

Proof in Secondary Classrooms Project (PISC) – PI: Michelle Cirillo, University of Delaware
NCTM Annual Meeting April 27, 2018 8:00 AM – 9:00 AM

Coordinating Geometric Modalities

When engaging in proof students translate ideas among diagrams, geometric notation, and language. The following task provides students an opportunity to coordinate geometric modalities prior to doing proof.

Student Task

Write a verbal statement for the following geometric notation. Then sketch, label, and mark a diagram with the given features.

- $\overline{KL} \perp \overline{PQ}$ and $\overline{KL}$ bisects $\overline{PQ}$

Defining

Students who do not know their geometric definitions are unlikely to be successful in doing proof. The following task provides students opportunities to engage with other students' reasoning and revise a definition.

Student Task

Emma and Jake are arguing over who has the better definition of complementary angles. Read their argument. Who do you agree with? Justify who has the better definition or write a new definition of your own.

Emma: Complementary angles add up to 90° .

Jake: Complementary angles are two angles, or a pair of angles, that may or may not be adjacent whose sum, or measures add, to 90° or form a right angle.

Emma: My definition is better, because it doesn't have so many words. Your definition has too much stuff in it.

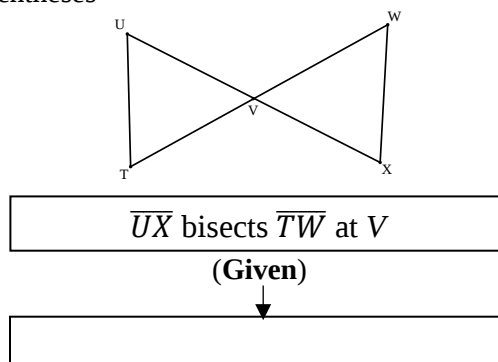
Jake: Well my argument includes what the geometric object is and what's special about that object.

Drawing Conclusions

The following task supports students in using definitions to draw valid conclusions prior to attempting to reason through full proofs.

Student Task

- Complete the flow chart by drawing a conclusion from the "Given" statement. Include the reason for your conclusion in parentheses



Starting Proof off Right: Engaging Students with Research-Based Strategies in Geometry

Common Sub-Arguments (PISC Project; Cirillo, 2018)

Learning Common Sub-arguments supports students in remembering shorter chains of reasoning that frequently appear in geometry proofs. In this sense, students are engaging in repeated reasoning.

GIVEN	DIAGRAM	A SUB-ARGUMENT AND RESULTING CONCLUSION	
A line segment bisects another line segment. e.g., $\overline{AB}$ bisects $\overline{DE}$		$\overline{AB}$ bisects $\overline{DE}$ C is the midpoint of $\overline{DE}$ $\overline{DC} \cong \overline{EC}$	- Given <i>Definition of Line Segment Bisector</i> – The bisector of a line segment is any line or subset of a line that intersects the segment at its midpoint. <i>Definition of Midpoint</i> – The midpoint of a line segment is the point of that line segment that divides the segment into two congruent segments.
Two lines are perpendicular e.g., $\overline{BD} \perp \overline{AC}$		$\overline{BD} \perp \overline{AC}$ $\angle BDA$ and $\angle BDC$ are right angles $\angle BDA \cong \angle BDC$	- Given <i>Definition of Perpendicular Lines</i> – Perpendicular lines are two lines that intersect to form right angles. <i>Theorem</i>. If two angles are right angles, then they are congruent.
Two lines intersect e.g., $\overline{ABD}$ and $\overline{CBE}$		$\overline{ABD}$ and $\overline{CBE}$ $\angle ABD$ and $\angle CBE$ are vertical angles $\angle ABD \cong \angle CBE$	- Given <i>Definition of Vertical Angles</i> – Vertical angles are two angles in which the sides of one angle are opposite rays to the sides of the second angle. <i>Theorem</i>. If two angles are vertical angles, then they are congruent.
Two lines are parallel e.g., $\overline{AB} \parallel \overline{DC}$		$\overline{AB} \parallel \overline{DC}$ $\angle 1$ and $\angle 2$ are alternate interior angles $\angle 1 \cong \angle 2$	- Given <i>Definition of Alternate Interior Angles</i> – Alternate interior angles are a pair of interior angles on opposite sides of the transversal, not sharing a common vertex. <i>Theorem</i>. If two parallel lines are cut by a transversal, then the alternate interior angles formed are congruent.
A straight angle and a ray whose endpoint is the vertex of the straight angle are drawn. e.g., $\overline{ABC}$ and $\overline{BD}$		$\overline{ABC}$ and $\overline{BD}$ $\angle 1$ and $\angle 2$ form a linear pair $\angle 1$ and $\angle 2$ are supplementary $m\angle 1 + m\angle 2 = 180^\circ$	- Given <i>Definition of Linear Pair</i> – A linear pair of angles are two adjacent angles whose sum is a straight angle. <i>Theorem</i>. If two angles form a linear pair then they are supplementary <i>Definition of Supplementary Angles</i> – Supplementary angles are two angles the sum of whose degree measures is 180.
Definition Source: Keenan, E. P., & Dressler, I. (1990). <i>Integrated mathematics: Course II</i> (2nd ed.). New York: Amsco School Publications, Inc.			

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