Proof in Geometry Using Transformations

The Common Core Mathematics Standards, and many individual state standards, now define congruence and similarity in terms of transformations and encourage teachers to integrate reasoning with transformations in proof arguments. What does that mean? This session will discuss the rationale and potential advantages for this approach using several illustrative examples.

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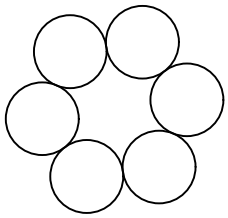
Introductory Task

Suppose you have a piece of string and a pile of beads of two different colors (red and white). If you put six beads on the string and then tie it off to create a necklace, how many different designs can you make with the two different color beads? Make sure you can justify your answer.

Note: Assume that, once the necklace is created, you cannot tell where it is tied. That is, there is no "beginning" to the necklace.



Properties of Rigid Transformations

8.G.A.1. Verify experimentally the properties of reflections, rotations, and translations.

- Points are mapped to points, lines are mapped to lines, line segments are mapped to line segments, etc. (***geometric "type" is preserved***)
- The distance between two points is the same as the distance between their images. (**distance/*length is preserved***)
- Angles are mapped to angles with the same measure. (***angle measure is preserved***)
- Parallel lines are taken to parallel lines.

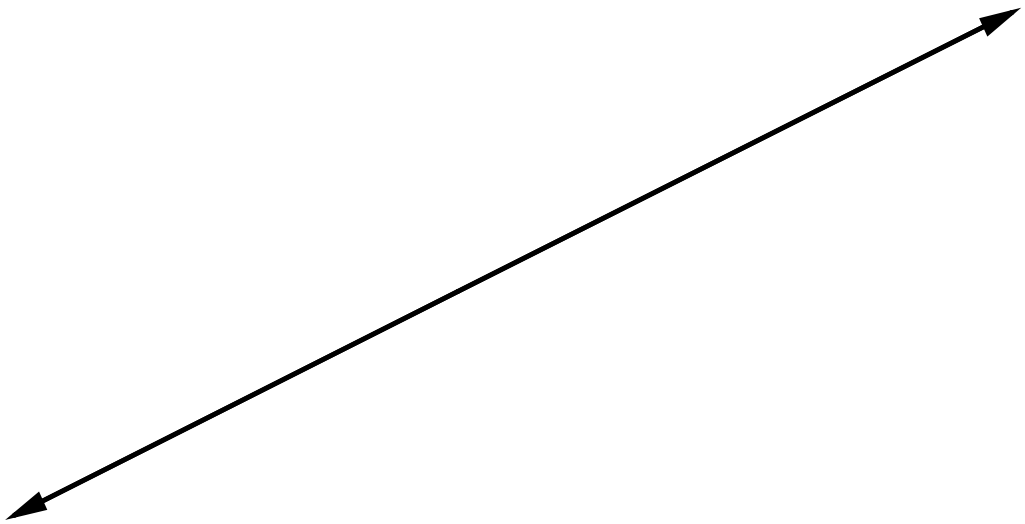
Congruence and Congruent

A **congruence** is a composition of a finite number of rigid transformations of the plane.

Two figures are said to be **congruent** if …

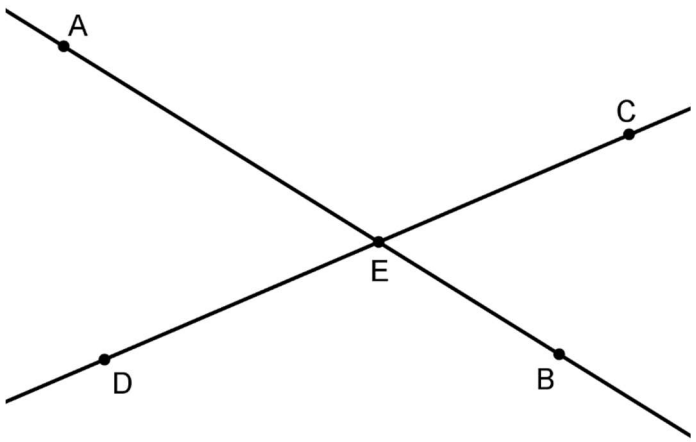
180° Rotations

9. What happens to a line in the plane when we perform a 180° rotation centered at a point?
   a. Before doing anything else, make a conjecture.
   b. Explore the question and verify or revise your conjecture(s).
   c. Convince us that you are right!



Vertical Angles

When two lines intersect at a point they form two pairs of **vertical angles** with vertices at the intersection point. Vertical angles are opposite one another (non-adjacent).



In the given diagram $\angle AED$ and $\angle CEB$ form one pair of vertical angles and $\angle AEC$ and $\angle BED$ form the other pair of vertical angles.

Examine the diagram carefully, then complete the following.

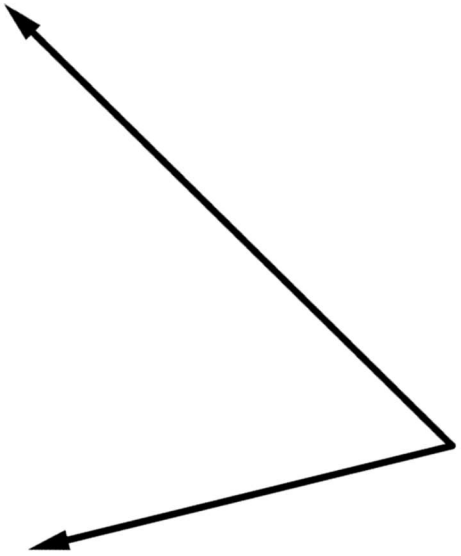
a. How are the measures of vertical angles related to each other? Make a conjecture.

b. Rotate the plane 180° with E as the center of rotation. How can you use this transformation to justify your conjecture in part (a)?

A Triangle with Two Congruent Angles

I want to create a triangle that has exactly two interior angles with the same measure. What must be the properties of those angles? Discuss with a partner and then create an argument to convince us that you're right!

Use one or more transformations to create a triangle with two angles that have the same measure as the given angle.



List every property of the triangle you've created and justify each conclusion using ***ONLY*** the definitions of transformations of the plane, properties of rigid transformations, or anything else we've discussed so far today.

Why does the previous activity provide a proof for all of the stated properties for ***any triangle with two angles that have the same measure***? [In other words, why are the conclusions valid for all such triangles and not just for the triangle we drew?]

Constructing a Perpendicular Line

Constructing a Kite