

Mathematical Modeling Goes to College: An Approach Inspired by the #MTBoS

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About me

Calling #1: I'm an active, leading
mathematical researcher, in topology.

Topologists study shapes and their interplay
through continuous functions.



About me



Calling #2: I'm devoted to the authentic practice of mathematics, by all.

<https://blogs.uoregon.edu/practiceofmathematics/>



About me



On my own campus, that has meant creating or revising

- “bridge” course to proof-based mathematics
- pre-service elementary math
- entry-level mathematical modeling.

See my MIT Electronic Mathematics Education Seminar.



About me

And I happily partner with anyone who has a similar vision, working at any level.



Think globally...


In 2012, the US President's Council of Advisors on Science and Technology, citing the negative impact of poor mathematics achievement on the STEM pipeline, made a host of recommendations, including having science professors teach math.

Think globally...



The CBMS (=AMS + NCTM + MAA +...) strongly recommends active learning at all levels.


The MAA promotes a Common Vision for both curricular and instructional reform at the college level.



Think globally...



The #MTBoS – aka math teacher twitter – popularizes student engagement through application.



Think globally...

Mathematical modeling is an ubiquitous mathematical practice, especially in physical and social science departments on college campuses, but it is hardly if at all represented in the curriculum.

Think globally...

Many states (TN, TX, OH, NH...) revising remedial – that is, non-credit-bearing – tracks. Cal State system eliminating them on a short timeline!

Act locally...

Students at the University of Oregon, and disproportionately those from underrepresented groups, too often find mathematics to be a barrier rather than an enabler.


Act locally...

Creation of a replacement for traditional non-credit-bearing math courses, in the college algebra (including STEM) track, grounded in mathematical modeling practices rather than skill remediation.

Outcome for this session



Understand the choices – curricular and instructional – for this new mathematical modeling course and reasoning for those choices.



Structure of this session



1. Introduction
 2. Activity Cycles
 3. Discussion of Course Outcomes
 4. Report of Results
 5. Comparison/ Connection with Other Approaches
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Activity Cycles

1. Task presented, including setting and curricular context.
2. Look closely at/ do activity.
3. Discuss design choices and their purposes with neighbors. Purpose includes key content, student practices and other outcomes.
4. Whole-group discussion of choices and purposes.

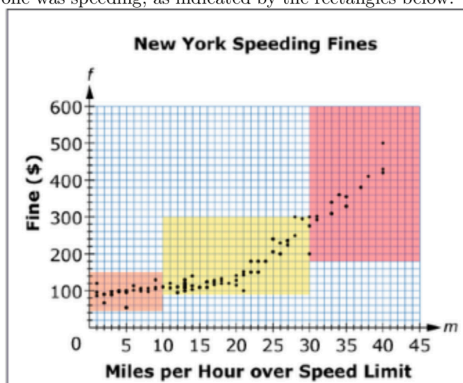
Activity – Speeding Tickets

Relatively early in course.

In-class worksheet, cycles where students work in groups, followed by whole-class discussion, along with a homework follow-up.

Activity – Speeding Tickets

The state of New York has guidelines for fines for speeding based on the range of how much one was speeding, as indicated by the rectangles below. The data points are actual fines given.



Activity – Speeding Tickets

- What do you notice? Name three features of this graph which catch your attention, and say what they mean in terms of the situation (speeding fines).
- Say in words what the current guidelines for fines are, and then translate that to mathematical notation by naming variables and writing inequalities.
- Referring to the data, argue why having a function which determines the amount of each speeding fine would be more fair than the current system.
- Find a *piecewise linear* function which fits the data well, with one linear function for under 20mph over the speed limit and another for over 20mph over the speed limit.
- Interpret each of the slopes and intercepts of the two lines you used to define the piecewise function in the previous part. Then say which of those four numbers is the least meaningful in this context.

Speeding Tickets – Choices and Purposes



Activity – Modeling Sales



Two-part “Quiz” format – individual work, with a “re-do” after prompting.

Administered after plenty of linear modeling experience, with mid-term on horizon.

Whole-class discussion follows.

15 minutes total.

Activity – Modeling Sales

Mini-Quiz

(a) A company had 1.2 million dollars in sales in 2012 and 1.8 million dollars in sales in 2016. Find a linear model for their sales as a function of time which fits these two data points, and interpret your model. Make sure to define your variables and name your function.

(b) Now consider: does it make more sense to predict and/or understand the amount of sales based on the year? Or does it make more sense to predict and/or understand what year it is based on sales?

(c) Re-read part (a) and based on your answer to (b), revise your answer to (a), if needed, below.

Modeling Sales – Choices and Purposes

- List here

Activity – Physiology Paper

After students have been using technology to fit data.

All in-class, cycles of group work followed by whole-class discussion.

Activity – Physiology Paper

β -Adrenergic or parasympathetic inhibition, heart rate and cardiac output during normoxic and acute hypoxic exercise in humans

Susan R. Hopkins, Harm J. Bogaard, Kyuichi Niizeki, Yoshiki Yamaya, Michael G. Ziegler and Peter D. Wagner

Department of Medicine, University of California, San Diego, La Jolla, CA 92093, USA

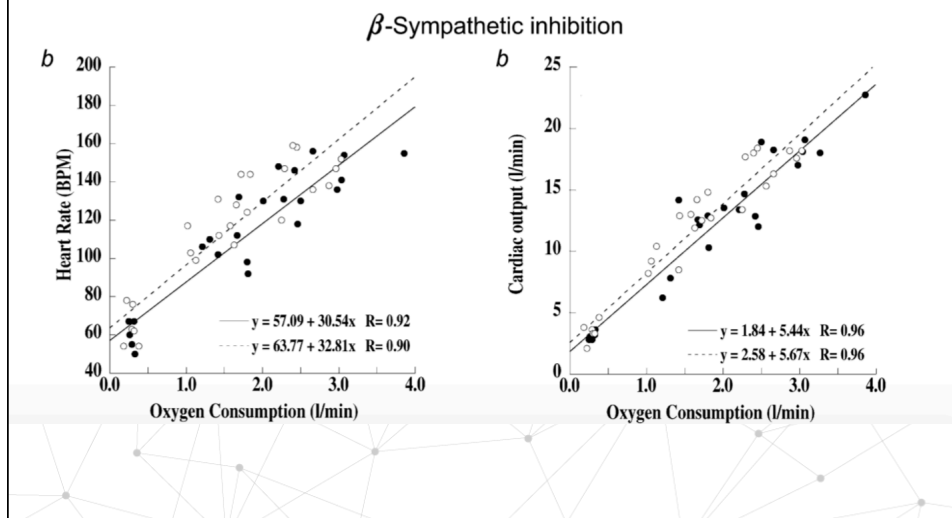
Acute hypoxia increases heart rate (HR) and cardiac output (\dot{Q}_t) at a given oxygen consumption (\dot{V}_{O_2}) during submaximal exercise. It is widely believed that the underlying mechanism involves increased sympathetic activation and circulating catecholamines acting on cardiac β receptors. Recent evidence indicating a continued role for parasympathetic modulation of HR during moderate exercise suggests that increased parasympathetic withdrawal plays a part in the increase in HR and \dot{Q}_t during hypoxic exercise. To test this, we separately blocked the β -sympathetic and parasympathetic arms of the autonomic nervous system (ANS) in six healthy subjects (five male, one female; mean \pm S.E.M. age = 31.7 ± 1.6 years, normoxic maximal \dot{V}_{O_2} ($\dot{V}_{O_{2,max}}$) = 3.1 ± 0.3 l min⁻¹) during exercise in conditions of normoxia and acute hypoxia (inspired oxygen fraction = 0.125) to $\dot{V}_{O_{2,max}}$. Data were collected on different days under the following conditions: (1) control, (2) after 8.0 mg propranolol i.v. and (3) after 0.8 mg atropine i.v. \dot{Q}_t was measured using open-circuit

Activity – Physiology Paper

Look briefly through the paper, and then focus on the figures on page 611. Note that the white/hollow data points and dashed trend line are for subjects who had less oxygen while the black/filled data points and solid trend line are for subjects under normal oxygen conditions.

- (a) What are the variables being measured? Read the METHODS section and summarize how the data is collected.
- (b) Does the data behave like a function? If so, explain why. If not, explain why and also how one could make a function from the data.

Activity – Physiology Paper



Activity – Physiology Paper

- (d) Interpret the slopes of the trend lines.
- (e) Why are the white/hollow data points and dashed trendline above the black/filled data points and solid trend line? (This is a basic physiology question.)
- (f) What does the “gap” between the two trend lines represent? (For example, what would a bigger or smaller gap mean?)
- (g) Why were the authors surprised by their interpretation that these gaps were the comparable across all conditions? What would it have meant if the gaps changed?

Physiology Paper – Choices and Purposes

- List here

Activity – Population Growth




Final exam review problem.

Students have done similar work in both worksheet and project settings.



Activity – Population Growth



4. The City Manager enters population data from the past fifteen years into a spreadsheet and uses software to find an exponential fit. The outcome is reported as $y = e^{1.0207x}$.
- (a) Rename the variables and state their units.
 - (b) Find an equivalent form which aids interpretation, and make that interpretation.
 - (c) How would you use the spreadsheet to calculate residuals?
 - (d) Explain why the city manager might care about the fitting function, and the residuals.
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Population Growth – Choices and Purposes.....→

- List here

Activity – Barbie Bungee Goes to College.....→

Culminating activity, bringing in units,
multivariable functions, and error analysis.

Scientific reports, written in standard format,
in addition to trial (of course).

Activity – Barbie Bungee Goes to College

Barbie Bungee, part II

You are working at a traveling amusement park – for toys – on the bungee jump. In part I of this project, each group found a linear model for their character's displacement as a function of the number of rubber bands. We will now pool our data and find a model which will work for any character, using that character's height and weight.

... in your report, you must also build on our in-class discussion to carefully address two aspects:

- Units: what are the units of all variables and all numerical quantities? Explain which are primary units and which are derived. Give the meaning of any derived unit.
- Error: Based on both residuals and your knowledge of the measurement process, estimate the error of your model. (Give your answer as a function of the number of rubber bands.) Compare that estimate with measurements for a new character. Use your error to find a “safe” number of rubber bands as a function of a character's height and weight.

Multivariable Bungee – Choices and Purposes

- List here

Outcomes

- List here

Target Outcomes

- Changes in student perceptions of the nature of mathematics (a relevant practice to better understand the world, to be integrated with language and “common sense” etc)
- Changes in student perceptions of math learning and themselves as math learners.

Target Outcomes

- Understand how functions relate quantities, including “measurement functions.”
- Understand function notation.
- Build mathematical models and calculate residuals, including with technology.
- Use successive value analysis to recognize linear and exponential models

A Function Definition

A function is a way someone, possibly with the aid of technology, takes a collection of numbers and produce a number.

In a measurement function, the input and output numbers are obtained by measurement.

A simple functional model is an approximate equality between a measurement function and a mathematical function, with level of approximation specified.

Plans

Three trials so far, run by two instructors, producing activities, notes, and instructional experience.

Fourth trial by third instructor in progress. Trending “more basic.”

Campus curriculum committee approval.

Plan to eliminate non-credit-bearing mathematics courses and replace in Fall 2018, with ten or so instructors and hundreds of students.

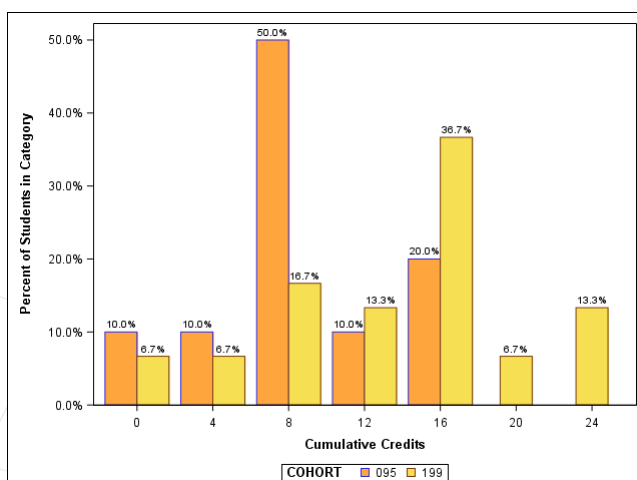
Results

Preliminary data is promising (3 trials, 72 students total).

Over 80% success rate (A/B/C/P) (vs. historical 65% for remedial math).

In first two trials, for students who took college algebra next term, a 75% success rate (in line with average).

Results – first cohort in Chem/ Math track



Compare contrast with other approaches for this population.

Historically, we were not “Do(ing) no harm”!

- Skill-based models (old and new – eg. emporium)
- “Stretch” college algebra
- Multiple pathways

A purely intellectual point

Mathematical modeling is a fundamental intellectual practice which has been severely underrepresented in college curricula.

We should remedy that, with modeling opportunities at all levels.

Thank you