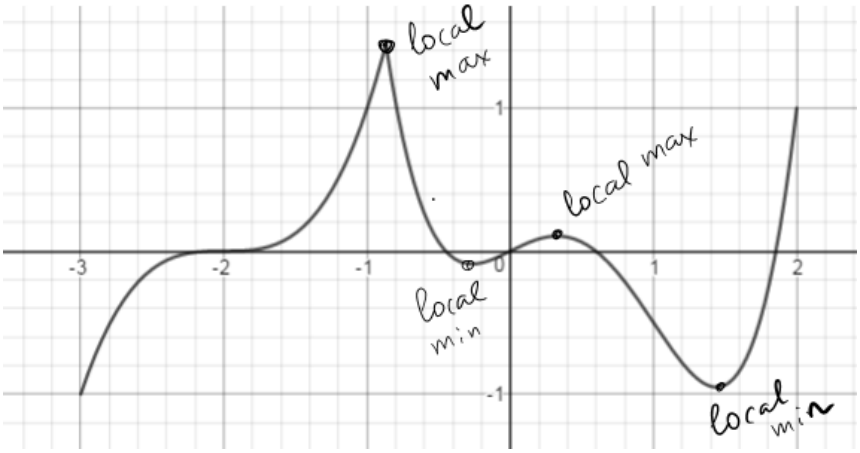
Calculus: Critical Points and Extrema

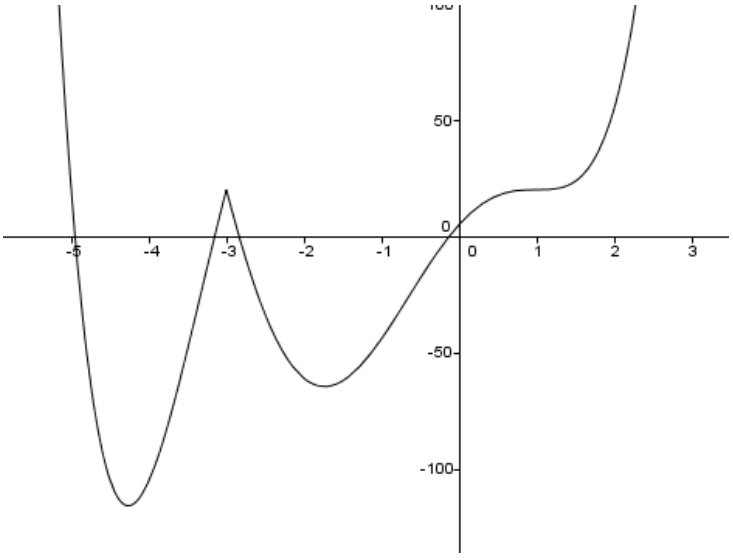
Introduction: Consider the following function on the interval [-3, 2].
Using the picture below, define what is a local maximum or minimum. It is also called **relative** maximum or minimum.



Before moving forward, make sure your definition is correct.
Note: endpoints are not considered local extrema. Make sure that your definition excludes endpoints.

A maximum or minimum of a function is called **extremum.**

Problem 1: Consider the function pictured below.



On the graph, find and label all points of local extremum.

Find (approximately) all values of x where f'(x)=0.

Find all values of x where f'(x) does not exist.

Find all intervals where f' is positive.

Find all intervals where f' is negative.

At a point of maximum, the graph goes up and then down. What must be true about f'(x) at a point of local minimum?

What must be true about f'(x) at a point of local minimum?

What must be true about f'(x) at a point of local extremum?

Is it possible that f'(x) = 0, but f(x) is not an extremum? Use the example above to explain.

Is it possible that f(x) is an extremum, but f'(x) $\neq$ 0? Use the example above to explain.

Problem 2: Given a function $f(x) = x^3 + 2x^2 + x$ find all local extrema analytically (i.e. without graphing). Specify which ones are minima and which ones are maxima.

(Hint: what must be true about f'(x) at a point of extremum?)

Check your answer by graphing.
If the graph does not match your answer, do not move on until you understand why.

Definition: Let f be defined at c. If f'(c)=0 or if f is not differentiable at c, then c is a **critical number** of f.

In the above examples find all critical numbers.

Outline a step-by step process for finding local extrema and determining which ones are minima and which ones are maxima. Use the term **critical number**.

This process is called **The First Derivative Test**
Make sure you got it right before moving on.

Problem 3: Find all local minima and maxima for $f(x) = x(x-2)^3$
Use the first derivative test to find and classify all local extrema. Follow the step-by-step process you outlined earlier. It is ok to use technology to simplify, factor, or solve an equation, but do not graph the function until you are done.