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CCSS Mathematical Practice (What STUDENTS Do)	NCTM Mathematics Teaching Practices (What TEACHERS Do)
1) Make sense of problems and persevere in solving them*	❑ Establish mathematics goals to focus learning
2) Reason abstractly and quantitatively	❑ Implement tasks that promote reasoning and problem solving
3) Construct viable arguments and critique the reasoning of others*	❑ Use and connect mathematical representations*
4) Model with mathematics*	❑ Facilitate meaningful mathematical discourse*
5) Use appropriate tools strategically	❑ Pose purposeful questions*
6) Attend to precision*	❑ Build procedural fluency from conceptual understanding
7) Look for and make use of structure	❑ Support productive struggle in learning mathematics
8) Look for and express regularity in repeated reasoning	❑ Elicit and use evidence of student thinking

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*	Graphic Supports*	Interactive Supports*	Verbal and Textual Supports
• Real-life objects (realia) or concrete objects • Physical models • Manipulatives • Pictures & photographs • Visual representations or models such as diagrams or drawings • Videos & films • Newspapers or magazines • Gestures • Physical movements • Music & songs	• Graphs • Charts • Timelines • Number lines • Graphic organizers • Graphing paper	• In a whole group • In a small group • With a partner such as *Turn-and-Talk* • In pairs as a group (first, two pairs work independently, then they form a group of four) • In triads • Cooperative learning structures such as *Think-Pair-Share* • Interactive websites or software • With a mentor or coach	• Labeling • Students' native language • Modeling • Repetitions • Paraphrasing • Summarizing • Guiding questions • Clarifying questions • Probing questions • Leveled questions such as *What? When? Where? How? Why?* • Questioning prompts & cues • Word Banks • Sentence starters • Sentence frames • Discussion frames • Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

MATHEMATICS TASK EVALUATION AND SELECTION GUIDE

Learning Goals (Mathematics AND Language):

TASK:

		EVIDENCE & NOTES
Task Potential	Try the task yourself and ask… ❑ What is problematic about the task? ❑ Is the mathematics interesting? ❑ What mathematical goals does the task address (and are they aligned to what you are seeking)? ❑ What strategies might students use? ❑ What key concepts and/or misconceptions might this task elicit?	
Problem Solving Strategies	Will the task elicit more than one problem solving strategy? ❑ Visualize ❑ Look for patterns ❑ Predict and check for reasonableness ❑ Formulate conjectures and justify claims ❑ Create a list, table, or chart ❑ Simplify or change the problem ❑ Write an equation	
Worthwhile Features	To what extent does the task have these key features: ❑ High cognitive demand ❑ Multiple entry and exit points ❑ Relevant contexts ❑ Opportunities for mathematical discourse ❑ Language development opportunities (L/S/R/W/Rep)	
Assessment	In what ways does the task provide opportunities for you to gain insights into student understanding through: ❑ Using tools or models to represent mathematics ❑ Student reflection, justification, and explanation ❑ Multiple ways to demonstrate understanding	

Adapted by. Lisa Boté from Van De Walle, J.A., Karp, K.S., Bay-Williams, J.M, (2014). *Elementary and Middle School Mathematics*: *Teaching Developmentally* (9th Ed). Boston, MA: Allyn & Bacon.

Levels of Demands

Lower-level demands (memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

Fig. 2 Characteristics of mathematical instructional tasks

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OPPORTUNITIES FOR LANGUAGE DEVELOPMENT IN MATHEMATICS LESSONS (OLDML) TOOL

ACTIVITY/TASK: _____

		LISTENING	SPEAKING	READING	WRITING	REPRESENTING
Phase 1: Before ***Engage & Launch***	*Opportunities for Language Development*					
	Language Development Supports (for different ELD levels)					
Phase 2: During ***Explore*** **(i.e., Central Task)**	*Opportunities for Language Development*					
	Language Development Supports (for different ELD levels)					
Phase 3: After ***Discuss & Summarize***	*Opportunities for Language Development*					
	Language Development Supports (for different ELD levels)					

Adapted from Aguirre, Julia M., and George C. Bunch, "What's Language Got to Do with It? Identifying Language Demands in Mathematics Instruction for English Language Learners." In Beyond Good Teaching: Advancing Mathematics Education for ELLs, edited by Sylvia Celedón-Pattichis and Nora G. Ramirez. Reston, Va.: National Council of Teachers of Mathematics, 2012.

Linguistic Supports by Domain and ELD Level

English Language Development Level	Comprehension (L/R)	Communication (S/W)
Bridging		
Expanding		
Developing		
Emerging		
Entering		