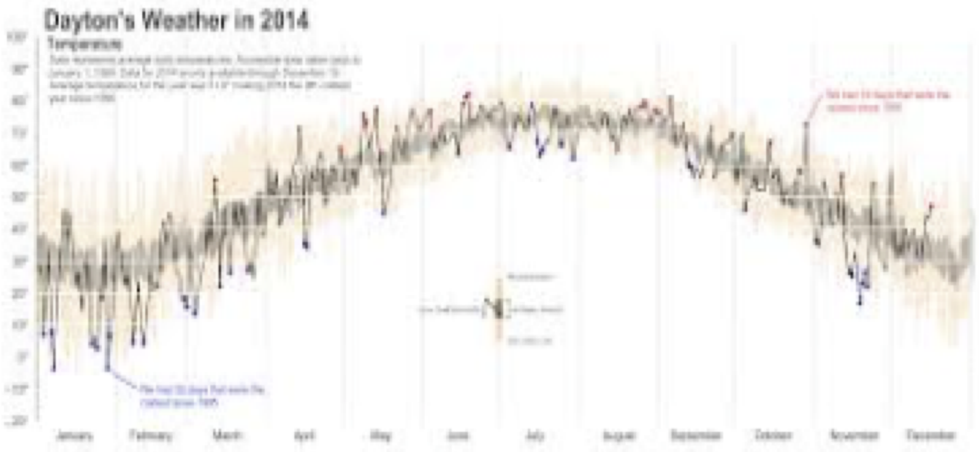
E.C. Functions model real situations …



Write three sentences to tell the story described by the function in each of the graphs.

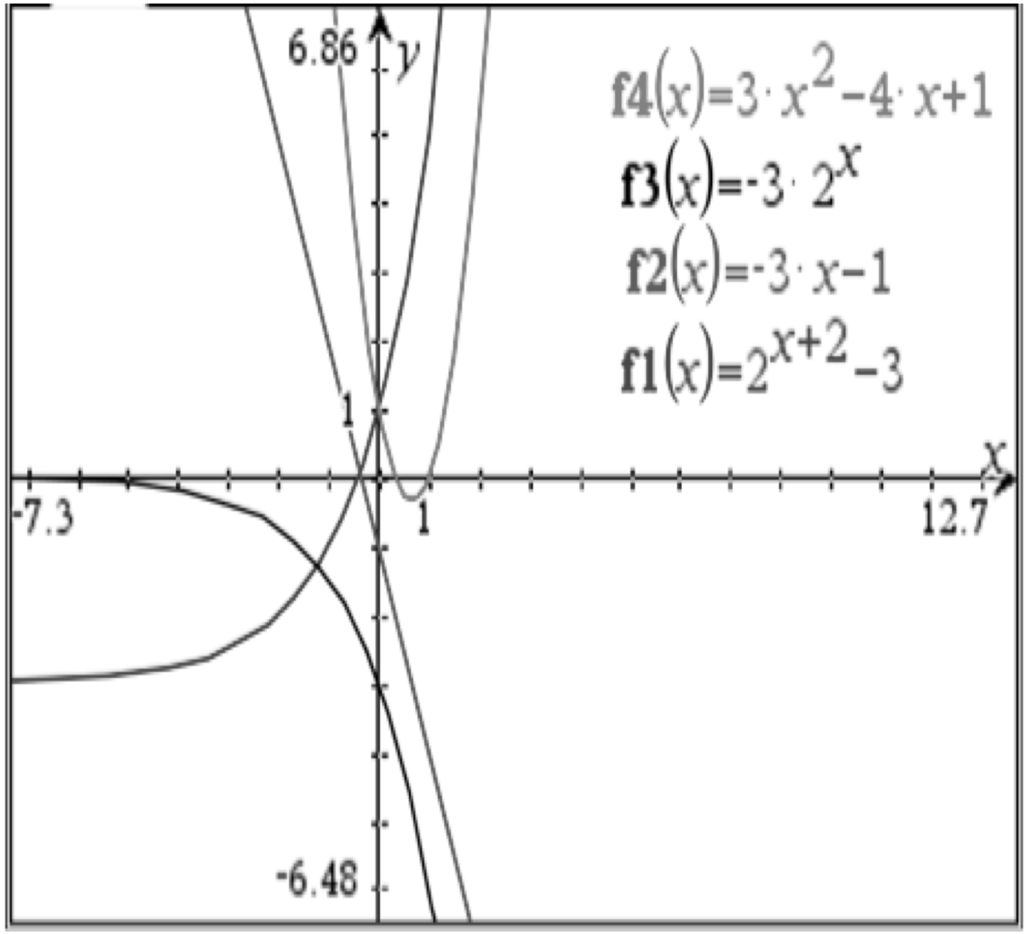






E.C. … considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.

Match the graphs, verbal descriptions and algebraic representations.



(i) The function is increasing for all values of x.

(ii) The function has an absolute minimum.

(iii) The function is decreasing for all values of x.

(iv) The rate of change $(f(x_1) - f(x_2))/(x_1 - x_2)$ is constant for all $x_1 \neq x_2$.

(v) The function has a horizontal asymptote at $y = 0$.

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Essential Concepts in Functions

- Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.

- Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.

- Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change, and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.

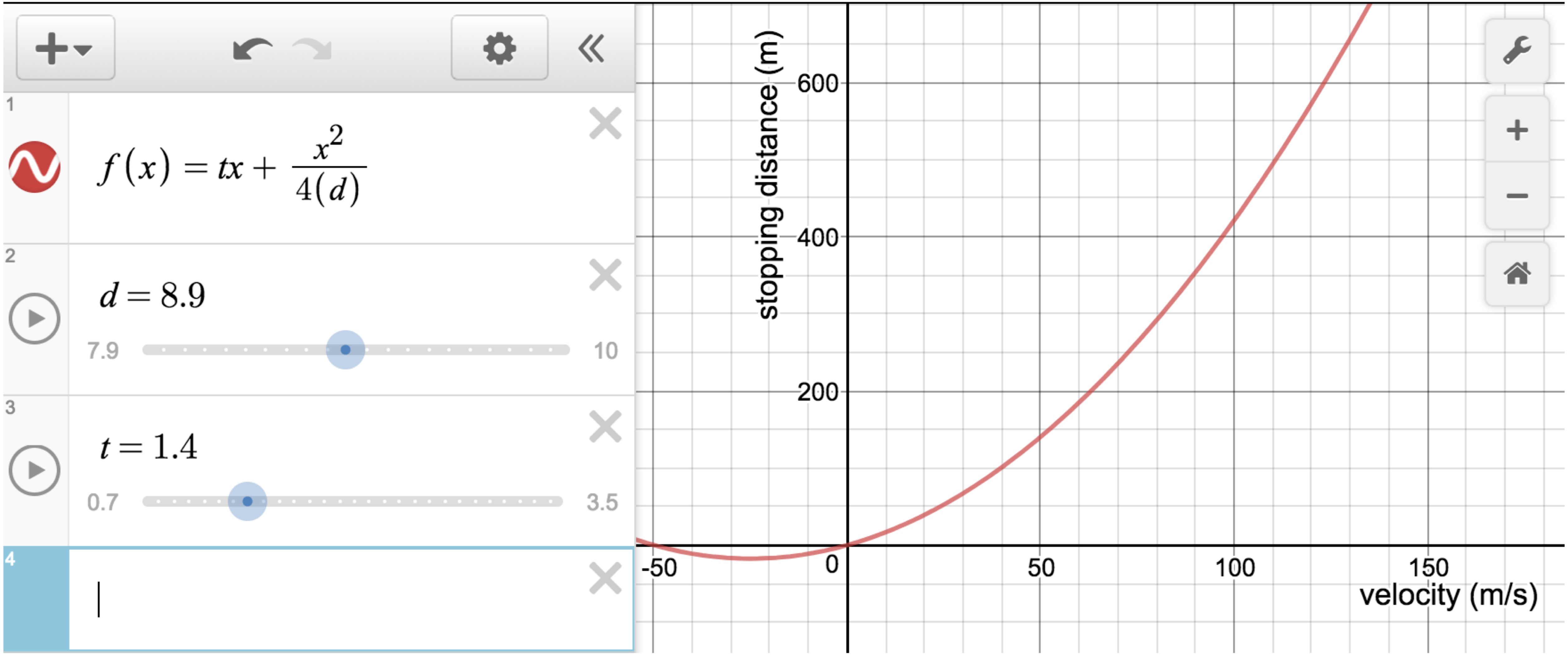
- **Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.**

Functions model real situations … Braking Distance

Braking Distance

- What questions about braking distance could we use math to answer?
- What factors affect braking distance?
- What *simplifying assumptions* could we make?

Braking Distance



Let’s reflect on the task

- What did you have to know to start the problem?
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Key Recommendation

- Each and every student learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.
- Discontinue the practice of tracking teachers and students into qualitatively different or dead-end course pathways.
- Instruction consistent with research-informed and equitable teaching practices.
- Offer continuous four-year pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts.

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Initiating Critical Conversations

- High schools are complex and good work is happening
  - What strategies are in place in your classroom, school, district, state, province that ensure all students have access to rigorous curriculum around algebra and functions?

Initiating Critical Conversations

What can you do, what conversations can you start, when you get back to work on Monday with regards to …

- the Essential Concepts and curriculum?
- professional development for teachers around equitable teaching practices?
- the role of technology?
- course pathways?
- other?

Additional Resources

- More4U (access code in book)
  - Section-specific content
  - MyNCTM