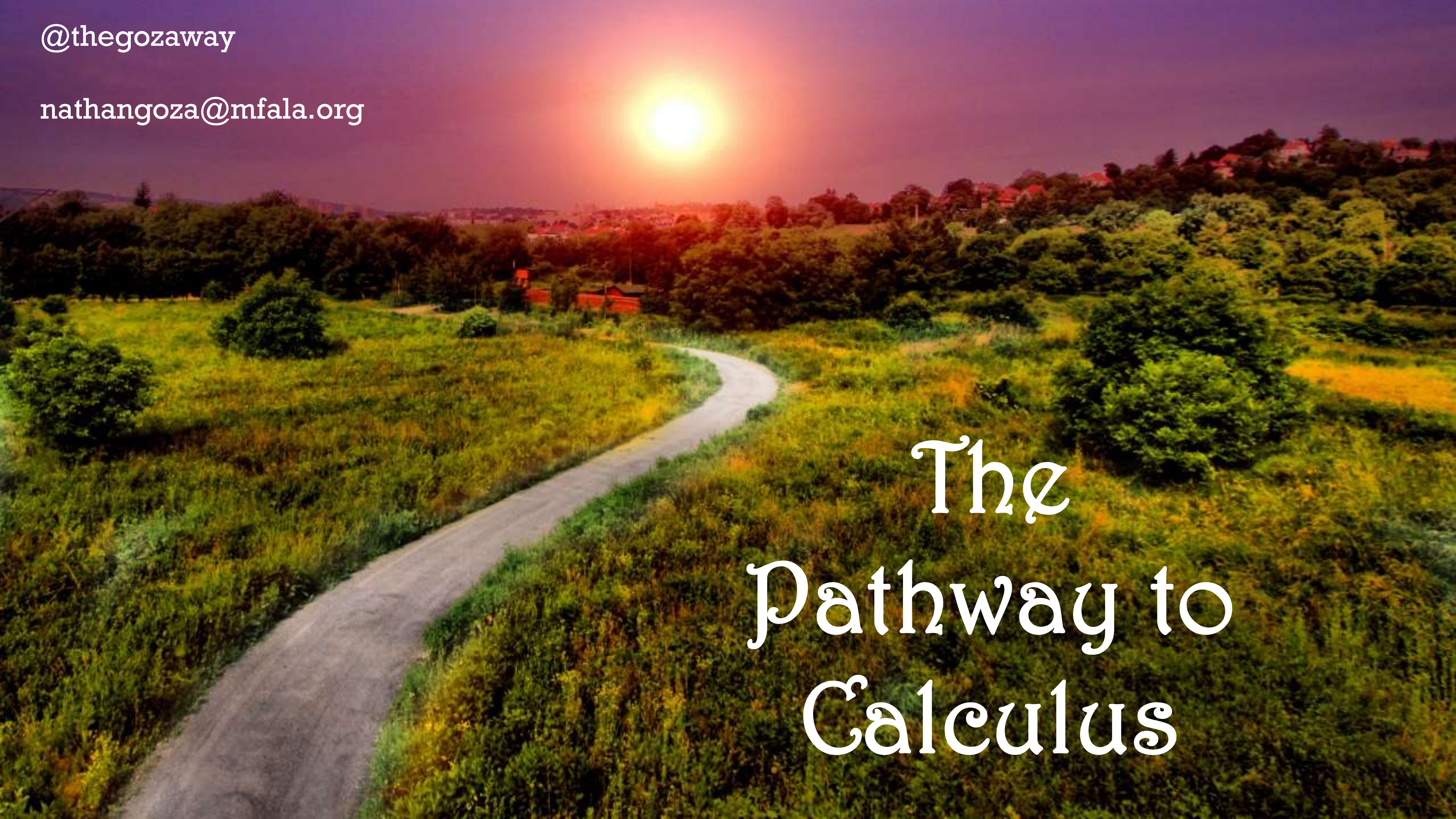


@thegozaway

nathangoza@mfala.org

The Pathway to Calculus

Who is this "we" you speak of?

Liem'nNate

@thegozaway

nathangoza@mfala.org

Nate Goza
&
Liem Tran

- **Attended UCLA for Undergraduate and Graduate School**
- **Teachers at Orthopaedic Medical Magnet High School in South Los Angeles**
- **Master Teacher Fellows for Math for America Los Angeles**
  - **Project Title**: ***Creating and Implementing a Pathway to Calculus via Common-Core Aligned Curriculum***
  - ***Grant provides monthly professional development as well as a common planning period that allows us to work on this project. (For 5 years)***

Math for America
Los Angeles

Year	Taking AB	Passing AB	Rate	Taking BC	Passing BC	Rate
2008	30	1	Bad			
2009	25	2	Low			
2010	15	1	Sheesh			
2011	18	10	56%			
2012	26	8	31%			
2013	31	23	74%			
2014	35	20	57%			
2015	36	24	66%	12	5	42%
2016	34	27	79%	21	13	62%
2017	52	34	65%	25	17	68%
2018	60			32		

Year	Taking AB	Passing AB	Rate	Taking BC	Passing BC	Rate
2008	30	1	Bad			
2009	25	2	Low			
2010	15	1	Sheesh			
2011	18	10	56%	Access??		
2012	26	8	31%			
2013	31	23	74%			
2014	35	20	57%			
2015	36	24	66%	12	5	42%
2016	34	27	79%	21	13	62%
2017	52	34	65%	25	17	68%
2018	60			32		

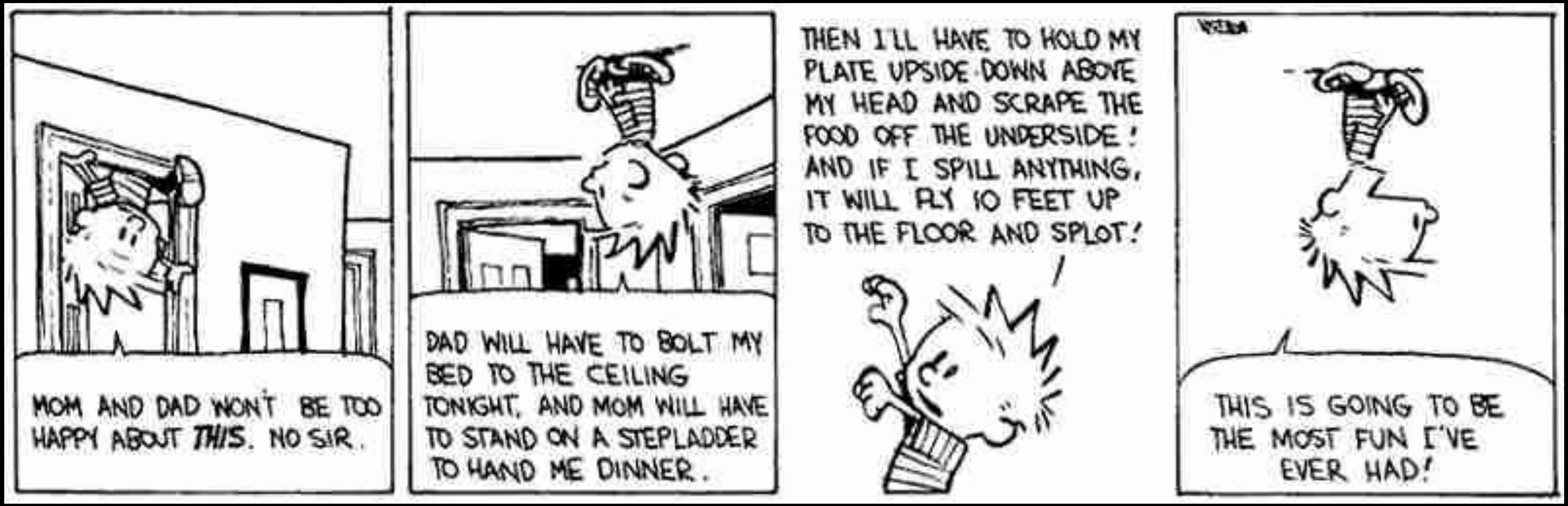
Goals:

Bring as many students as possible "on board."

Increase students' ability to think and reason mathematically.

Reduce the number of times Calculus students get stuck on non-Calculus.

We think Calculus is kinda Ceiling-y



If we prepare students the right way ahead of time, they can be comfortable on the Ceiling.

What We Want to Share

The Curriculum as a Whole

The Individual Tasks that are Highlights

The Guiding Principals used to Create the Curriculum

The Strategies that we use to Implement the Curriculum

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Year 1: The Story (and stories) of Algebra

Year 2: The Bridge to Calculus (aka Turning Up the Math)

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For Us

Year 1: Algebra 1

Year 2: Honors Advanced Math

Year 1: The Story (and stories) of Algebra

Year 2: The Bridge to Calculus (aka Turning Up the Math)

Could be:

Year 1: Algebra, Year 2: Algebra 2

Year 1: Algebra 1, Year 2: Pre-Calc

Year 1: Algebra 2, Year 2: Pre-Calc

Year 1: The Story (and stories) of Algebra

A Function-Based, Scenario Filled Approach

“Numbers are Useful & Important”

Year 1: Algebra 1

0) **Numbers (Origins and Number Sense)**
1) Relations & Functions
2) Linear Expressions & Functions (growing by a constant amount)
3) Arithmetic Sequences & Series
4) Exponential Expressions & Functions (growing by a constant factor)
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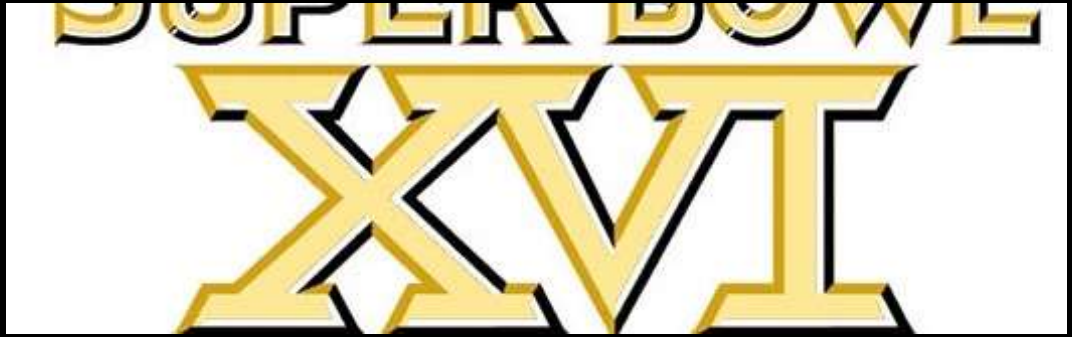
Which One Doesn’t Belong



100 50 37 5

How would you make:

1) 32
2) 94
3) 44
4) 103
5) 156

"Sometimes Sets of Numbers are Related to Each Other"

Approximate Years of Education (After High School)	Median Yearly Income ($)
0*	34,736
1	38,532
2	41,184
4	57,252
6	68,952
8	85,228

Year 1: Algebra 1

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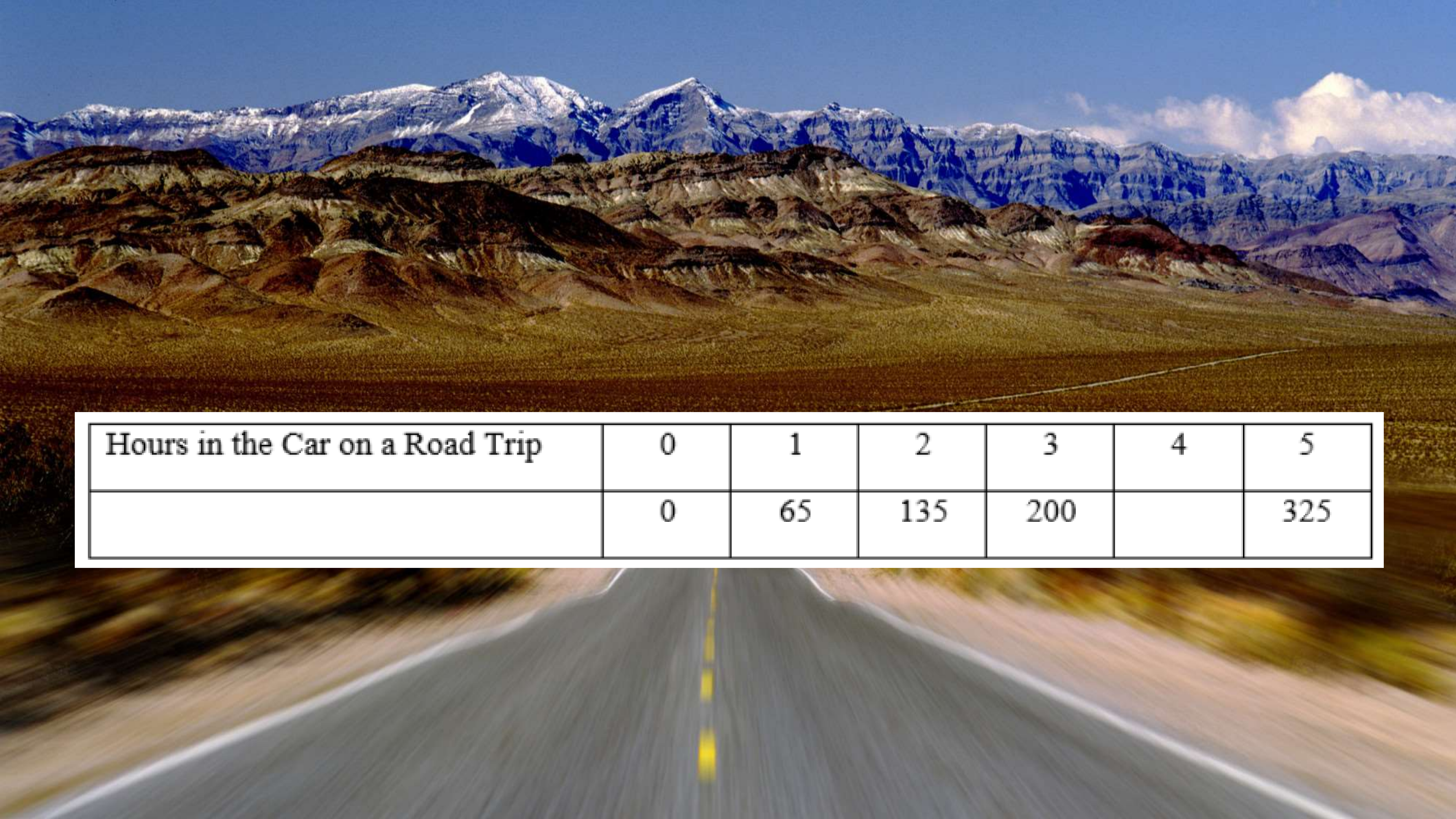
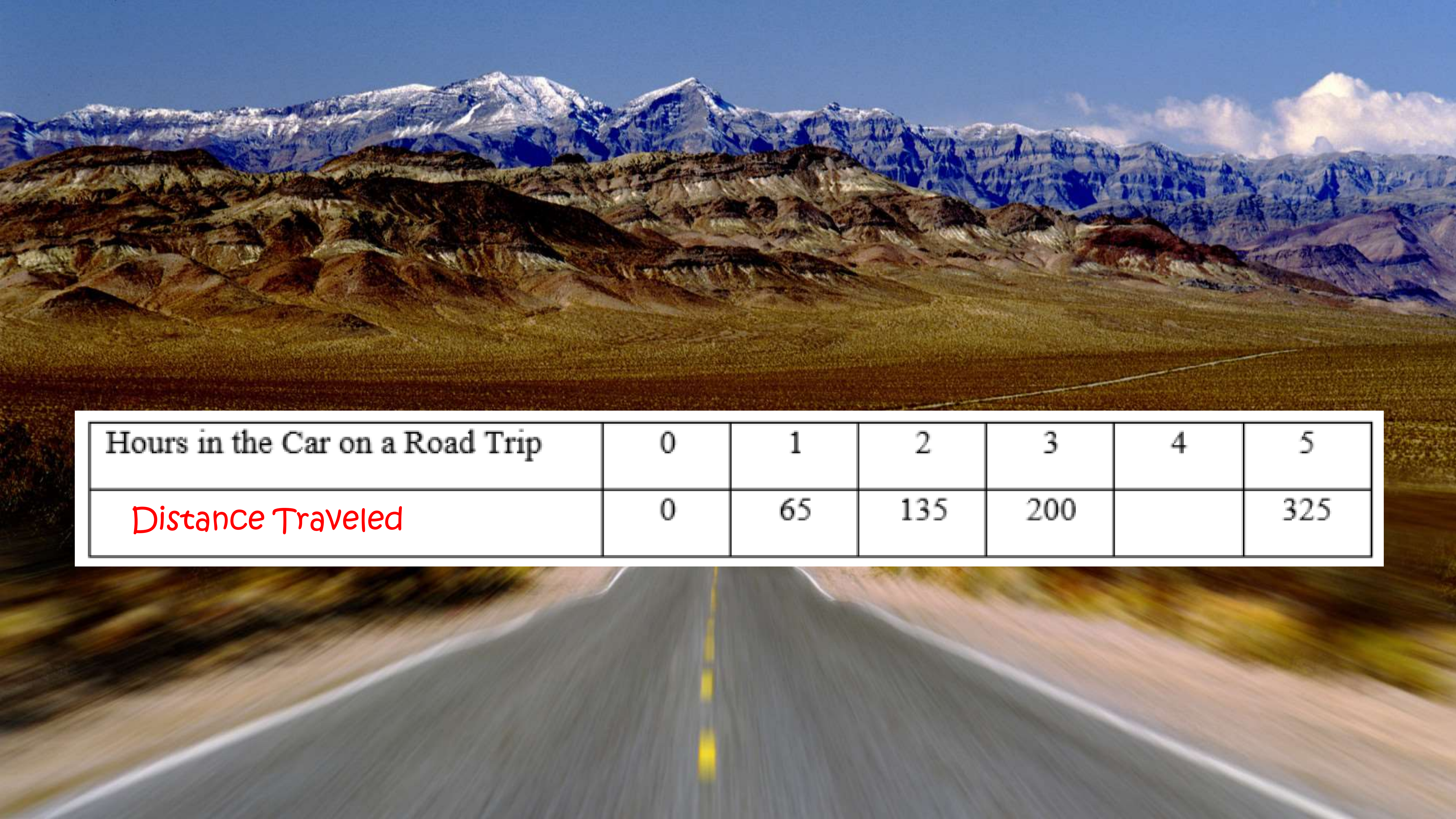
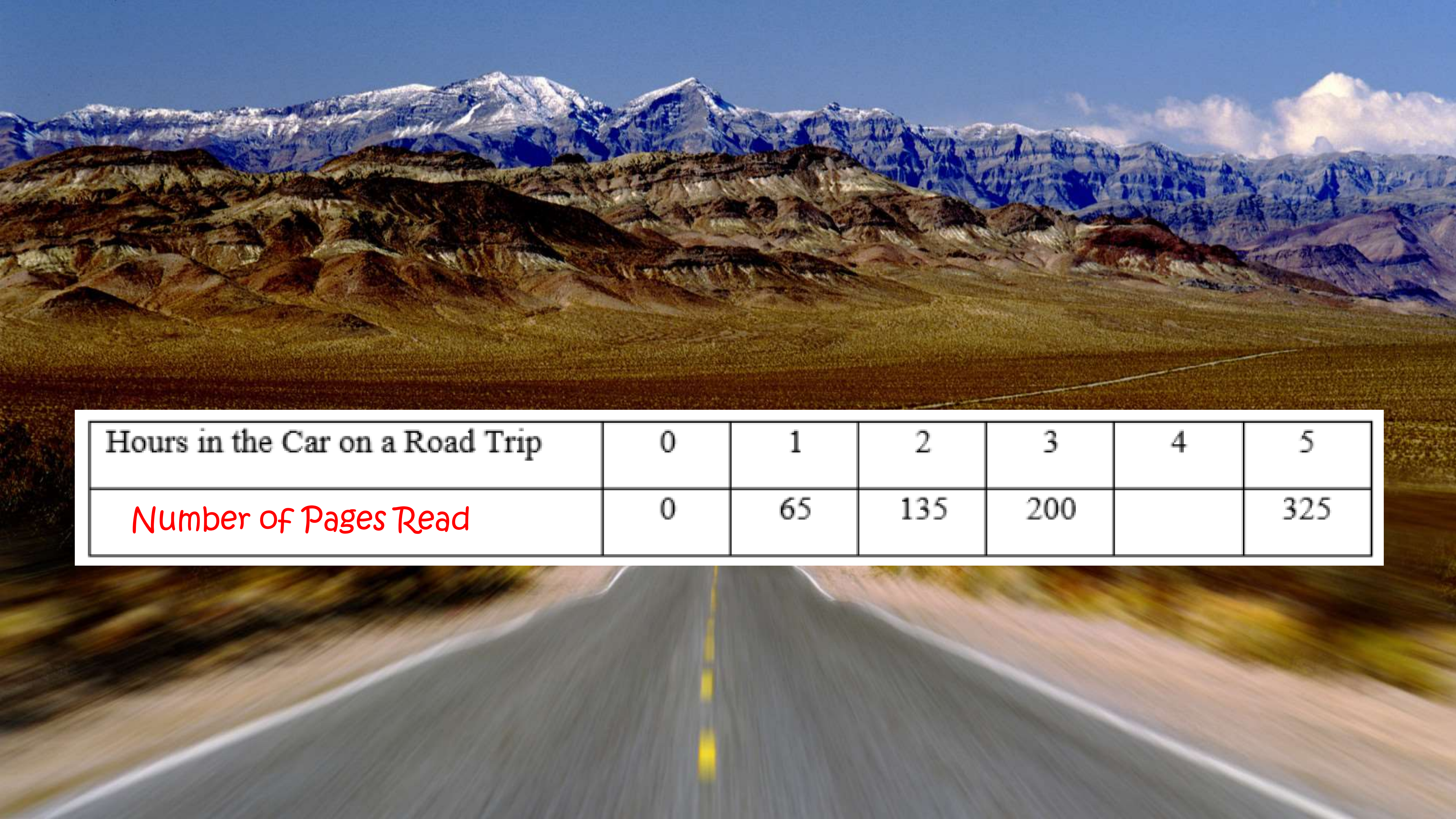


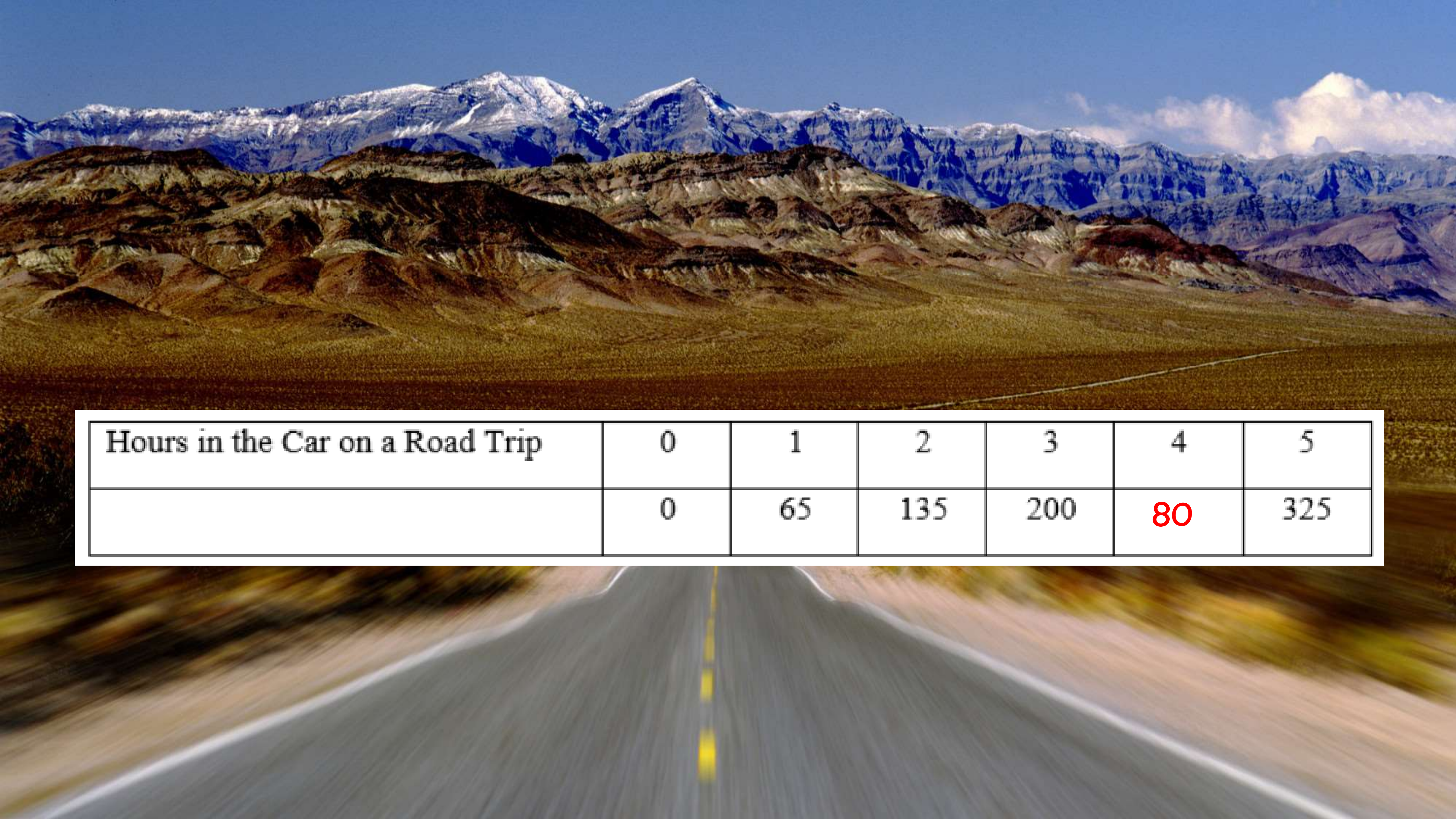
Hours in the Car on a Road Trip	0	1	2	3	4	5
	0	65	135	200		325



Hours in the Car on a Road Trip	0	1	2	3	4	5
Distance Traveled	0	65	135	200		325



Hours in the Car on a Road Trip	0	1	2	3	4	5
Number of Pages Read	0	65	135	200		325



Hours in the Car on a Road Trip	0	1	2	3	4	5
	0	65	135	200	80	325

Hours in the Car on a Road Trip	0	1	2	3	4	5
Miles per hour	0	65	135	200		325

	1	2		3	4	5	6	7	8	9	10	11	12
	31	28	29	31	30	31	30	31	31	30	31	30	31

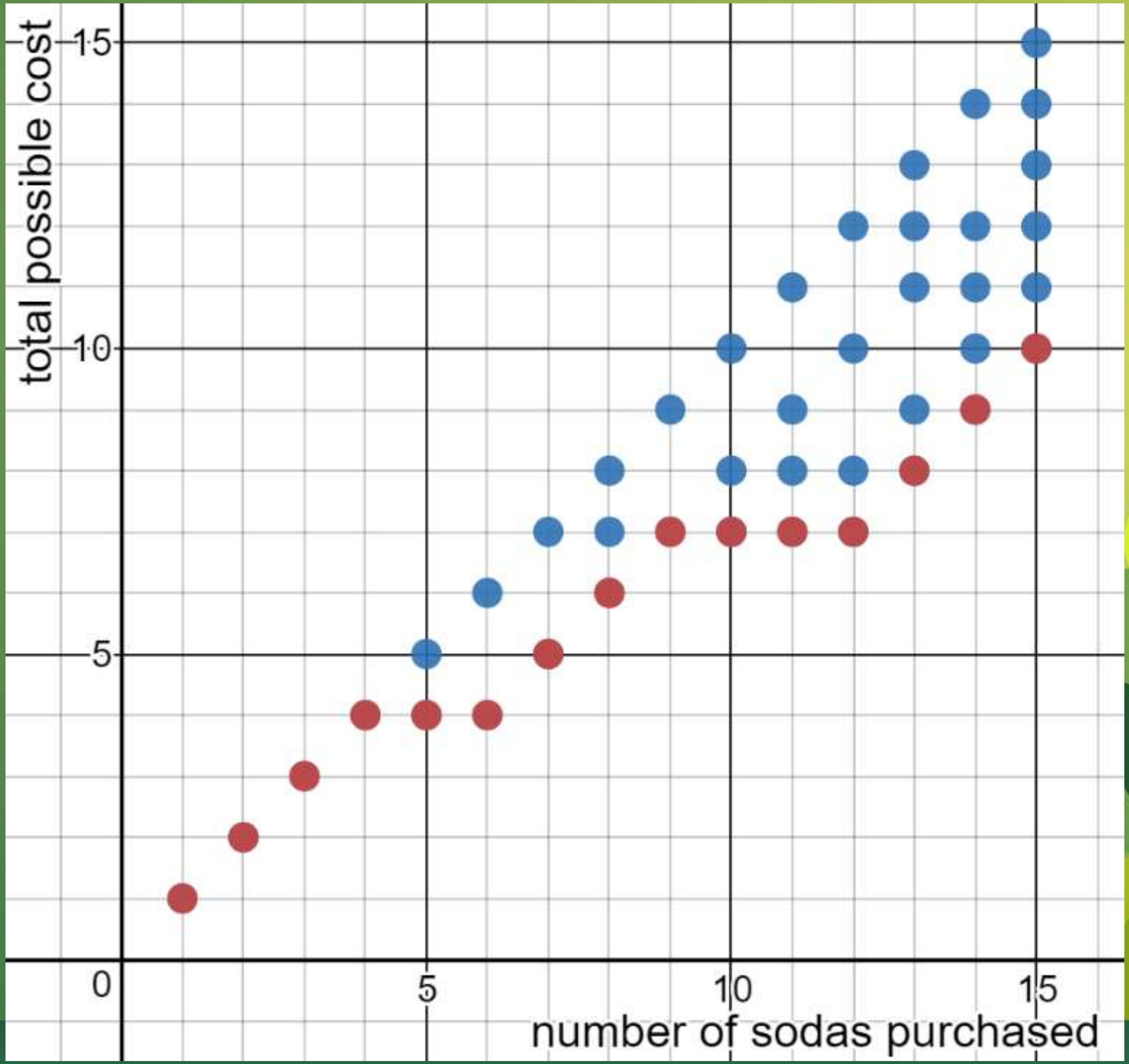
Month in the Year	1	2		3	4	5	6	7	8	9	10	11	12
Days in the Month	31	28	29	31	30	31	30	31	31	30	31	30	31

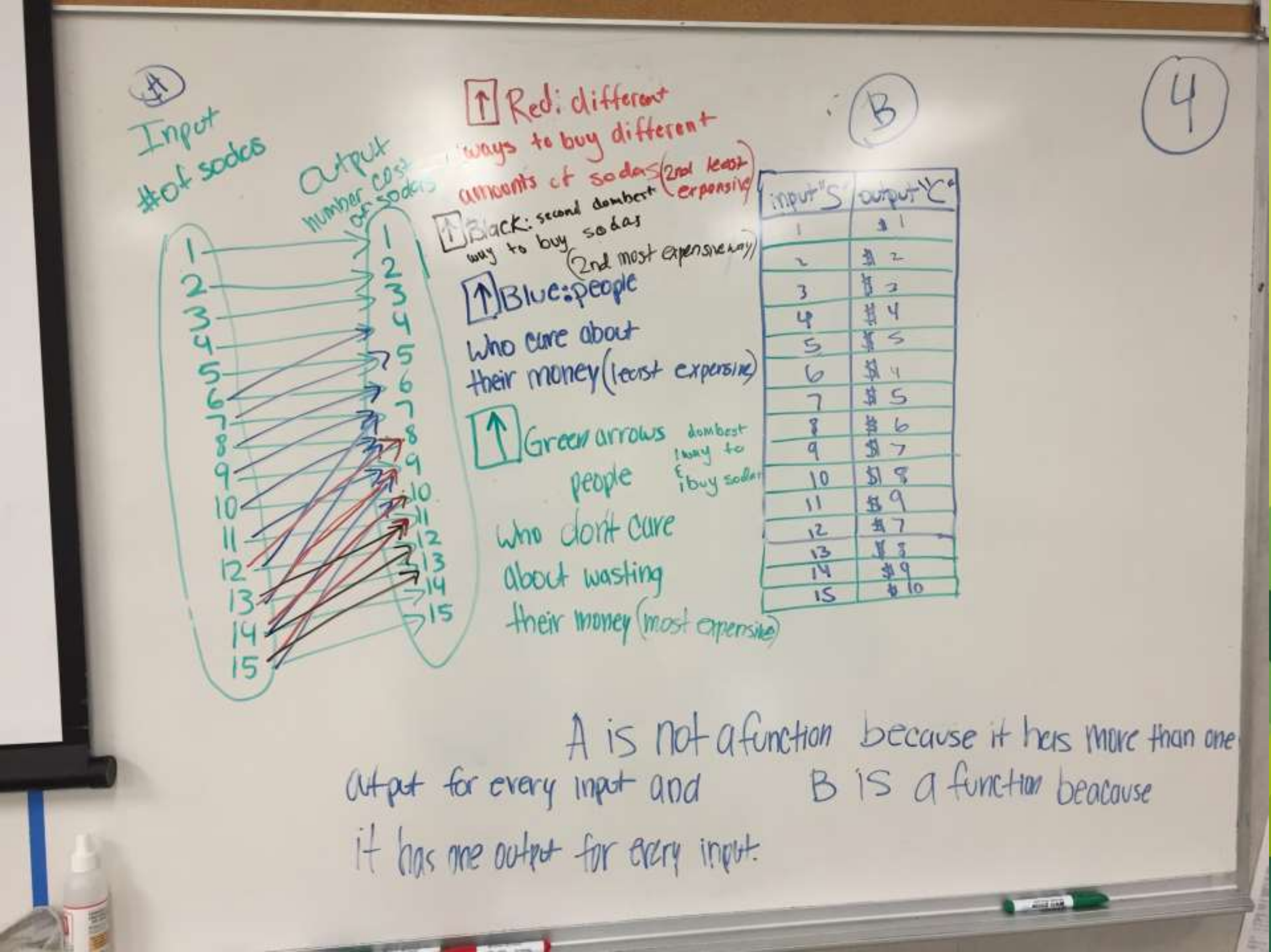
1.6: Mini Mart Madness aka A Mountain Doozey

A local Mini-Mart sells sodas in different ways:

Individual Sodas	**$1**
Six-Packs	**$4**
Twelve-Packs	**$7**

What is the lowest total cost of 13 sodas?
What are the other possible costs?


total possible cost
15
10
5
0
5
10
15
number of sodas purchased


4
A
Input
#of sodas
Output
number cost of sodas
Red: different ways to buy different amounts of sodas (2nd least expensive)
Black: second dombest way to buy sodas (2nd most expensive way)
Blue: people who care about their money (least expensive)
Green arrows dombest Iway to buy sodas people who don't care about wasting their money (most expensive)
B
input "S" | output "C"
1 | $1
2 | $2
3 | $3
4 | $4
5 | $5
6 | $4
7 | $5
8 | $6
9 | $7
10 | $8
11 | $9
12 | $7
13 | $8
14 | $9
15 | $10
A is not a function because it has more than one output for every input and B is a function beacause it has one output for every input.

Mountain Dew

1.11: Drink More Water

I drink water by filling my water bottle *and refilling it* throughout the day.

DOMAIN:

Day of the week (Monday is 1)	Fluid ounces of water consumed
1	40
2	27
3	60
4	20
5	40

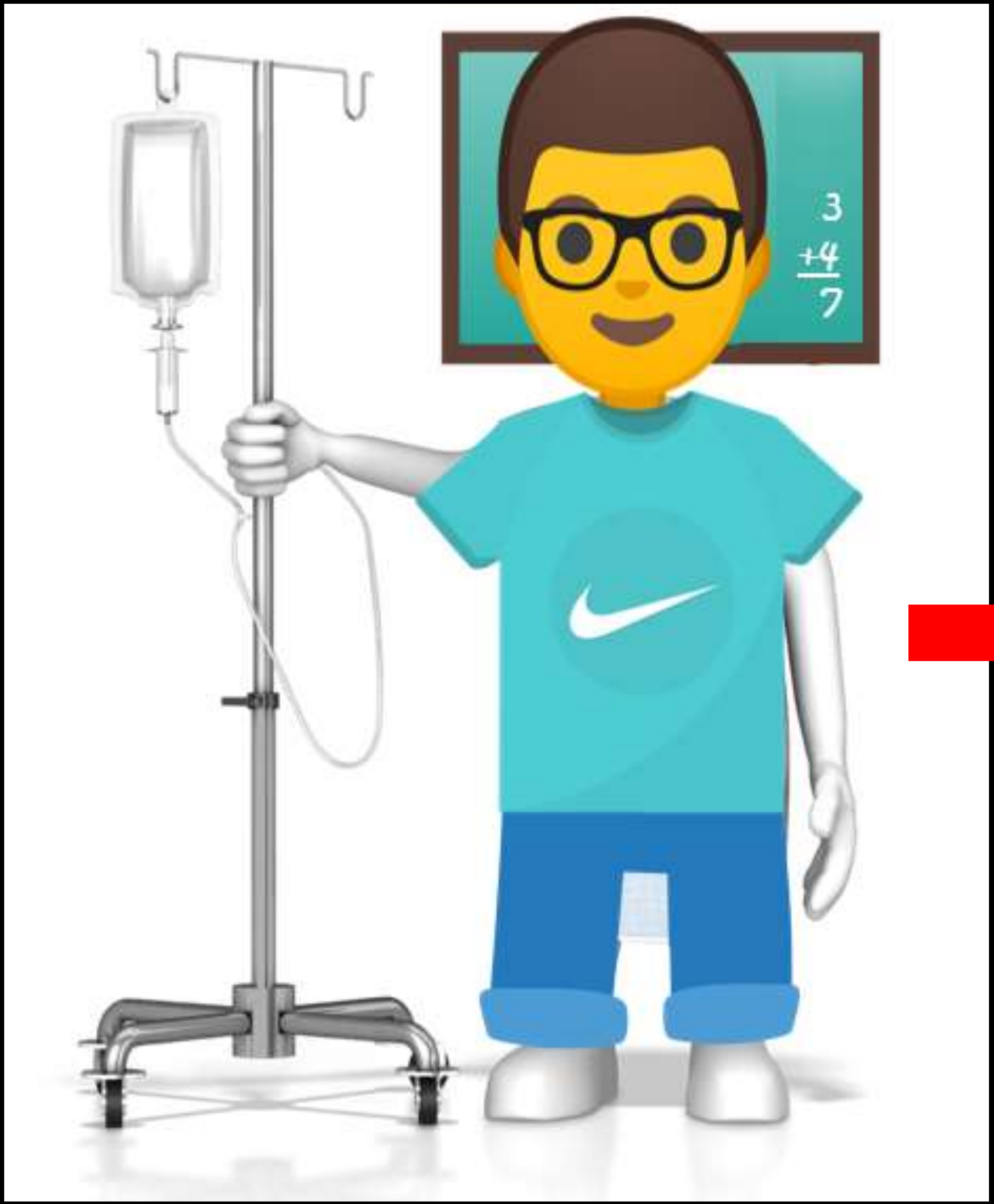
RANGE:

The table below shows the amount of water in my bottle at various times throughout 3rd period.

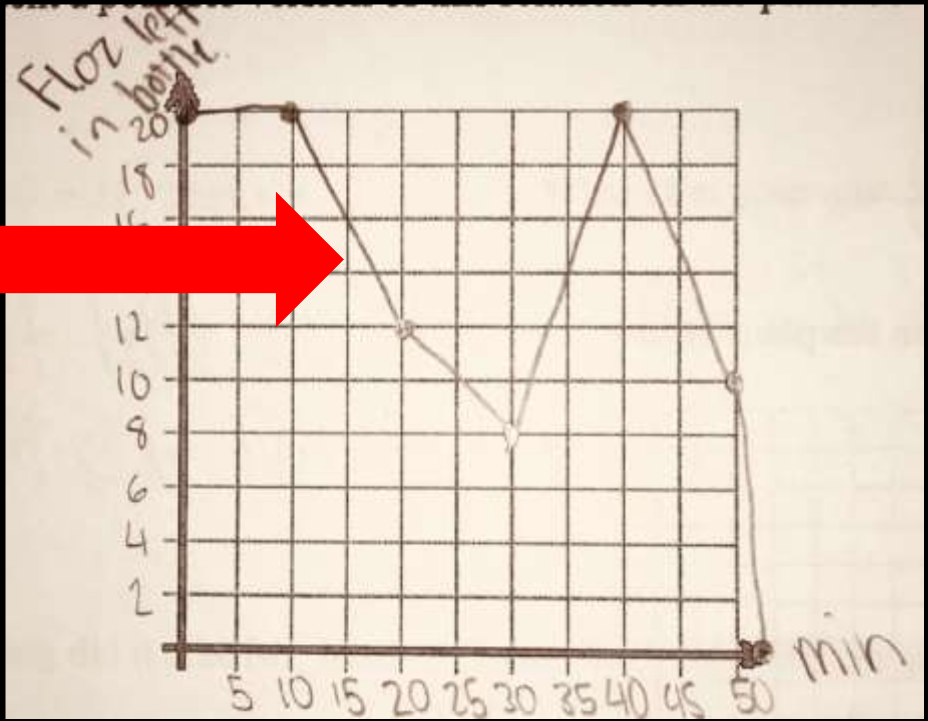
DOMAIN:

Minutes into 3rd Period	Fluid ounces of water in the bottle
0	20
10	20
20	12
30	8
40	20
50	10
53	0

RANGE:

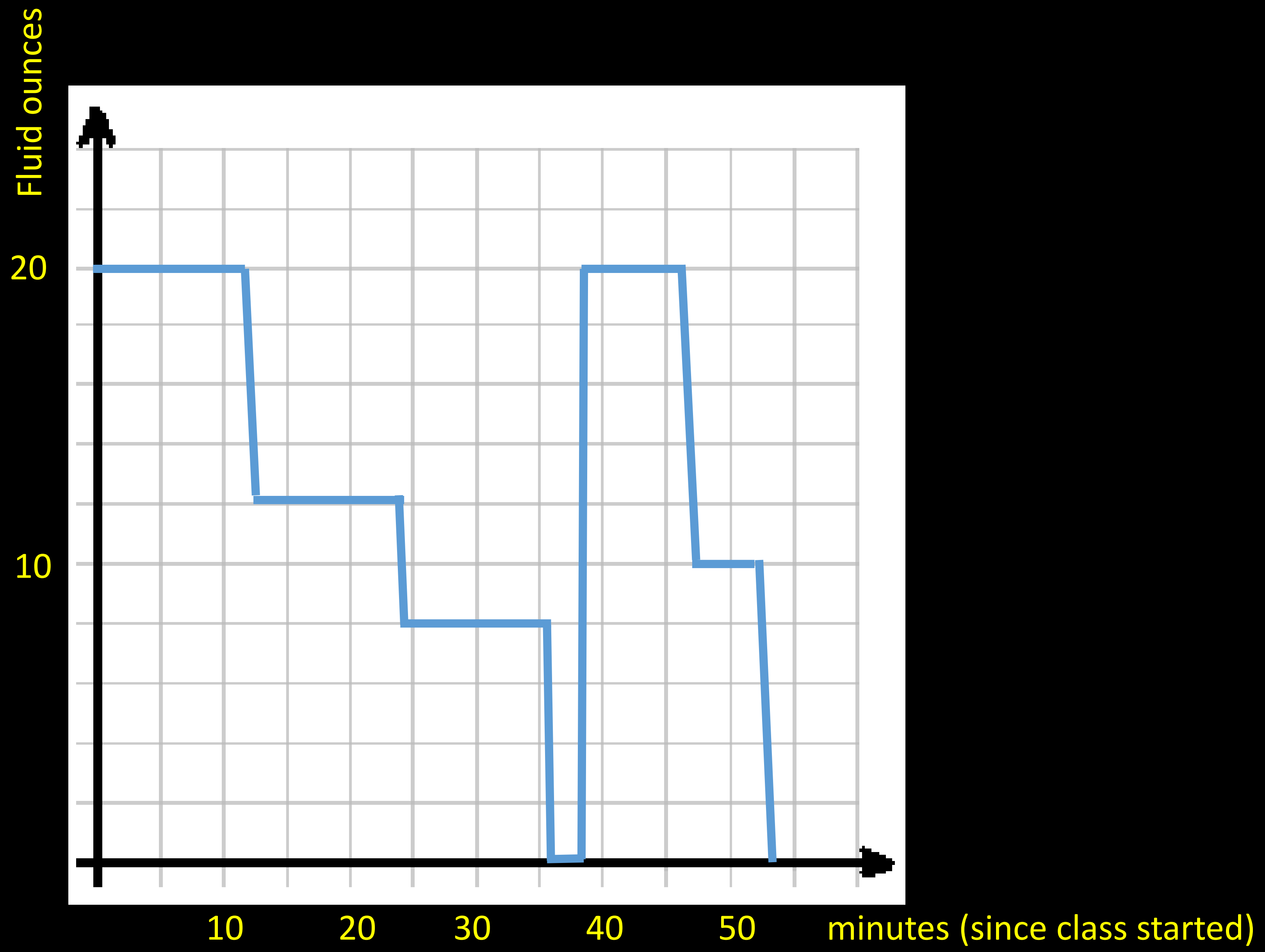


Minutes into 3rd Period	Fluid ounces of water in the bottle
0	20
10	20
20	12
30	8
40	20
50	10
53	0

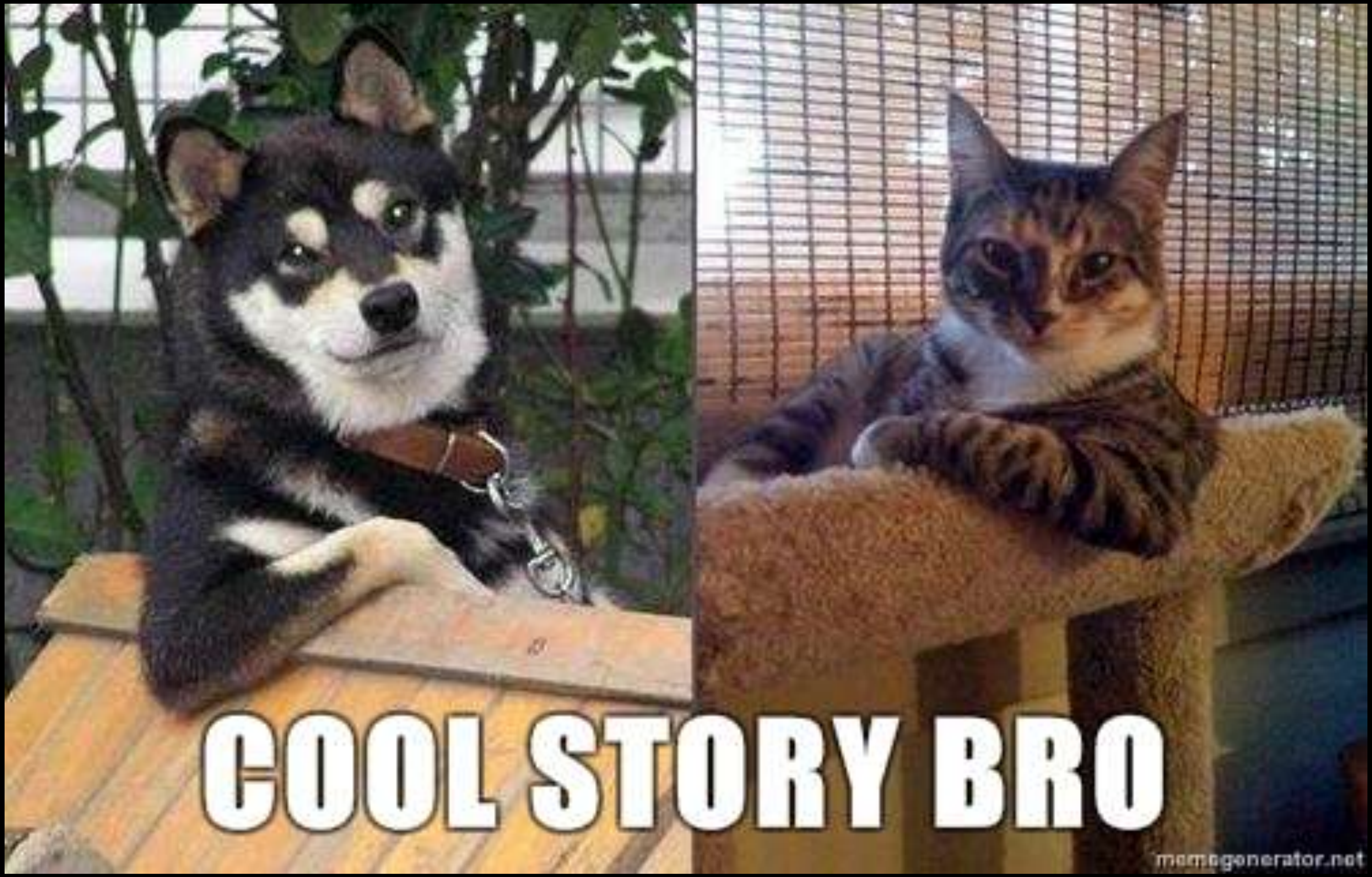


Domain: $[0, 53]$

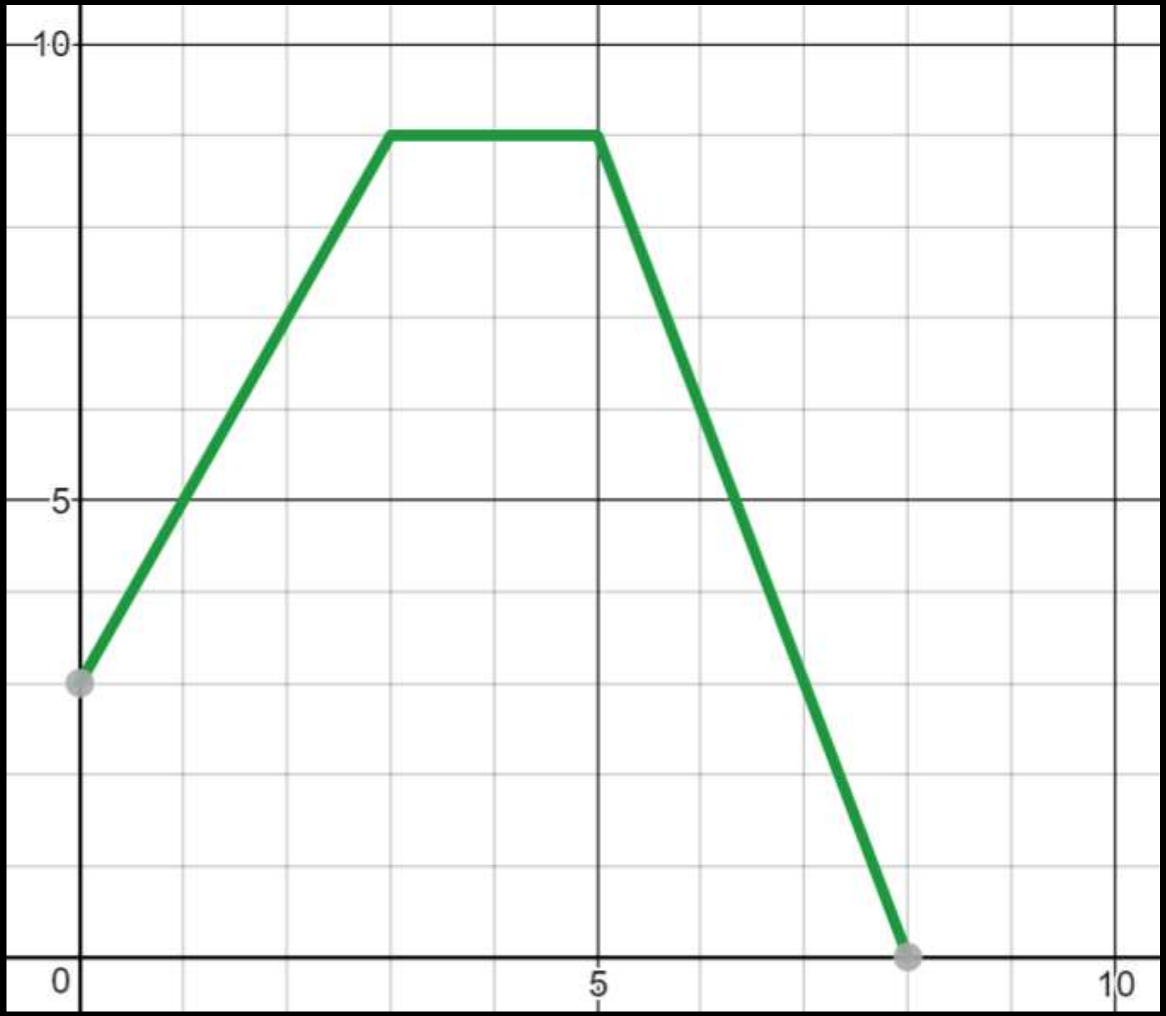
Range: $[0, 20]$

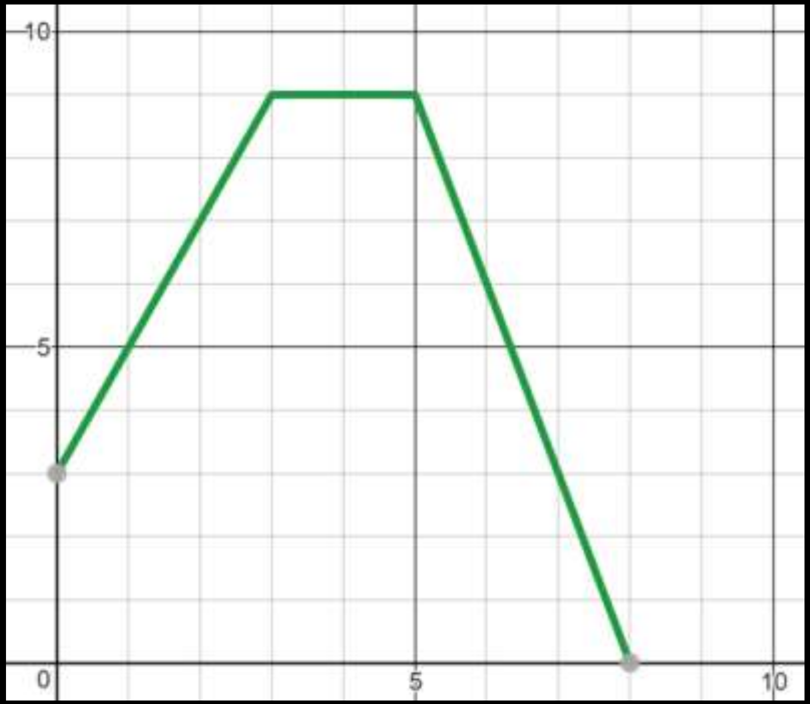


Fluid ounces
20
10
10
20
30
40
50
minutes (since class started)


COOL STORY BRO
memegenerator.net

Create a scenario that can be modeled by the graph below. The input axis should be some unit of time. Make sure to label both axis to match your story.



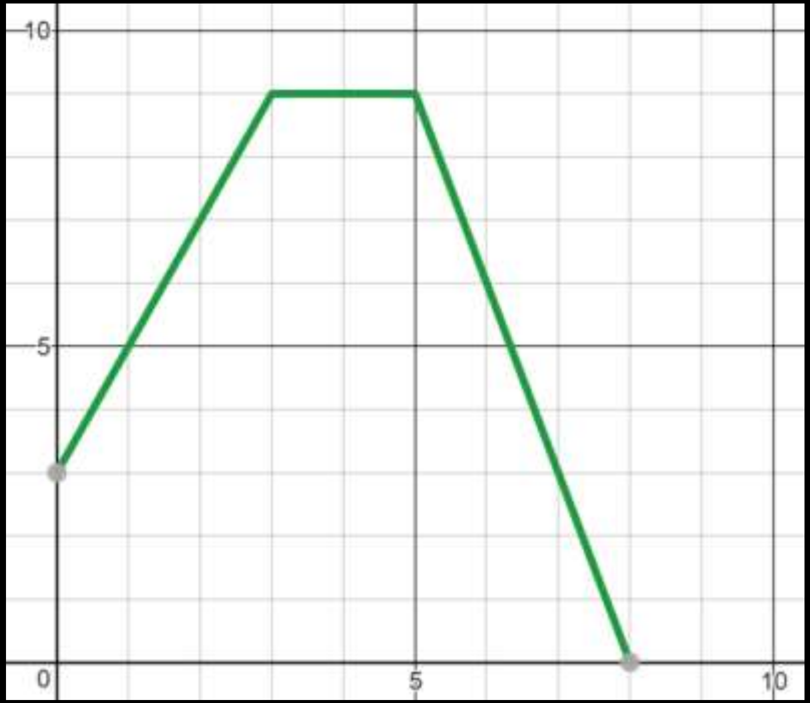


What is the **domain**?

What is the **range**?

$$G(2) = \_\_\_\_\_\_\_\_$$

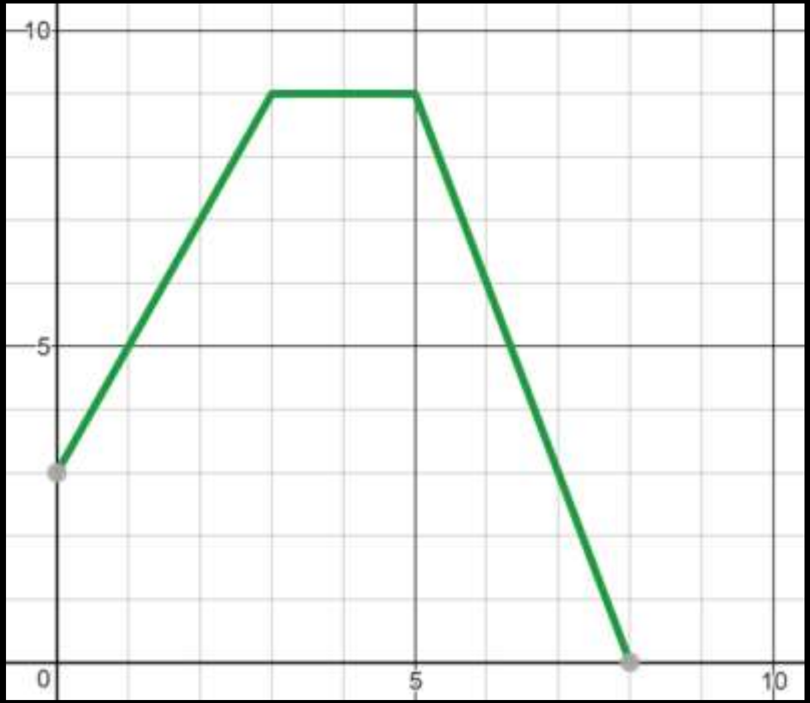
What does this **equation** tell us in terms of your story?



On which interval is the graph increasing?
What does this tell us in terms of your story?

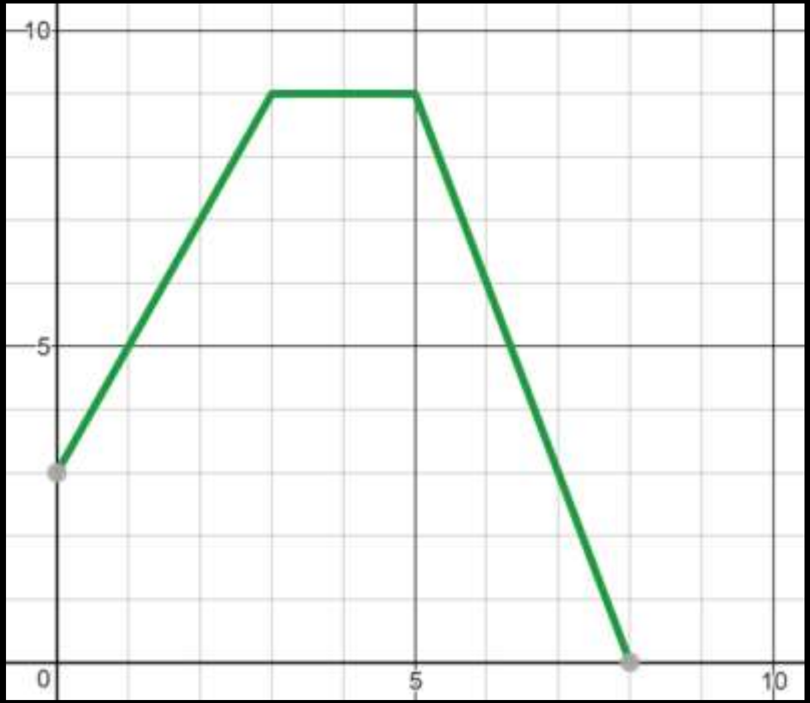
On which interval is the graph decreasing?
What does this tell us in terms of your story?

Explain why, in terms of your story, the graph stays constant on the interval [3,5].



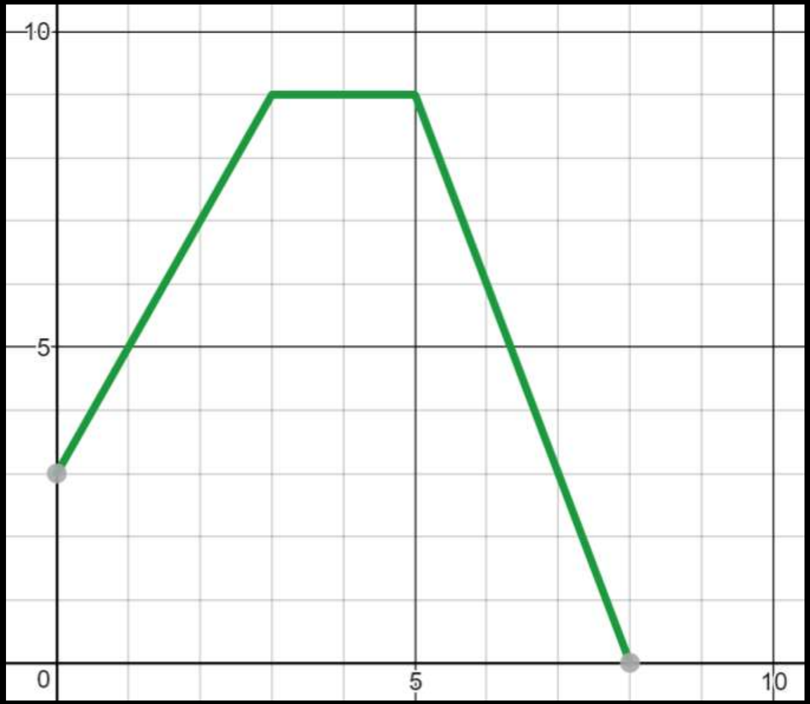
What is the average rate of change of G on the interval $[0,3]$?

What does your answer tell us?



What is the maximum value of G?
What does it tell us in terms of your story?

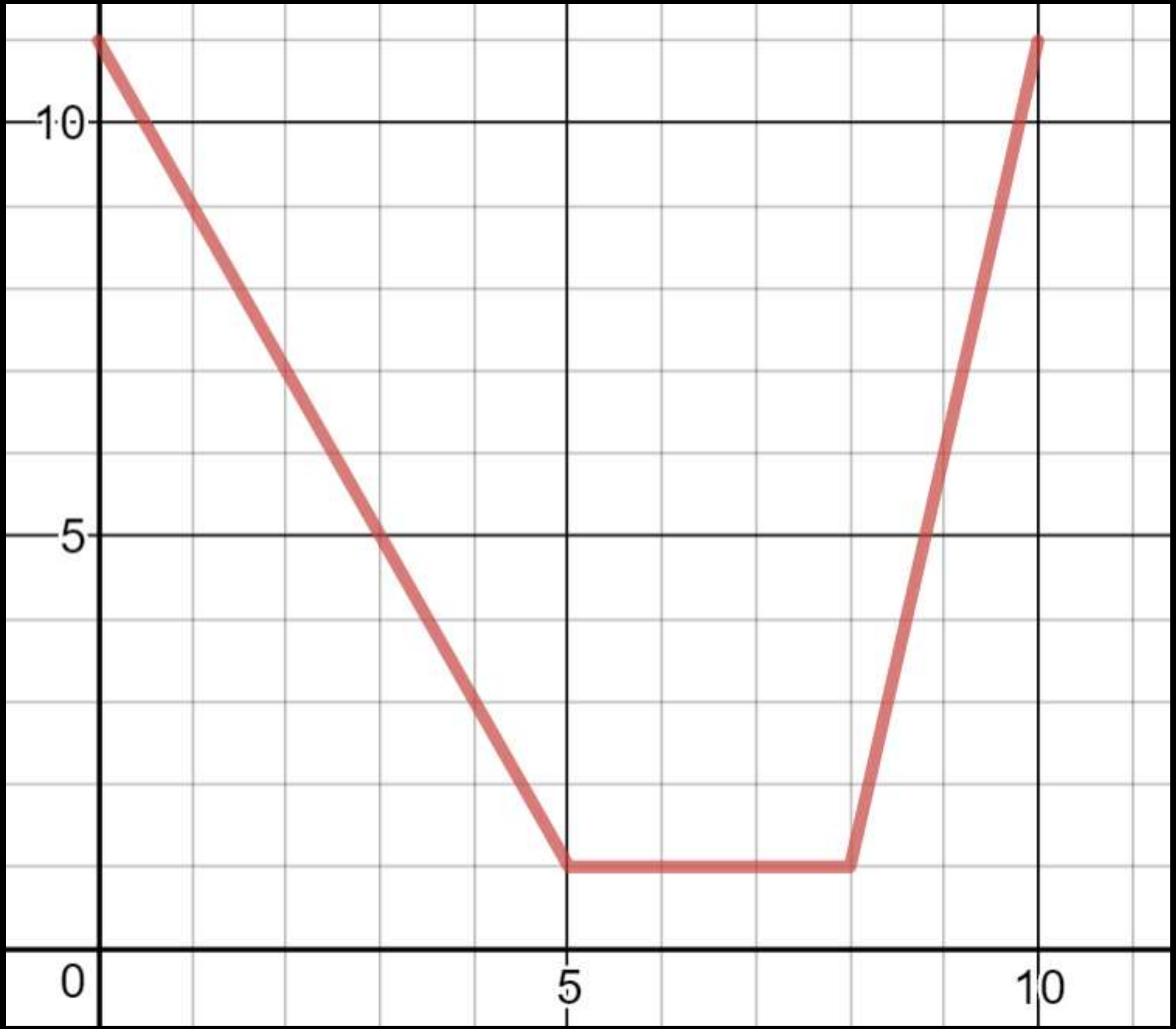
What are the roots of G?
What do they tell us?



What are the **solutions** to the equation $G(t) = 6$?
What do the answers represent?

Explain why the equation $G(t) = 9$ has an **infinite number of solutions.**

What could this graph represent? Explain your reasoning.



Year 1: Algebra 1

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2.1: Straight (Lines) Outta Compton

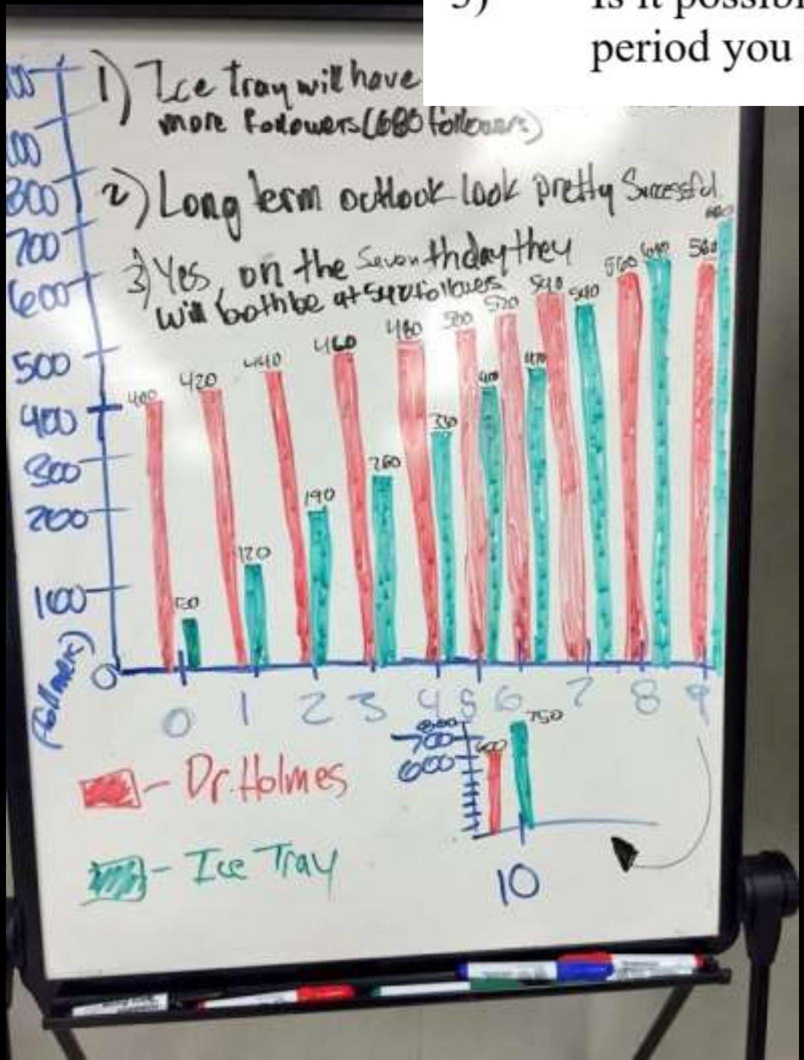
Two new hip-hop artists from Compton are starting to gain popularity. One way to see this is by tracking the number of followers they have on Instagram.

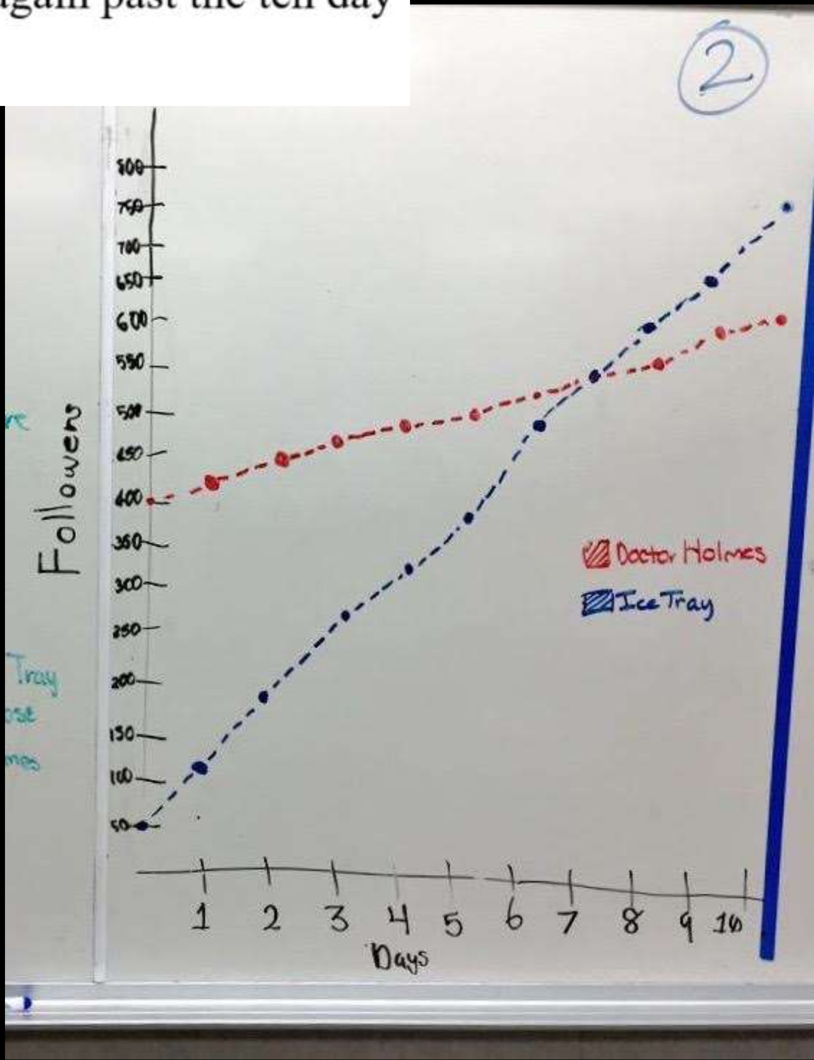
Doctor Holmes has been producing for a while so he already had 400 followers, and now every day he is getting 20 new followers. Ice Tray just released his first single so his popularity is growing much faster. He only has 50 followers, but every day 70 new people follow him.

1) For each of the next 10 days who will have more followers?

2) Will there be a time when both artist have the same number of followers?

3) Is it possible for them to have the same number of followers again past the ten day period you have explored? Explain?





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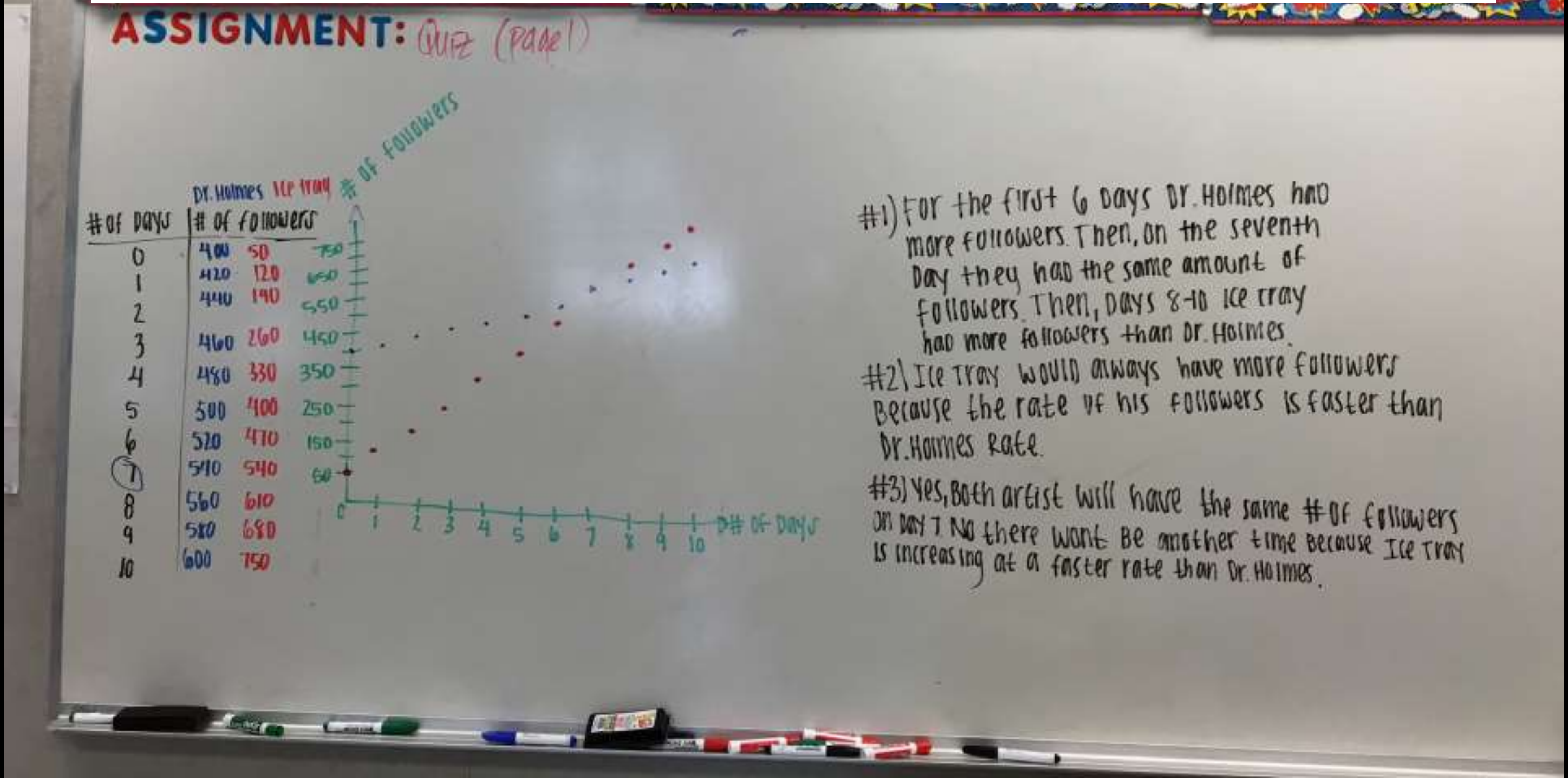


1) Complete each table below for the number of followers each artist has after *d* days.

d (days)	Number of Ice Tray followers
0	50
1	120
2	190
3	260
4	330
5	400
6	470
7	540
8	610
9	680
10	750

d (days)	Number of Dr. Holmes followers
0	400
1	420
2	440
3	460
4	480
5	500
6	520
7	540
8	560
9	580
10	600

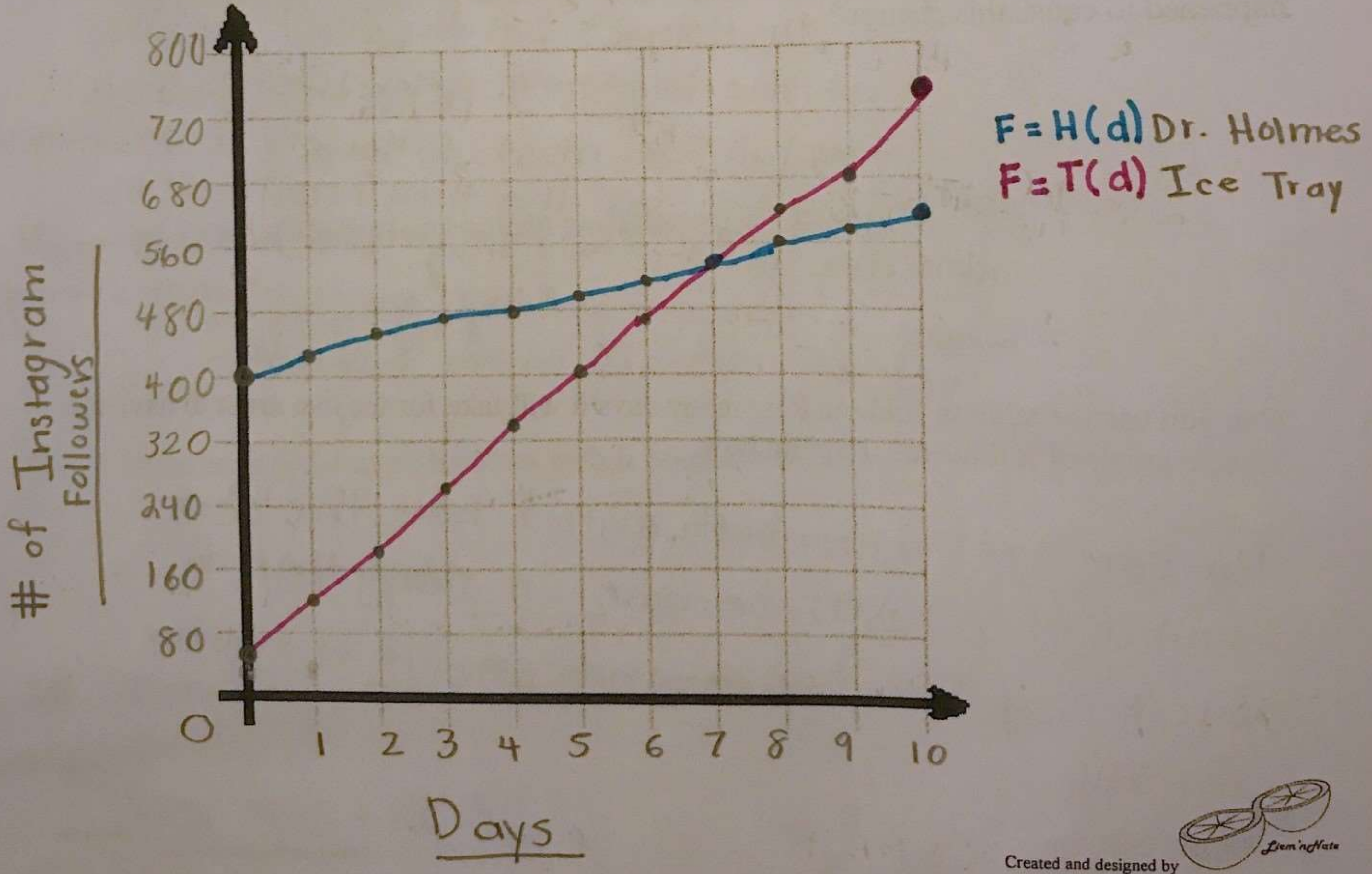
2) Who has more followers in the beginning? Who has more followers after 10 days? What happened to cause this change?

Dr. Holmes has more followers in the beginning. Ice Tray has more followers ~~ft~~ after 10 days. His rate of getting more followers is greater than Dr. Holmes.

3) Can you use the table to find out how many days it will take for the two artist to have the same number of followers? Explain how.

Yes, ~~because~~ you just have to check when they both have the same number of followers.

Created and designed by



Created and designed by Liem'nNate

16) Doctor Holmes and Ice Tray will have the same number of twitter followers when the two expressions are equal (when $T(d) = H(d)$). Set up and solve **an equation** to find the number of days it will be before they have the same number of followers.

$$50 + 70d = 400 + 20d$$
$$-20d \qquad -20d$$
$$50 + 50d = 400$$
$$-50 \qquad -50$$
$$\frac{50d}{50} = \frac{350d}{50}$$
$$7 = d$$

On day 7, they will have the same number of followers.

$$T(d) + H(d)$$

$$(70d + 50) + (20d + 400)$$

$$90d + 450$$

$$T(d) - H(d)$$

$$(70d + 50) - (20d + 400)$$

$$50d - 350$$

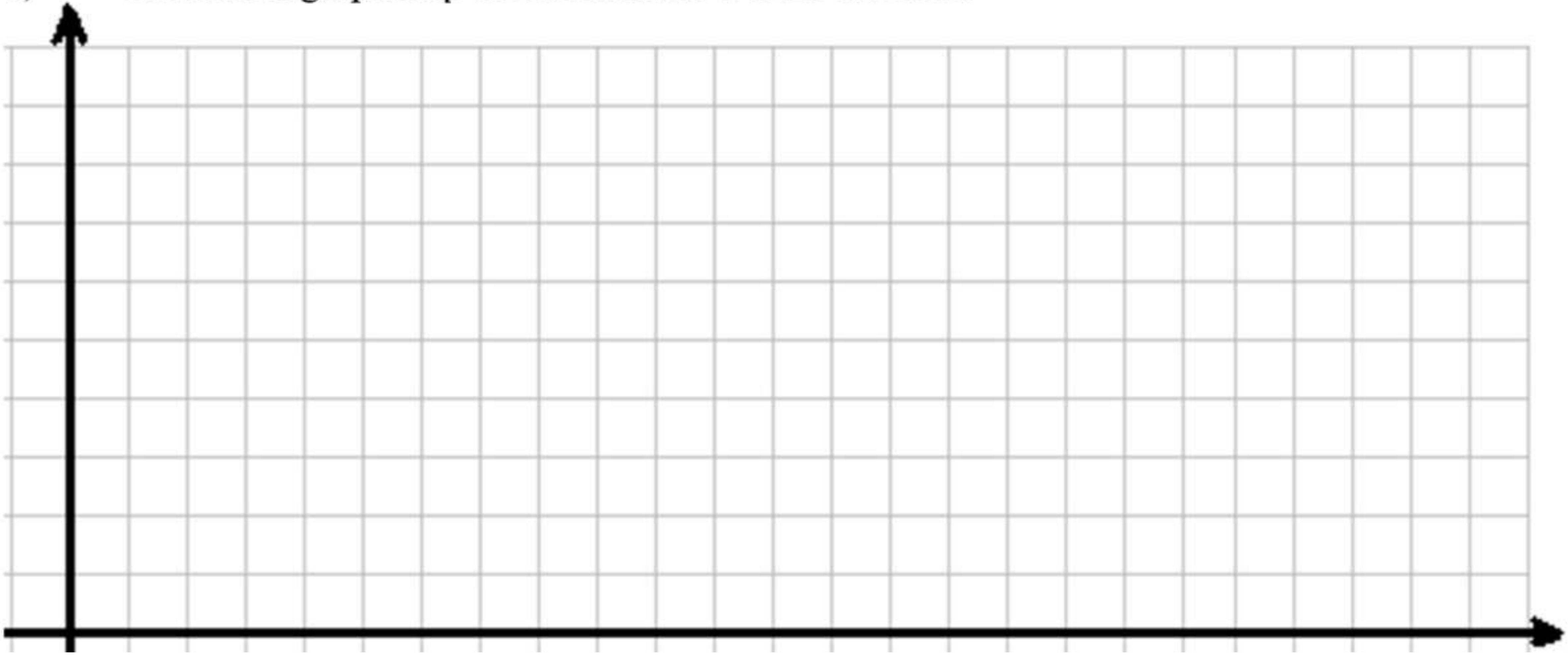
2.15: Pool Full of Water (and We Divin')

Mr. Tran is filling up a plastic swimming pool in his backyard so that we can throw a pool party. He places a hose in the pool and it starts filling with water at a constant rate of 20 gallons per minute. The pool is full when it reaches 300 gallons.

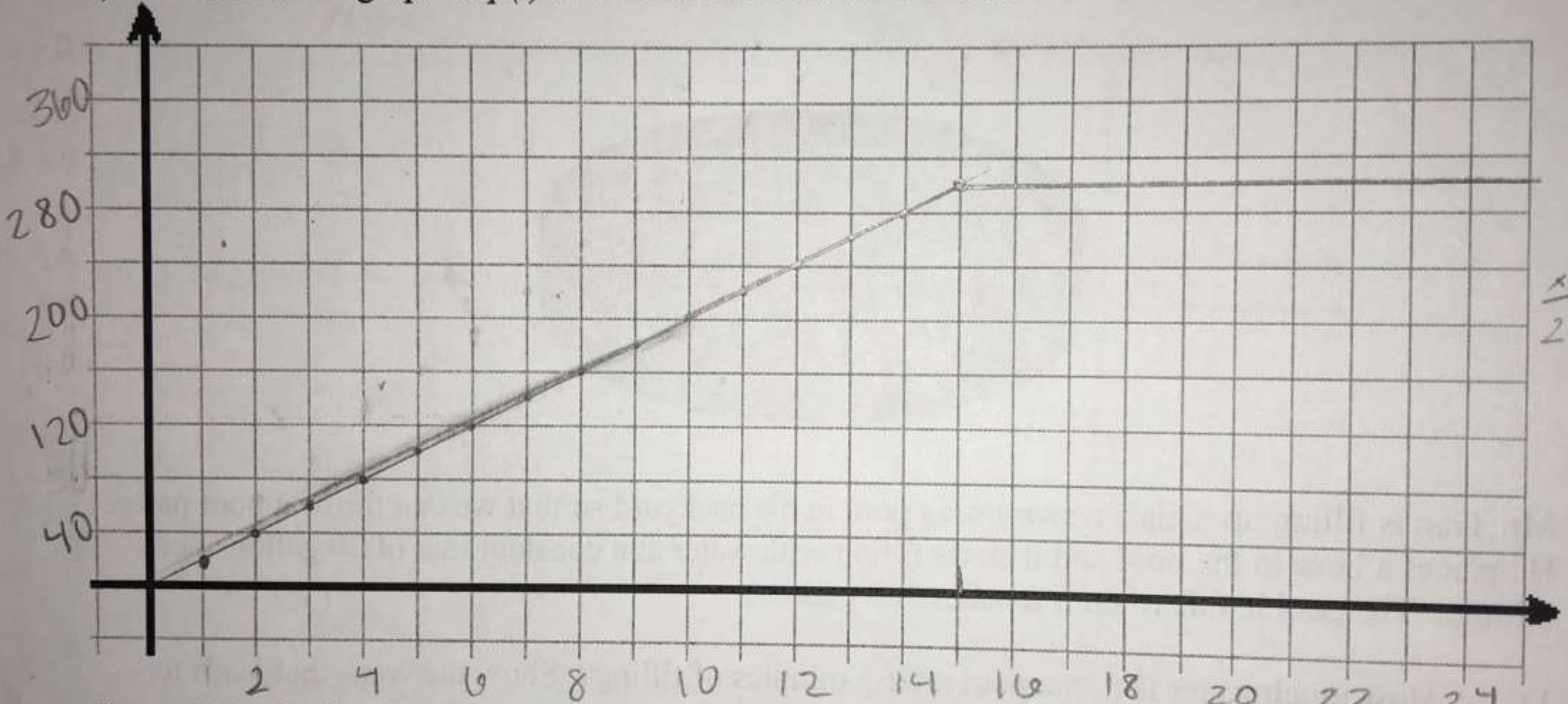
Assume that $p(t)$ represents the amount of water in the pool t minutes after the Mr. Tran starts to fill it.

3) What is $p(2)$? What does it represent?

4) Sketch the graph of p on the interval 0 to 25 minutes.



4) Sketch the graph of *p(t)* on the interval 0 to 25 minutes.



12
×20
00
240

5) What is the domain of *p(t)*?

(0, 15)

6) What is the range of *p(t)*?

[0,]

11) In order to write an equation for *p(t)* we need to write equations for two linear functions on two different time intervals. This is called a **piece-wise function**. Can you write the two equations for the two "pieces" of line in the graph?

P(t) = 20t → Yes because one equation shows the amount of → p(t) = 20t water (gallons) in the pool in t minutes, while the other equations

P(t) = 300

shows that no matter the time (minutes) pass by, the answer will equal to 300 gallons.

12) On which time interval does the first equation you wrote model the amount of water in the pool? On which interval does the second equation model the amount?

#1 equation
P(t) = 20t
(amount of water in pool)
[0, 15]

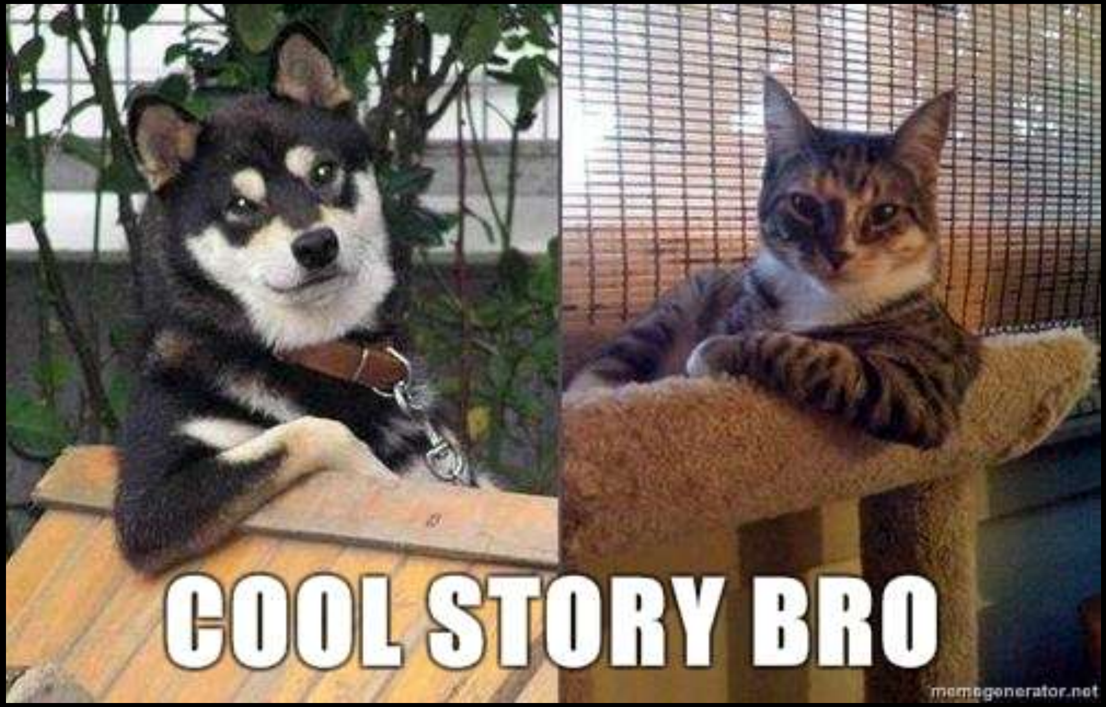
#2 equation
P(t) = 300
(amount)
[15, 25]

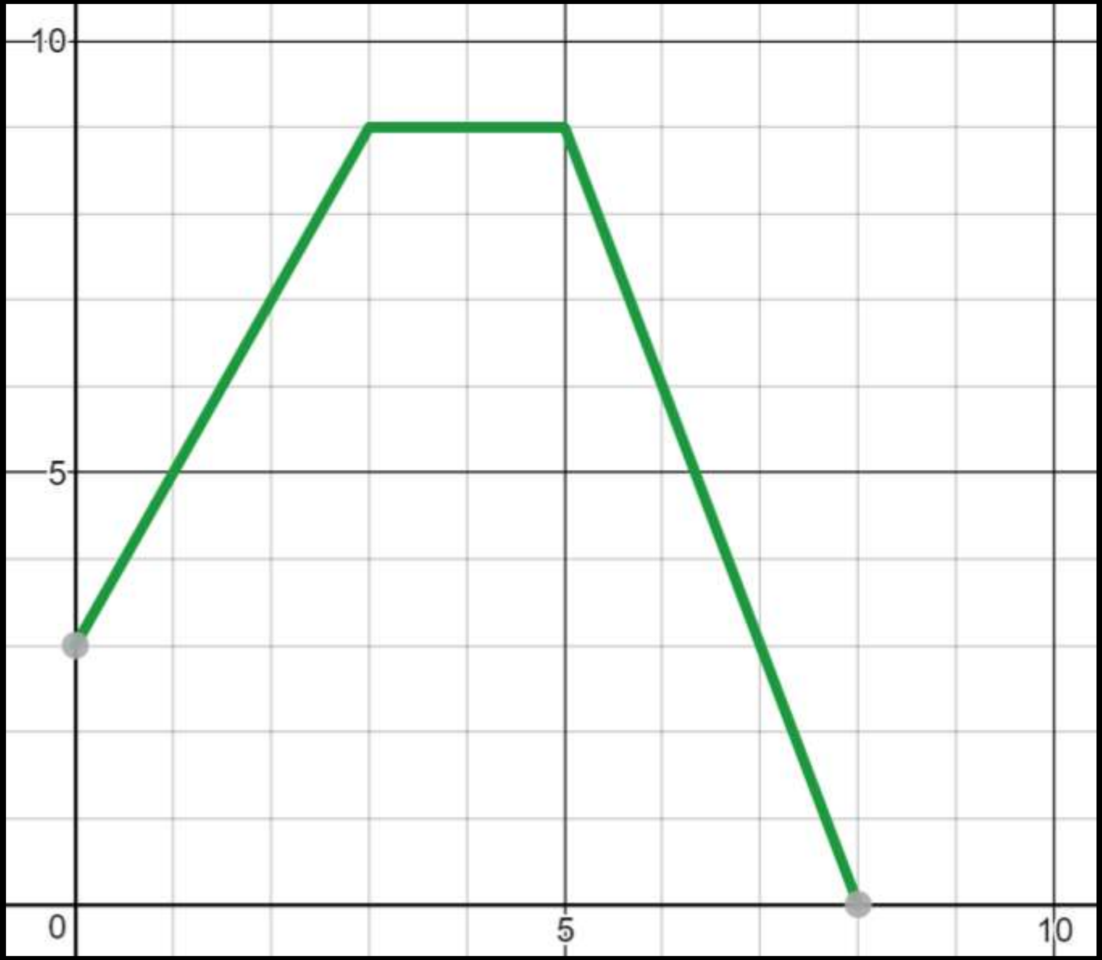
The correct notation used to write a **piecewise equation** to model $p(t)$ is given below.

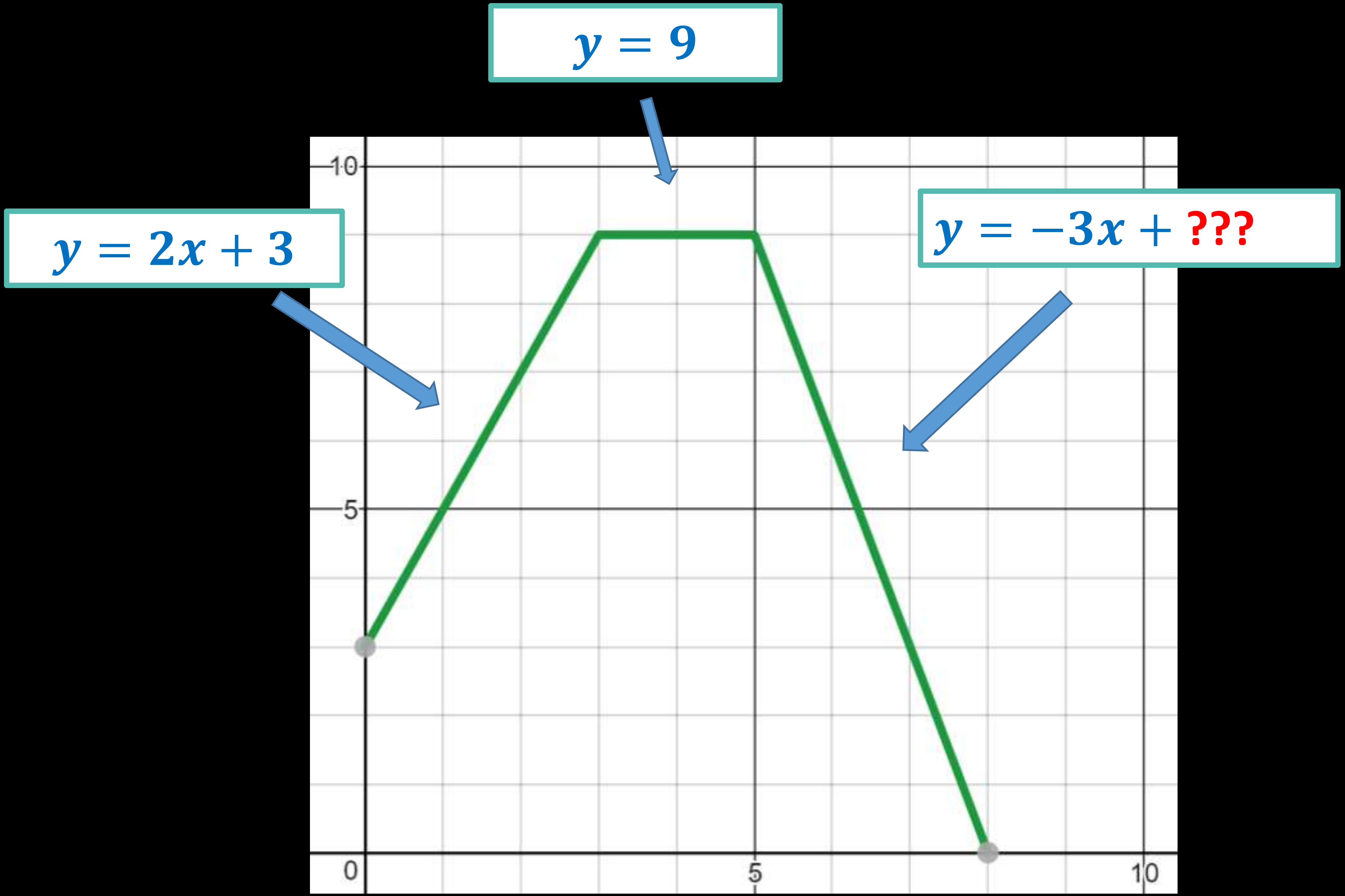
$$p(t) = \begin{cases} 20t & 0 \leq t \leq 15 \\ 300 & 15 < t \leq 25 \end{cases}$$

Explain how this notation makes sense to you.

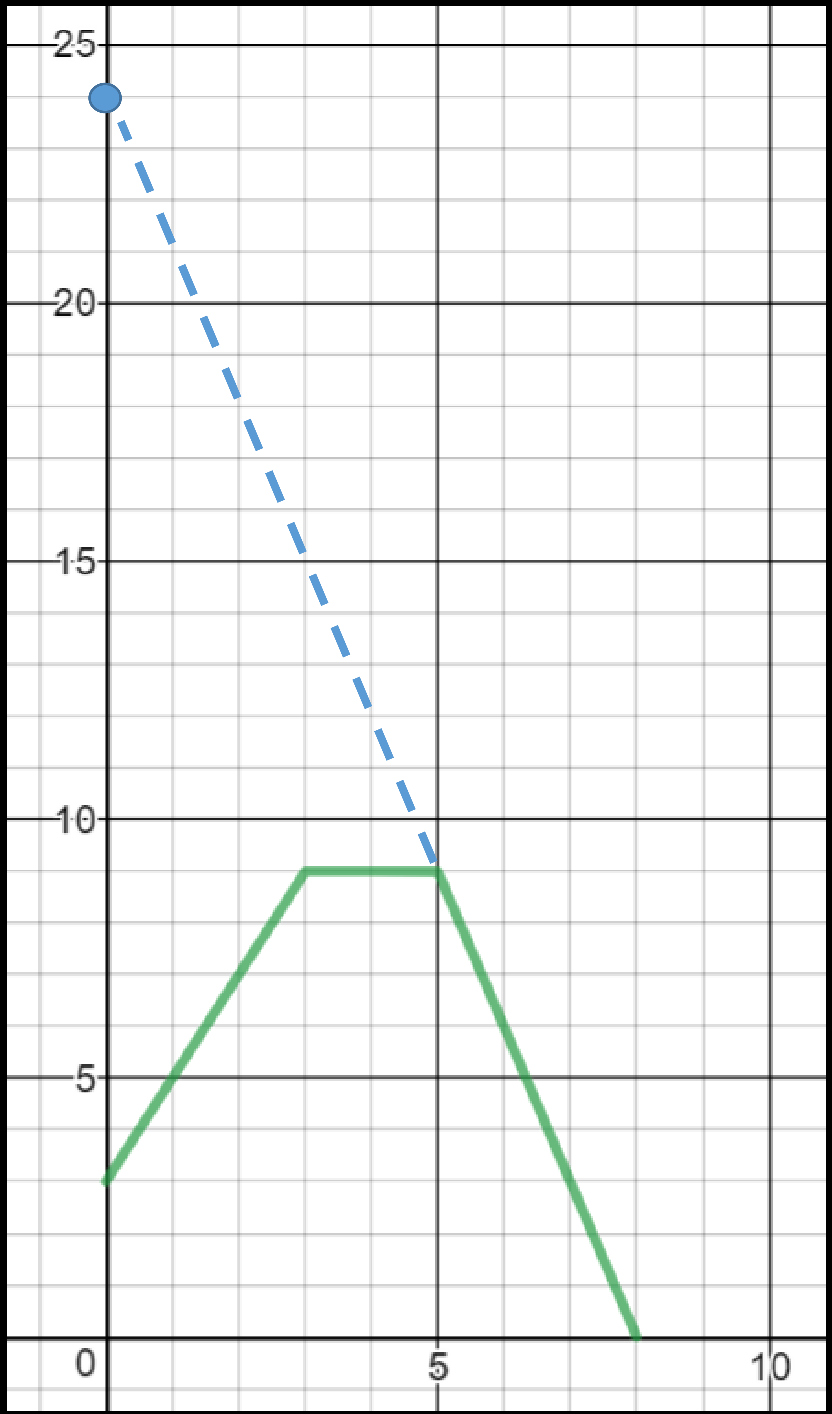
Now let's go back to this…







$y = 9$
$y = 2x + 3$
$y = -3x + ???$
10
5
0
5
10



$$y - y_1 = m(x - x_1)$$

$$y - 10 = -2(x - 6)$$

$$y - 10 = -2x + 12$$

$$y = -2x + 22$$

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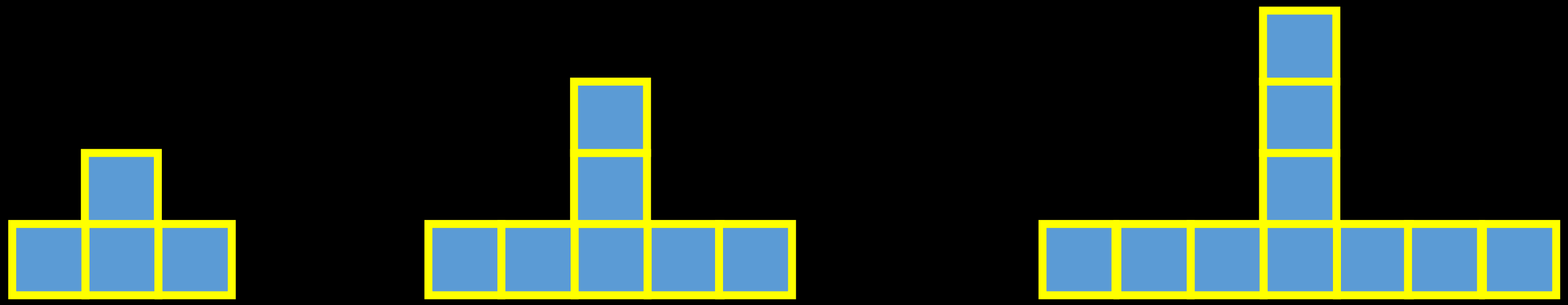
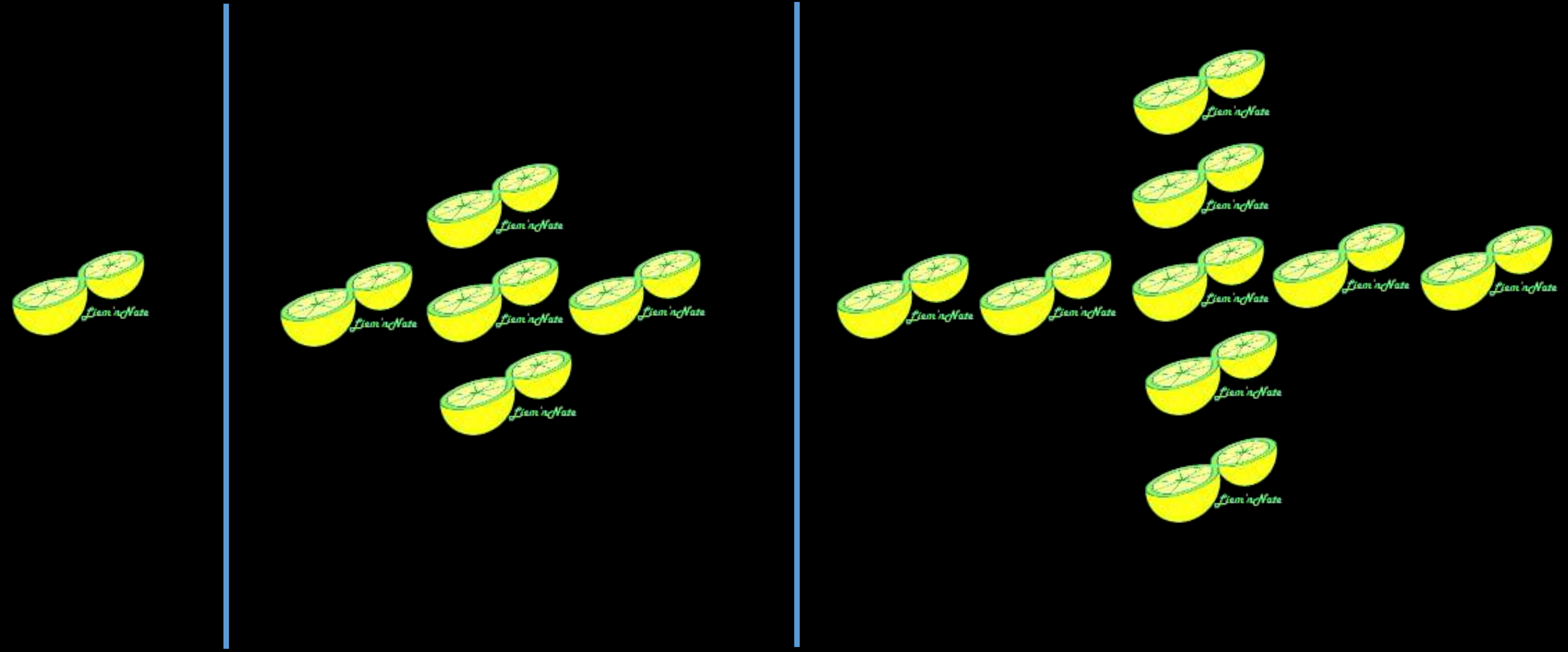


Draw the next figure in the pattern.

How many squares will there be in the 5th figure?
(You don't have to draw it!)

How many squares will there be in the 100th figure?



How many Liem'nNate logos will be in the next figure?

Write a rule for the n^{th} figure in the pattern: $a_n =$________________________

Use the rule to find a_{100}.

What do you Notice? What do you Wonder?

How many pennies will there be in the 100th row?

$$a_n = 2n + 1$$

$$a_{100} = 2(100) + 1$$

201 pennies

Eventually there will be a row that takes 101 pennies to make. Which row will that be?

$$101 = 2n + 1 \rightarrow 100 = 2n \rightarrow 50 = n$$

The 50th row will need 101 pennies.

How many total pennies will it take to build 100 rows?

$$\sum_{n=1}^{100} 2n+1 = \frac{100}{2}(3+201) = 50(204) = 10200$$

How many rows of this tower can you build with 120 pennies?

???

$$\sum_{n=1}^{x} 2n + 1 = 120$$

How many total pennies will it take to build x rows?

$$T(x) = \sum_{n=1}^{x} 2n + 1 = \frac{x}{2}(3 + 2x + 1) = \frac{x}{2}(2x + 4) = x^2 + 2x$$

How many total pennies will it take to build 100 rows?

$$T(x) = x^2 + 2x$$

$$T(100) = (100)^2 + 2(100) = 10200$$

How many rows of this tower can you build with 120 pennies?

$$T(x) = x^2 + 2x$$

$$120 = x^2 + 2x$$

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57. Bacteria Growth The number B of bacteria in a petri dish culture after t hours is given by

$$B = 100e^{0.693t}.$$

(a) What was the initial number of bacteria present? 100

(b) How many bacteria are present after 6 hours? ≈ 6394

What do we like about this problem?

What don't we like?

The number of bacteria in a petri dish is growing in such a way that its population quadruples every hour. If there are initially 20 bacteria in the population, *how much time will pass before there are 10,000 bacteria in the dish?*

At the start of 2018 the up and coming Math Education duo **@LiemnNate** had exactly 20 followers on Twitter.

Experts predict the number of followers will quadruple every year for the next few years.

How many followers will @LiemnNate have at the start of 2019?

Write an equation for $L(t)$, the predicted number of followers @LiemnNate will have t years from 1/1/18.

Evaluate $L(2)$. What does this answer represent?

At the beginning of which year will @LiemnNate to reach exactly 1,280 followers?

Evaluate $L\left(\frac{3}{2}\right)$. What does this answer represent?

Does the answer make sense?

How many followers is @LiemnNate predicted to have in two and a half years?

How many twitter followers is @LiemnNate predicted to have at the end of this month?

Explain why your answer seems reasonable (or does it?).

Right now the math education guru @ddmeyer has 64.3K followers.

How long will it take @LiemnNate to obtain that many followers?

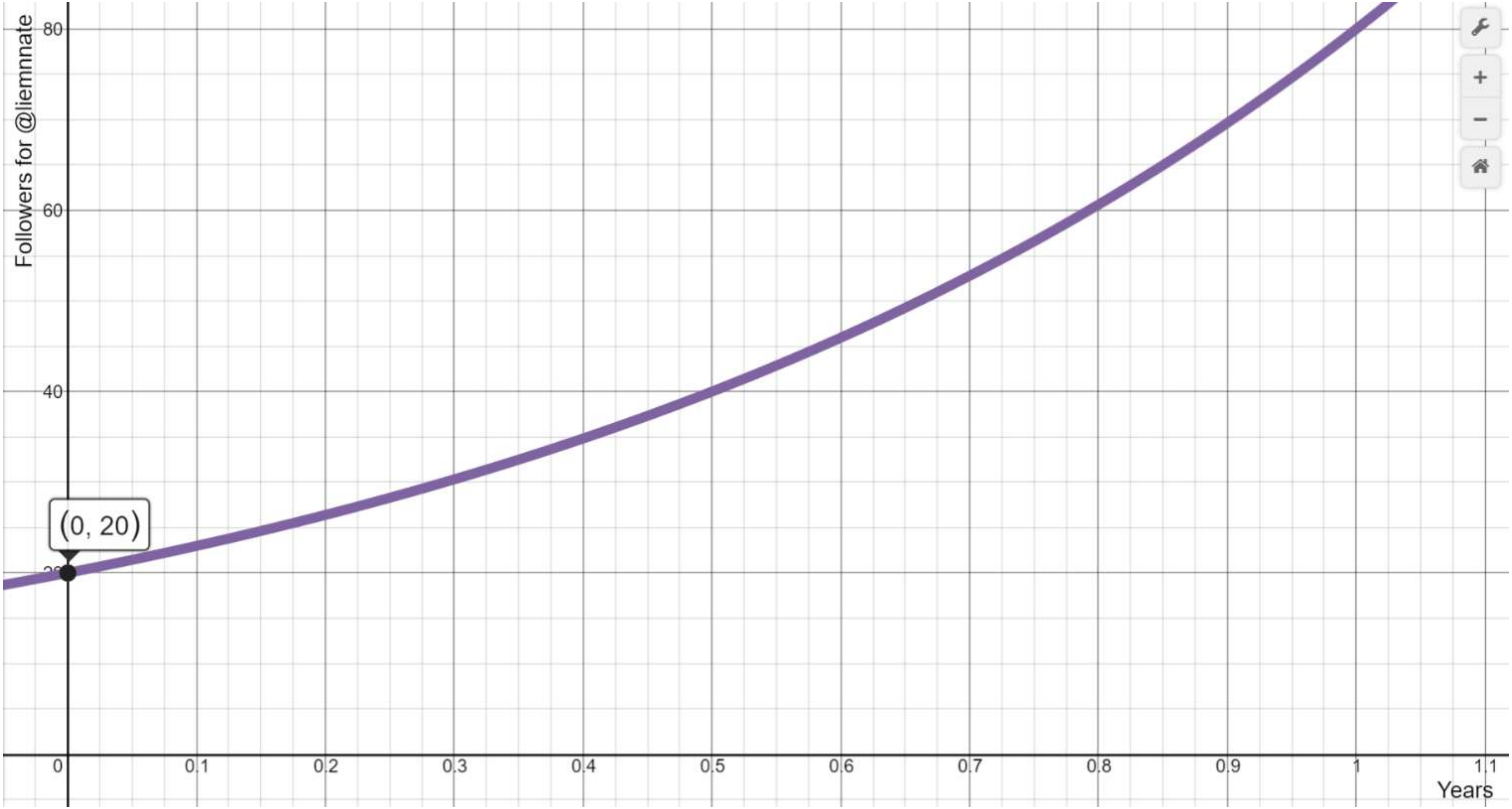
What is a reasonable domain for L?

How did you come to this conclusion?

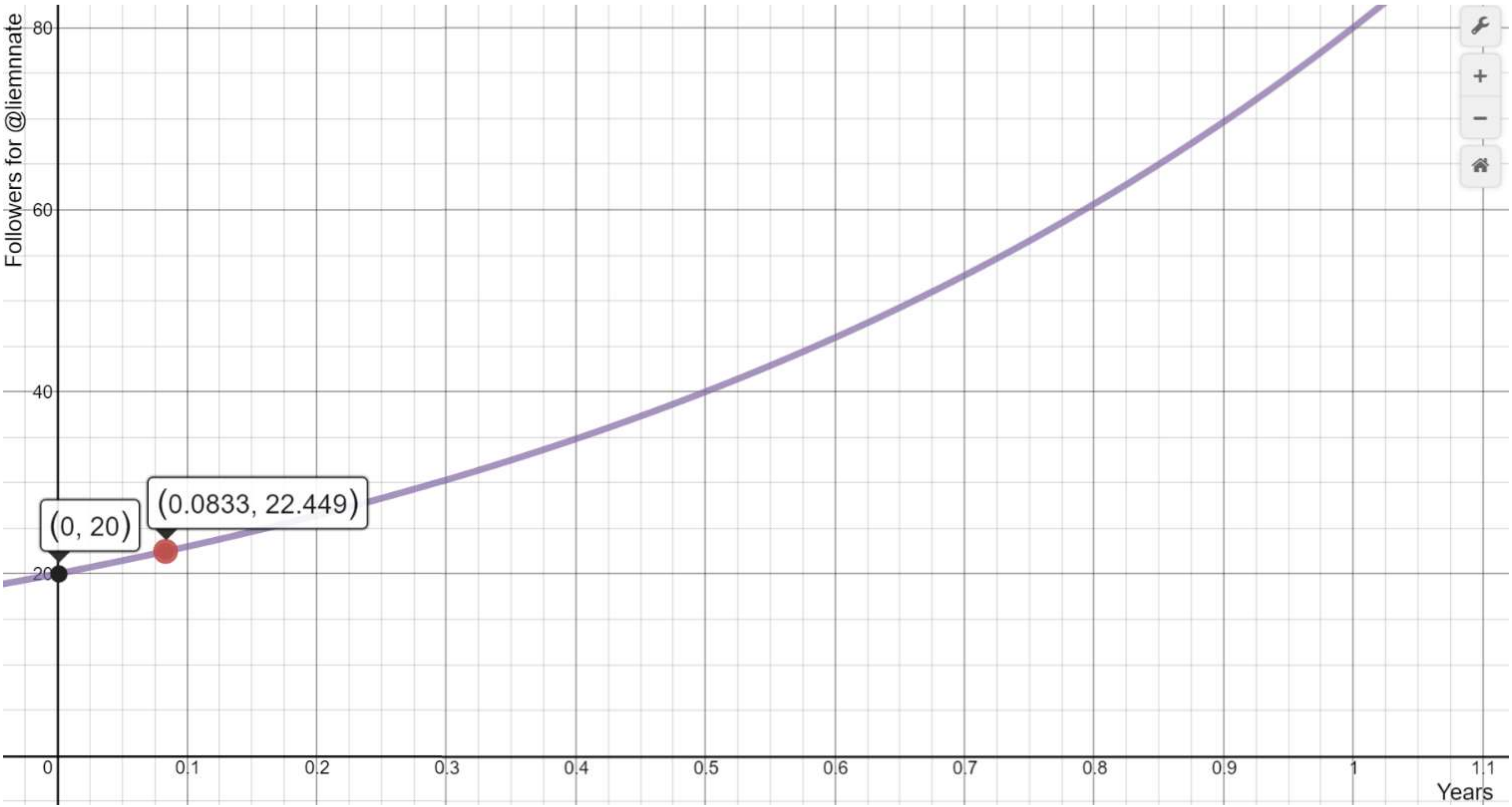
Is L discrete or continuous?

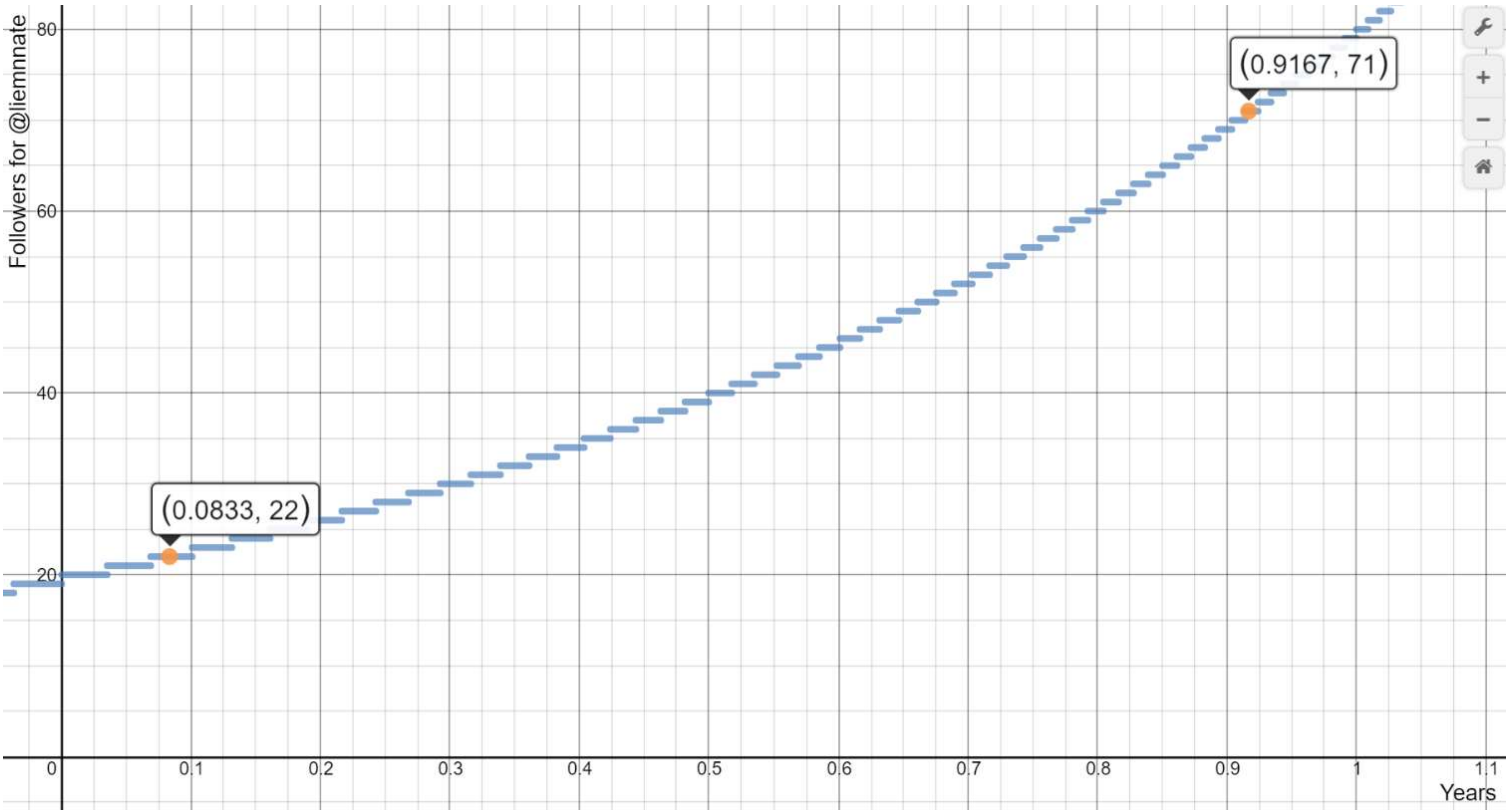
Explain your choice.

Followers for @liemnnate
6000
4000
2000
(0, 20)
0
1
2
3
4
5
6
Years

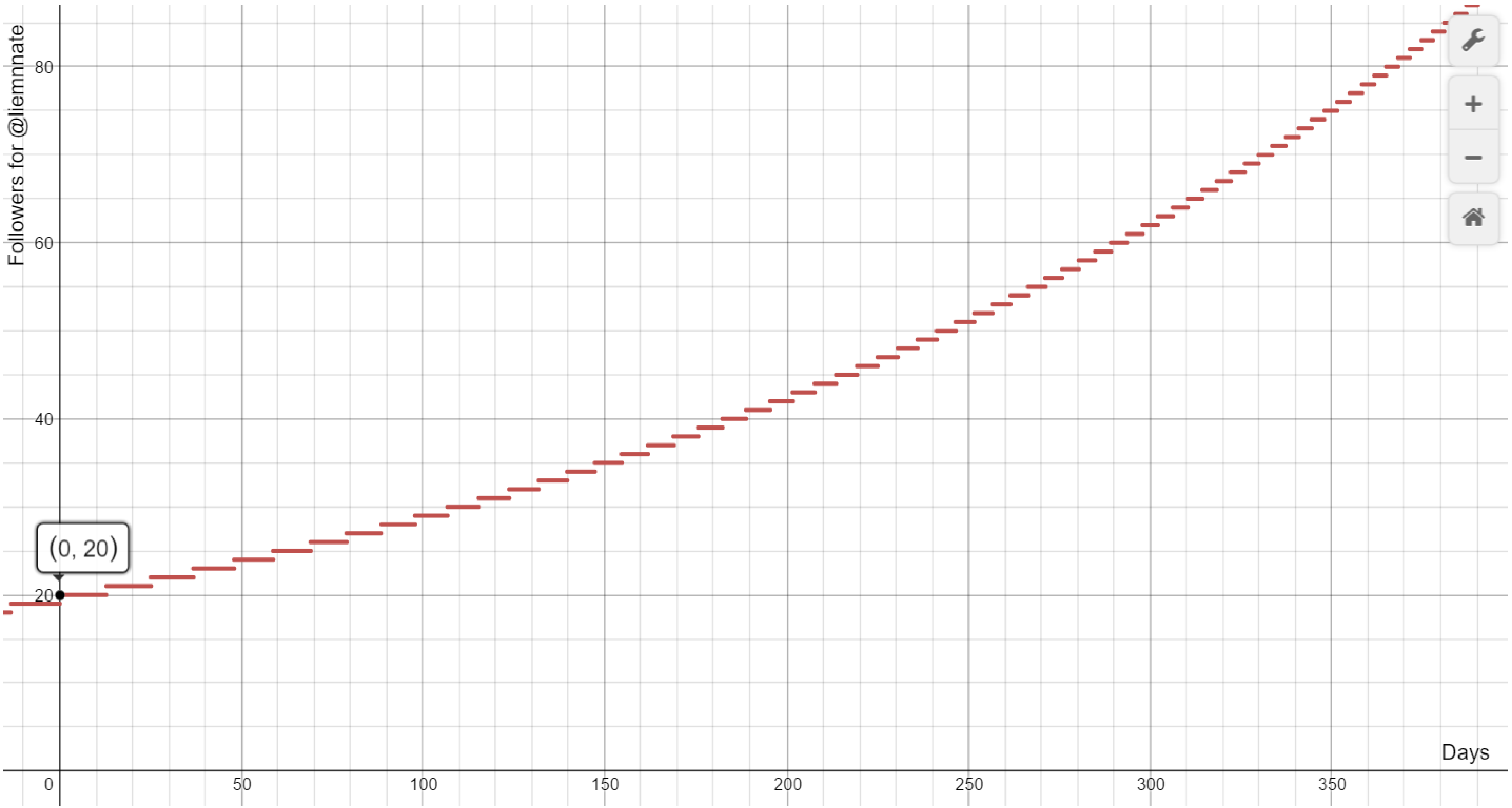


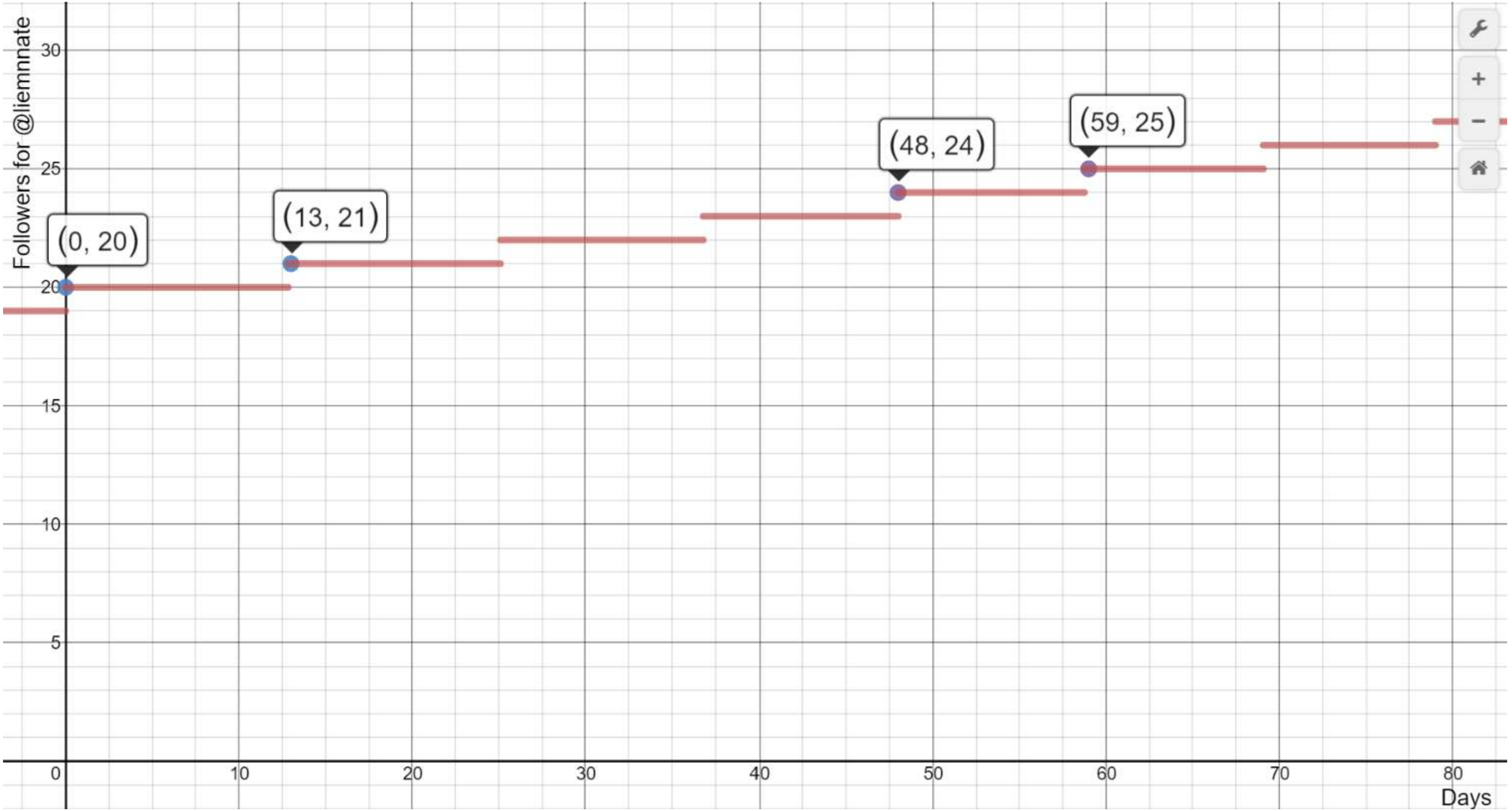
Followers for @liemnate
80
60
40
(0, 20)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
Years


Followers for @liemnnate
80
60
40
20
(0, 20)
(0.0833, 22.449)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
Years

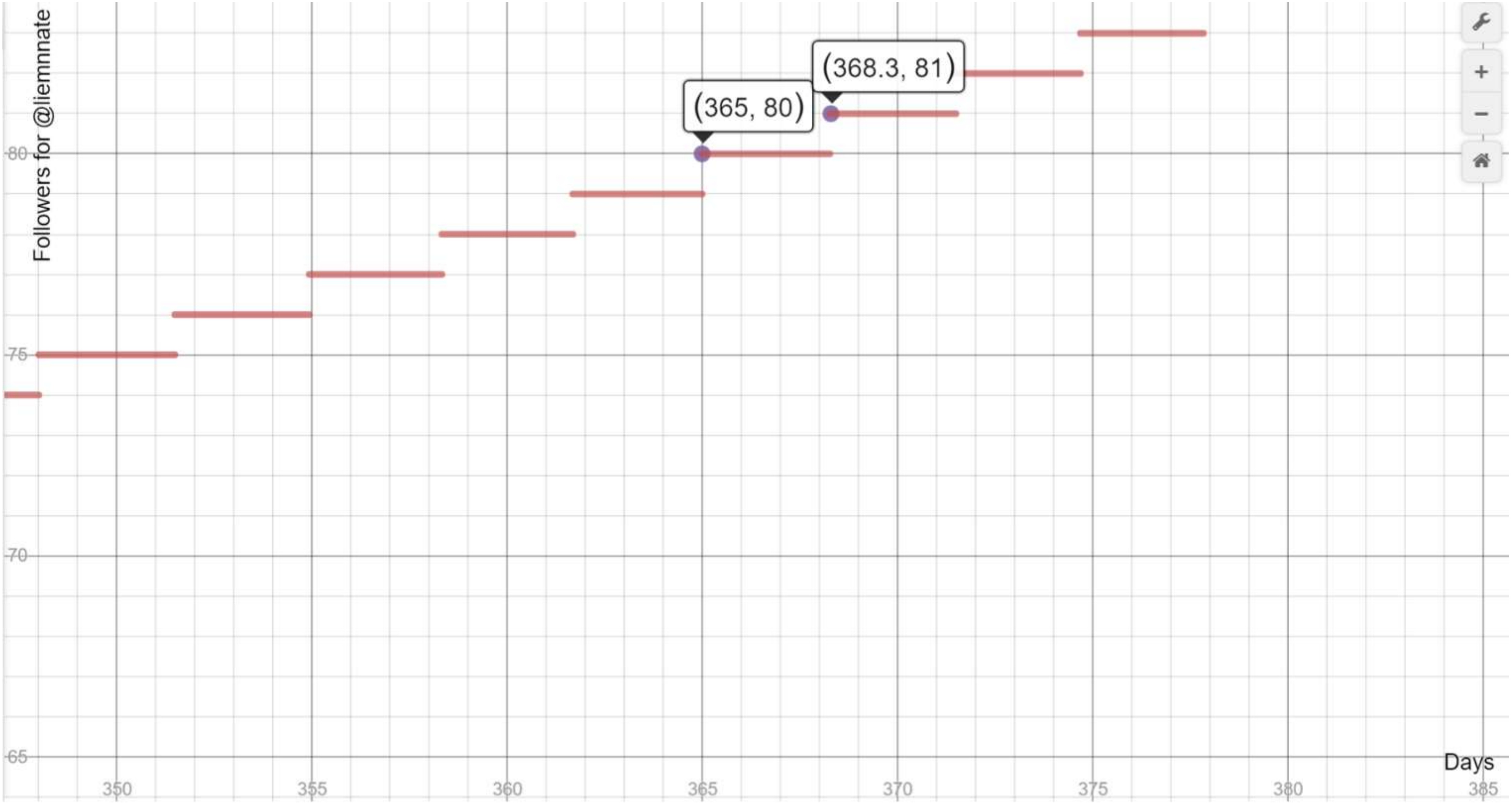


Followers for @liemnnate
80
60
40
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
Years
(0.0833, 22)
(0.9167, 71)


Followers for @liemnate
80
60
40
20
(0, 20)
0
50
100
150
200
250
300
350
Days



Followers for @liemnnate
(0, 20)
(13, 21)
(48, 24)
(59, 25)
30
25
20
15
10
5
0
10
20
30
40
50
60
70
80
Days



Followers for @liemnnate
(365, 80)
(368.3, 81)
80
75
70
65
350
355
360
365
370
375
380
385
Days

TIME MACHINE
PICTURE HERE

Use your equation to evaluate $L(-1)$.

What does this answer represent?

Does it make sense?

This is just a matter of time.

How long will it take for @LiemnNate to reach 1 million twitter followers?

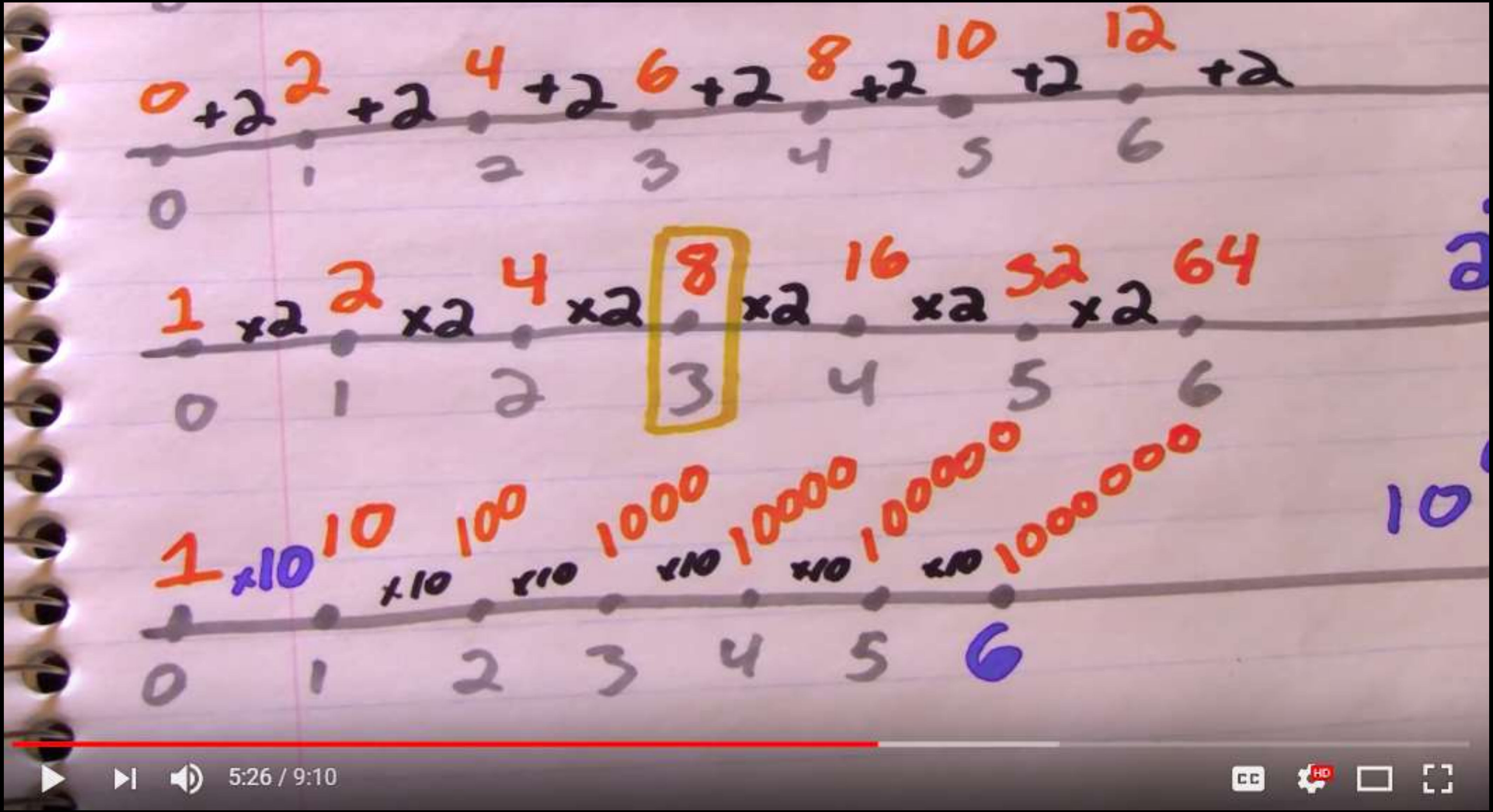
How long will it take for @LiemnNate to reach 1 million twitter followers?

$$L(t) = 1,000,000$$

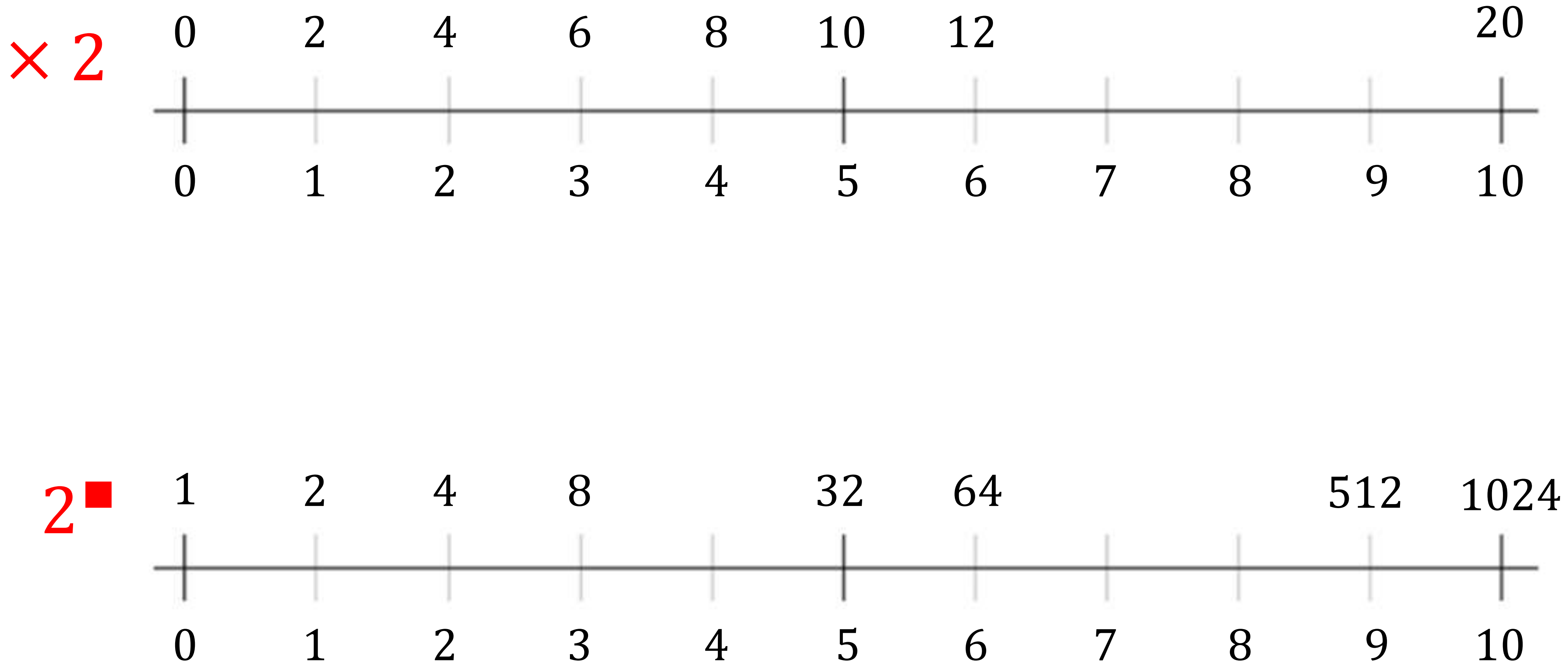
$$20(4)^t = 1,000,000$$

$$4^t = 50,000$$

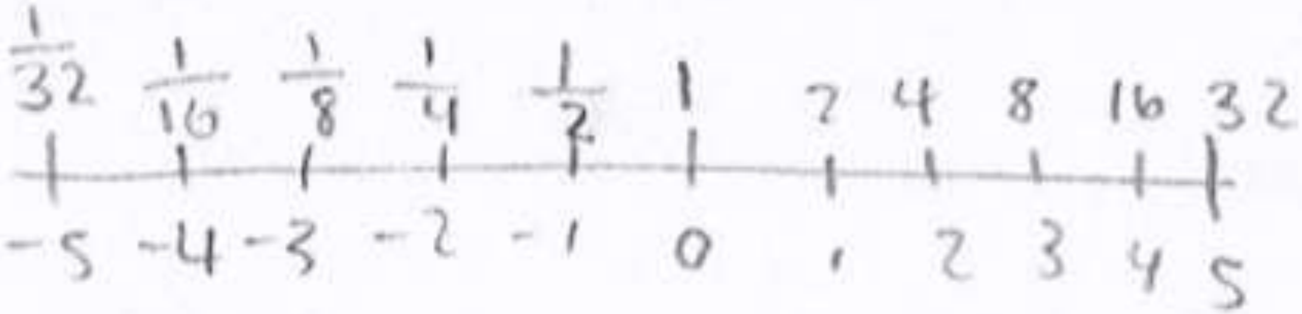
???



Vi – Hart: "How I Feel About Logarithms"


× 2
0 2 4 6 8 10 12 20
0 1 2 3 4 5 6 7 8 9 10
2■
1 2 4 8 32 64 512 1024
0 1 2 3 4 5 6 7 8 9 10

4) Create a number line to count by × 2. Have it go from -5 to 5.

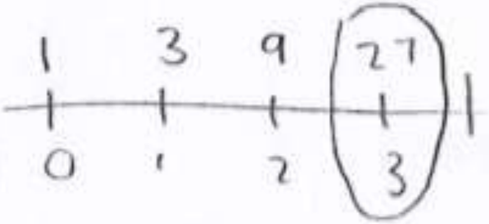


Use the number line to evaluate

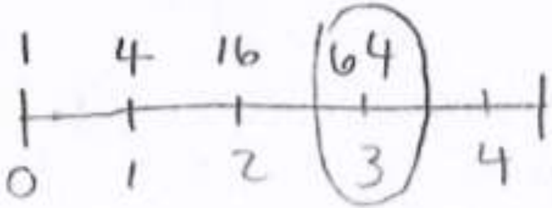
a) $\log_2 8 = 3$ b) $\log_2 16 = 4$ c) $\log_2\left(\frac{1}{4}\right) = -2$

d) $\log_2 32 = 5$ e) $\log_2\left(\frac{1}{32}\right) = -5$ f) $\log_2 1 = 0$

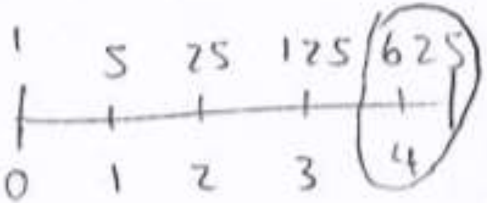
1) $\log_3 27 = 3$



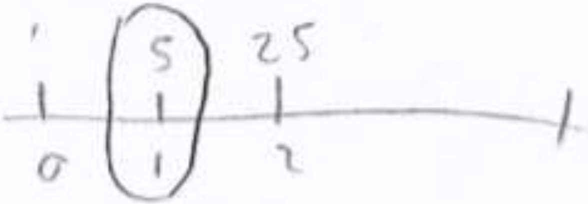
2) $\log_4 64 = 3$



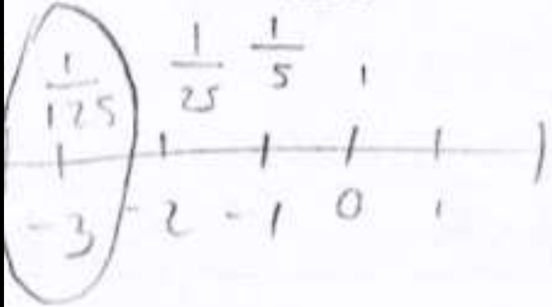
3) $\log_5 625 = 4$



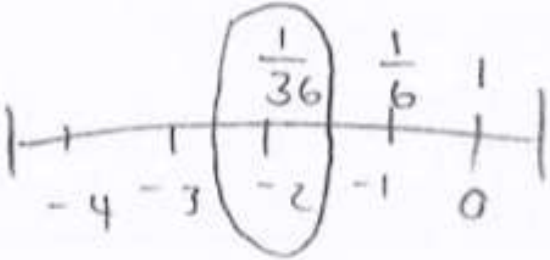
4) $\log_5 5 = 5$



5) $\log_5 \left(\frac{1}{125}\right) = -3$



6) $\log_6 \left(\frac{1}{36}\right) = -2$



Race to 1 Million

A new online game called "**Metropolis**" allows gamers to build cities.

The goal is to build your city as big as possible.

Right now some of the Ortho Teachers are playing *Metropolis* and Ms. Coley decides to reward the first teacher to build his/her population up to 1,000,000 people by giving them a week off.

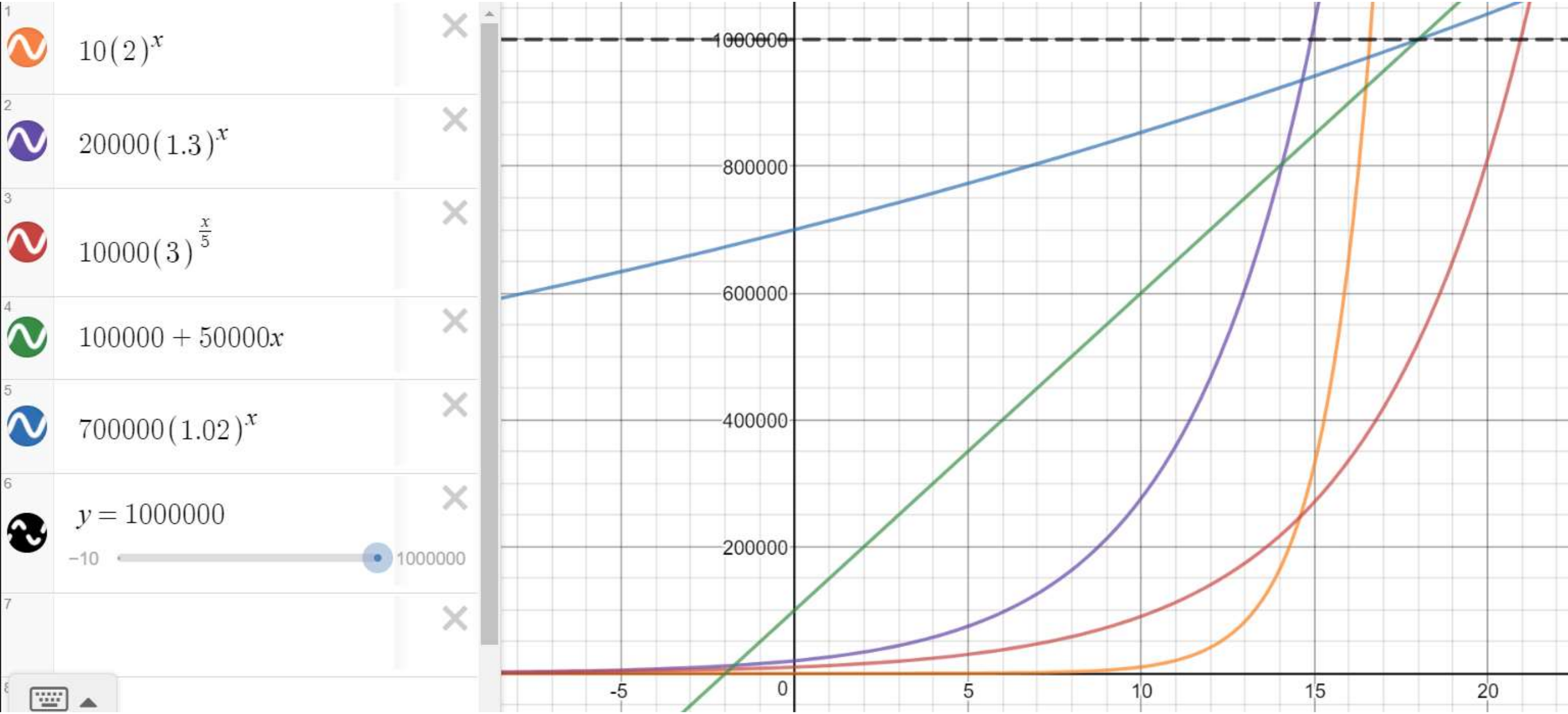
San Diegoza just started and has 10 people and is doubling in size every day.

Trancouver has 20,000 people and is growing exponentially at a rate of 30% each day.

Leung Beach has 300,000 people and is growing exponentially at a rate of 8% each day.

Baltimorales has 700,000 people and is growing exponentially at a rate of 2% each day.

Santi Barbrez as 100,000 people and is growing 50,000 per day. (Santibanez)



1
10(2)^x
2
20000(1.3)^x
3
10000(3)^(x/5)
4
100000 + 50000x
5
700000(1.02)^x
6
y = 1000000
−10
1000000
7
1000000
800000
600000
400000
200000
-5
0
5
10
15
20

1)

SAN DIEGOZA

$$\frac{10(2)^d}{10} = \frac{1{,}000{,}000}{10}$$

$$\log 2\ (2)^d = \log 2\ 100{,}000$$

$D \approx 17$ days

3)

LEUNG BEACH

$$\frac{10{,}000(3)^{d/5}}{10{,}000} = \frac{1{,}000{,}000}{10{,}000}$$

$$\log 3\ (3)^{d/5} = \log 3\ 100$$

$5 \cdot d/5 = 4.19 \cdot 5$

$d \approx 21$ days

1) Wode Islnd
2) San Diegoza
3) Baltimorales
4) TrancoUver
5) Leung Beach

proven

Wode Island

2.) $$\frac{20{,}000(1.3)^d}{20{,}000} = \frac{1{,}000{,}000}{20{,}000}$$

$$\log_{1.3}(1.3)^d = \frac{50}{\log 1.3}$$

days ≈ 15

4) Baltimorales

$$100{,}000 + 50{,}000(d) = 1000000$$

$$-100{,}000 \qquad -100000$$

$$\frac{50{,}000(d)}{50000} = \frac{900{,}000}{50000}$$

days ≈ 18

Trancover

5) 100% + 2% = 102

$$\frac{700{,}000(1.02)^d}{700{,}000} = \frac{1{,}000{,}000}{700{,}000}$$

$$\log 1.02\ (1.02)^d = \log 1.02\ 1.42$$

≈ 18 days

Year 1: Algebra 1

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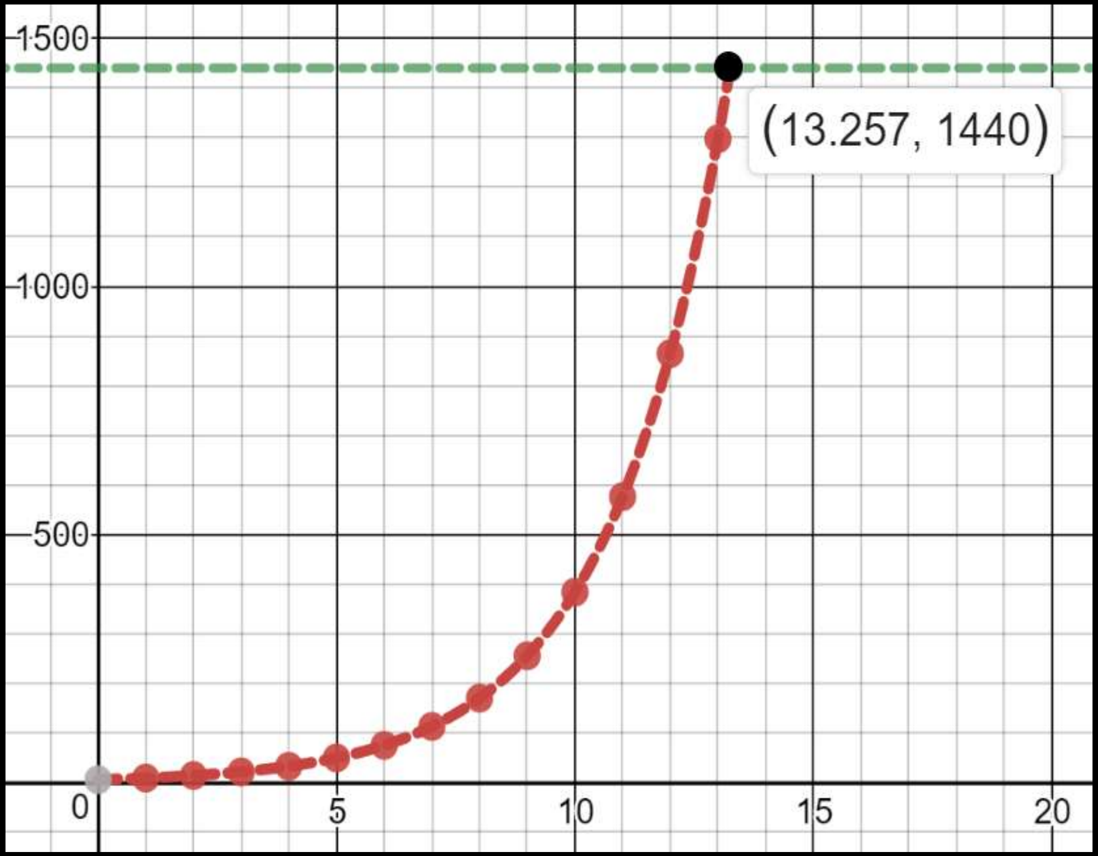
Ever since Clash Royale came out Roy has been playing more and more. On the first day he only played Royale for 10 minutes. He got frustrated and quit the training. On each of the following days he played for 1.5 times longer than the previous day.

Minutes of Royale played on day n:

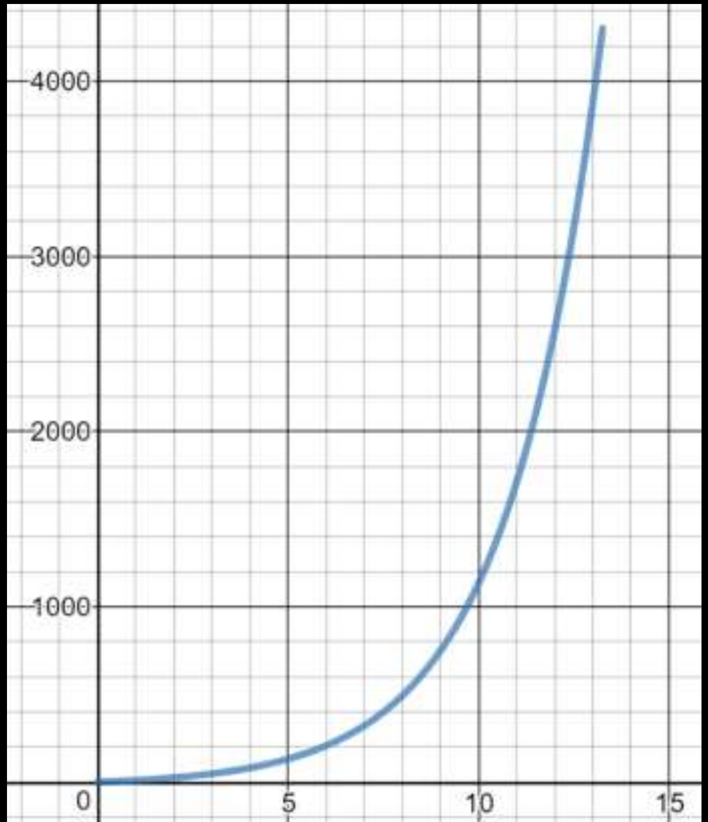
$$10(1.5)^{n-1}$$

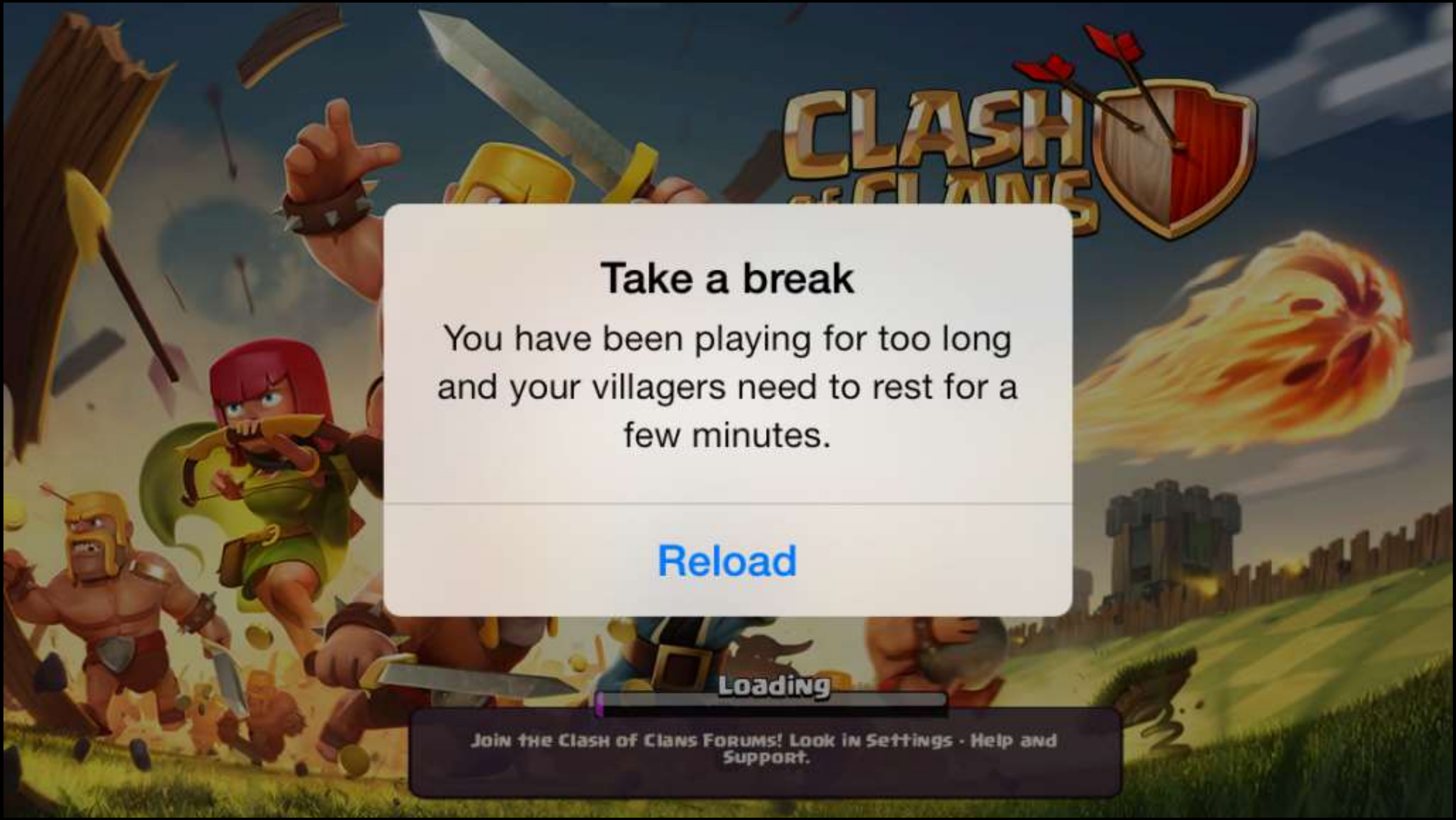


Total minutes of Royale played after d days:

$$\sum_{n=1}^{d} 10(0.5)^{n-1} = \frac{10(1-1.5^d)}{1-1.5}$$

$$= 20(1.5)^d - 10$$




CLASH of CLANS
Take a break
You have been playing for too long and your villagers need to rest for a few minutes.
Reload
Loading
Join the Clash of Clans Forums! Look in Settings - Help and Support.

Because Clash Royale was so fun, Roy had to cut back on Playing Clash of Clans. On the first day he played Clash Royale, Roy was still playing Clash of Clans for 400 minutes (that's almost 7 hours a day!). However, each day that followed he played half as many minutes of Clash of Clans as he had the day before.

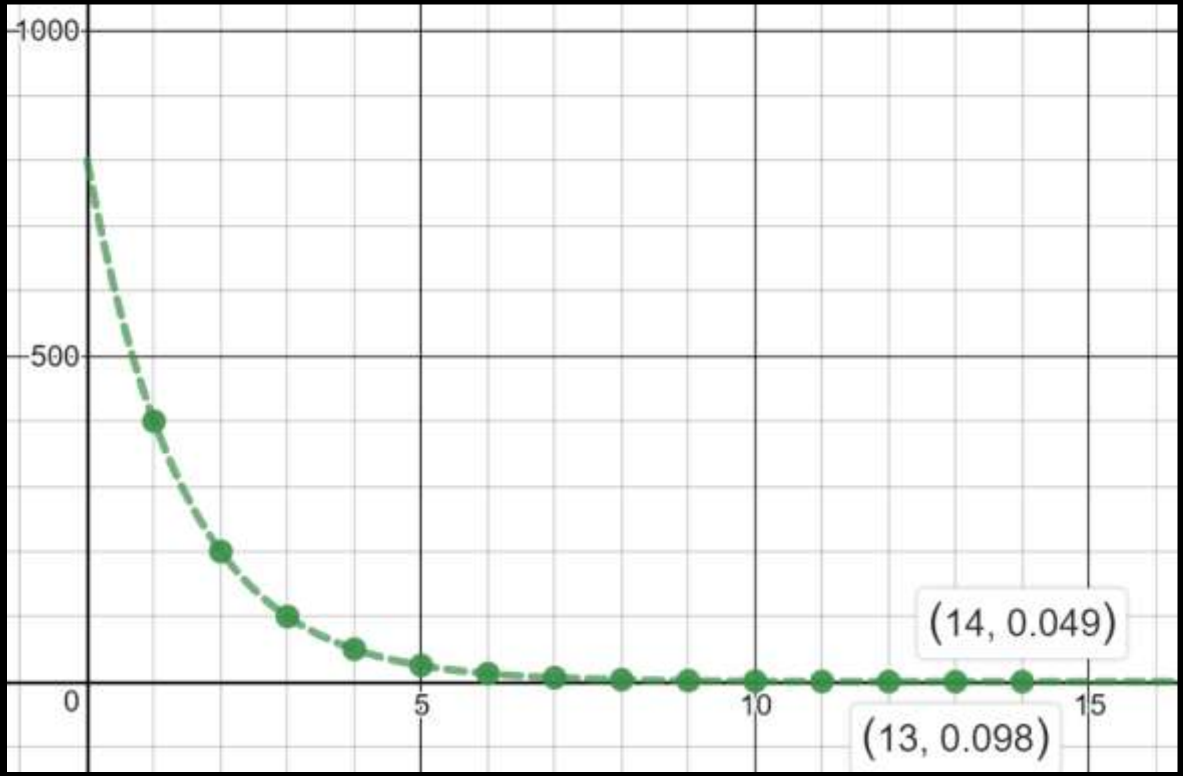
Minutes of Clash played on day n:

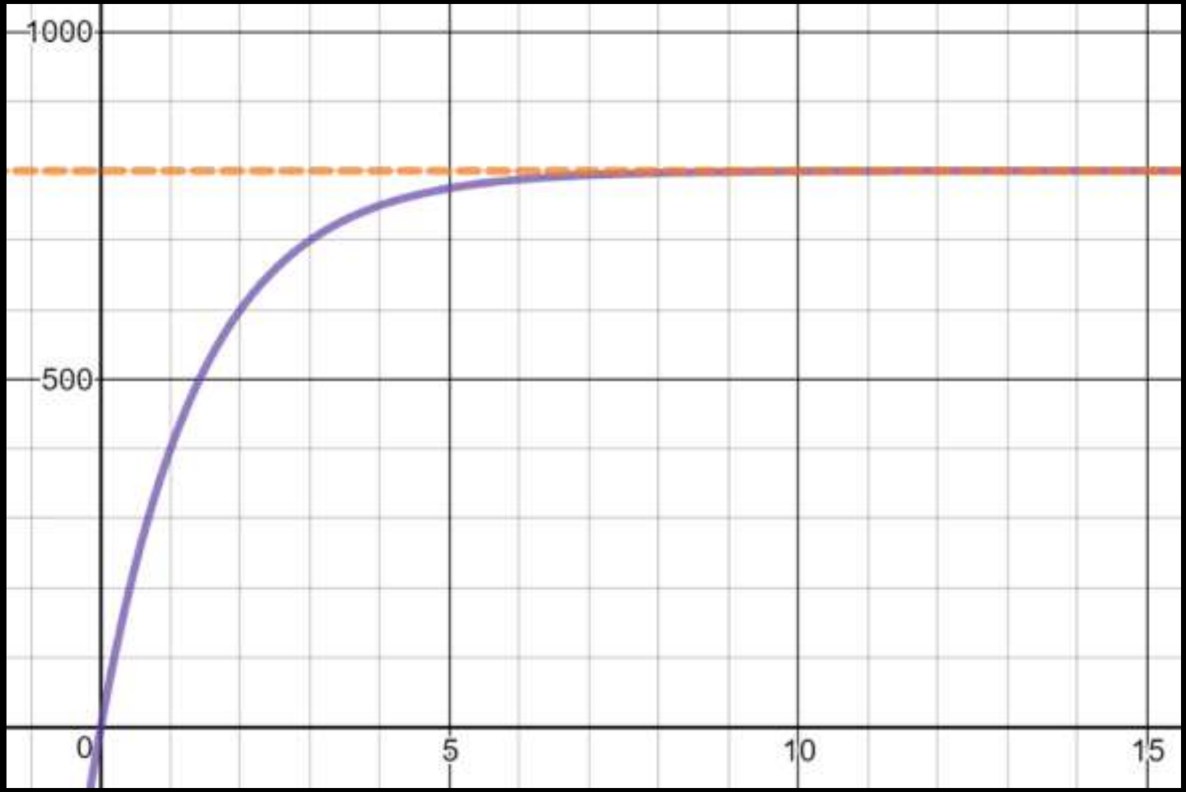
$$400(0.5)^{n-1}$$

Total minutes of Clash played after d days:

$$\sum_{n=1}^{d} 400(0.5)^{n-1} = \frac{400\left(1-0.5^{d}\right)}{1-0.5}$$

$$= 800 - 800(.5)^{d}$$

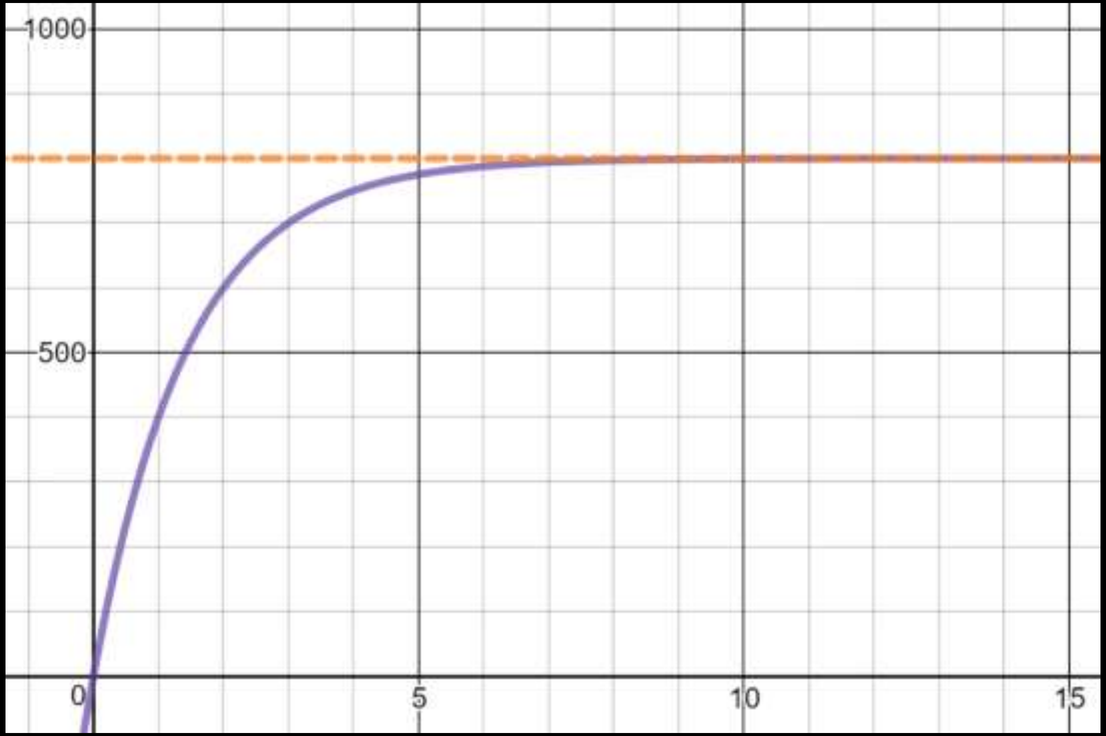




Is there a limit to how much Clash of Clans Roy will play for the rest of his life?

What is the limit? How do you know?

$$T(x) = 800 - 800(.5)^d$$



Quit For Love

He decided to try a slower approach by smoking 10% fewer cigarettes each month. He knew that when he got to the point when he smoked less than a pack a month he would be able to quit completely.

1 pack per day

30 days per month

20 cigarettes per pack

3 years to quit

1 marriage to save

Will he quit in time? How do you know?

1) Show a mathematical justification that my friend failed *or* that he was able to quit in time. Explain your answer in writing as well!

$f(n) = 600(.90)^{n-1}$

$f(n) = 600(.9)^{36-1}$

$f(n) = 600(.9)^{35}$

$f(n) = 600(0.02503155 5)$

$f(n) = 15$

$\frac{19}{600} = \frac{600(.90)^{n-1}}{600}$

$.031666667 = .9^{n-1}$

$\log_{.9} .0316667 = \log_{.9} .9^{n-1}$

$32.76 = n - 1$

$+1 \quad +1$

33.76

He succeeds because he only smokes 15 cigarettes a month. 15 is less than 20 so he will be able to quit. It only took him about 34 months to smoke less than 20 cigarettes.

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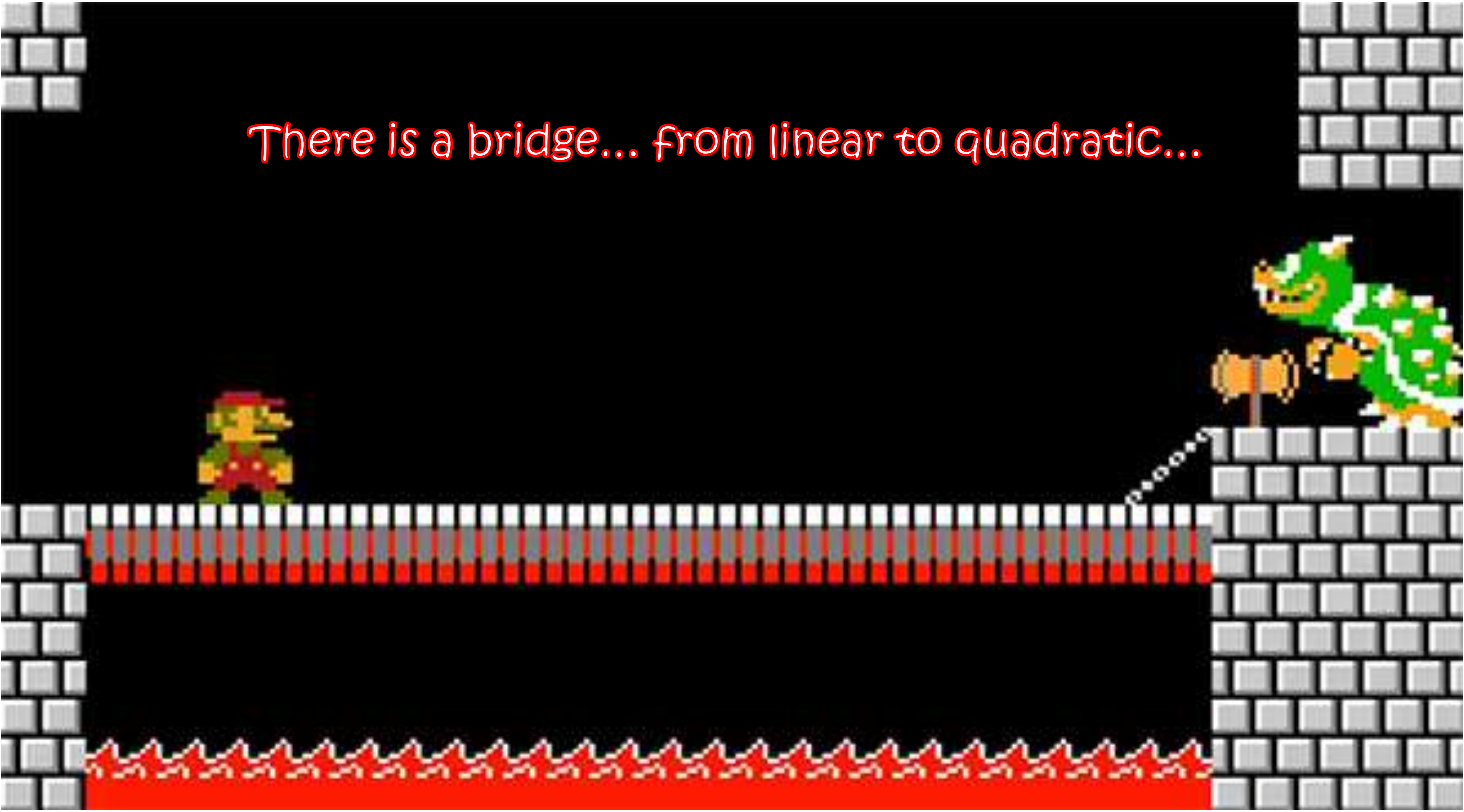
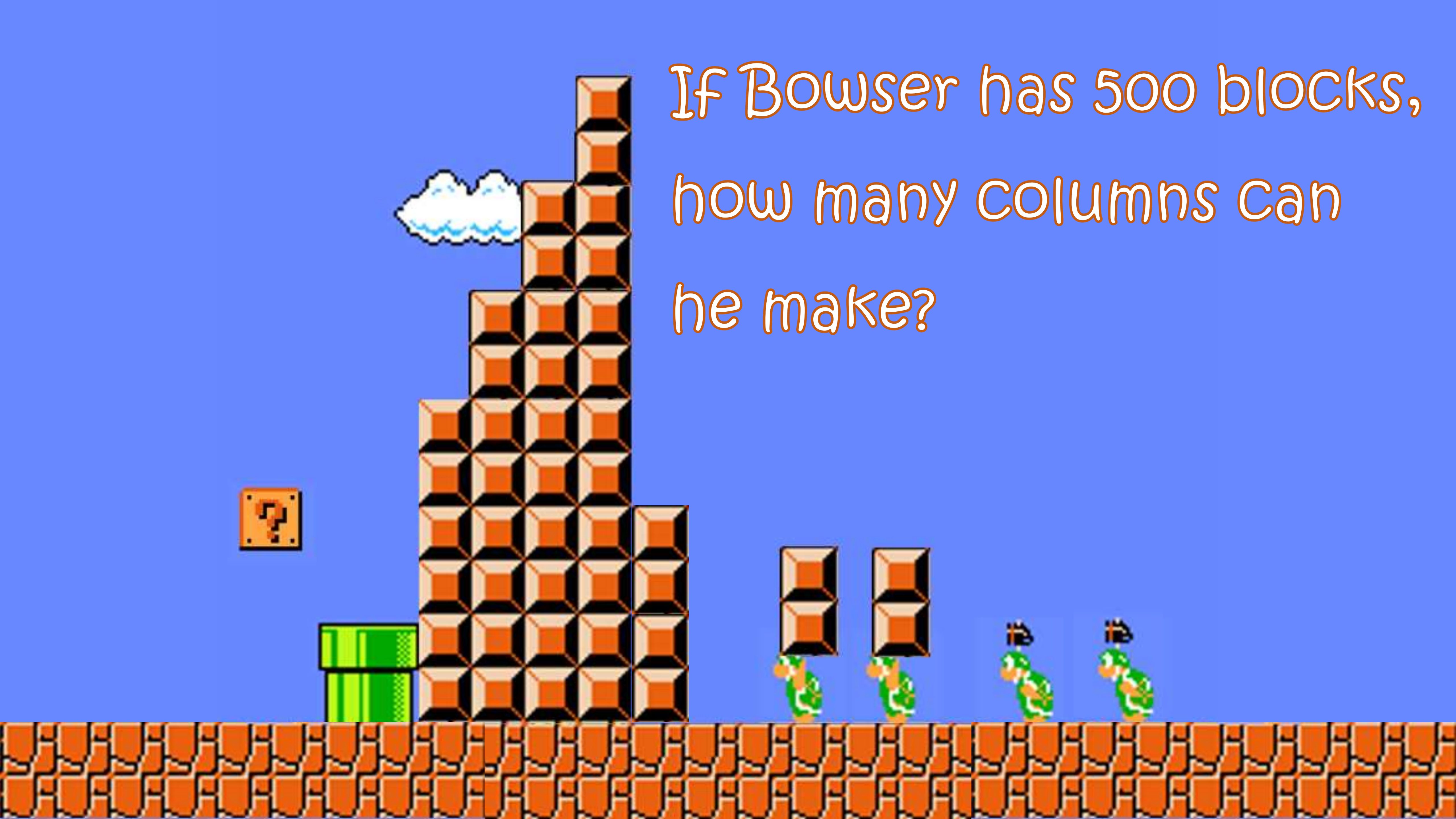

There is a bridge... from linear to quadratic...

If Bowser has 500 blocks, how many columns can he make?

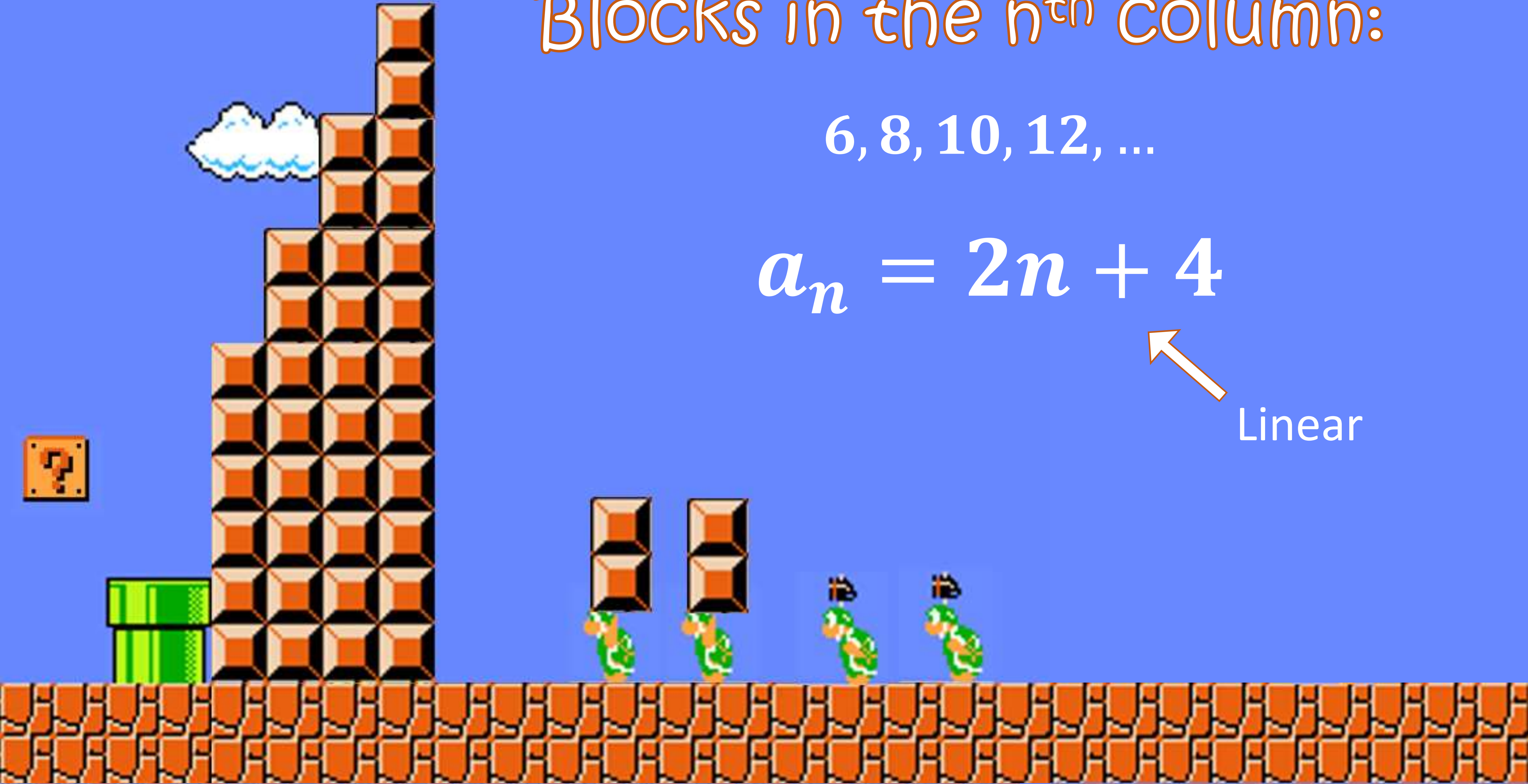


Blocks in the n^th column:

6, 8, 10, 12, …

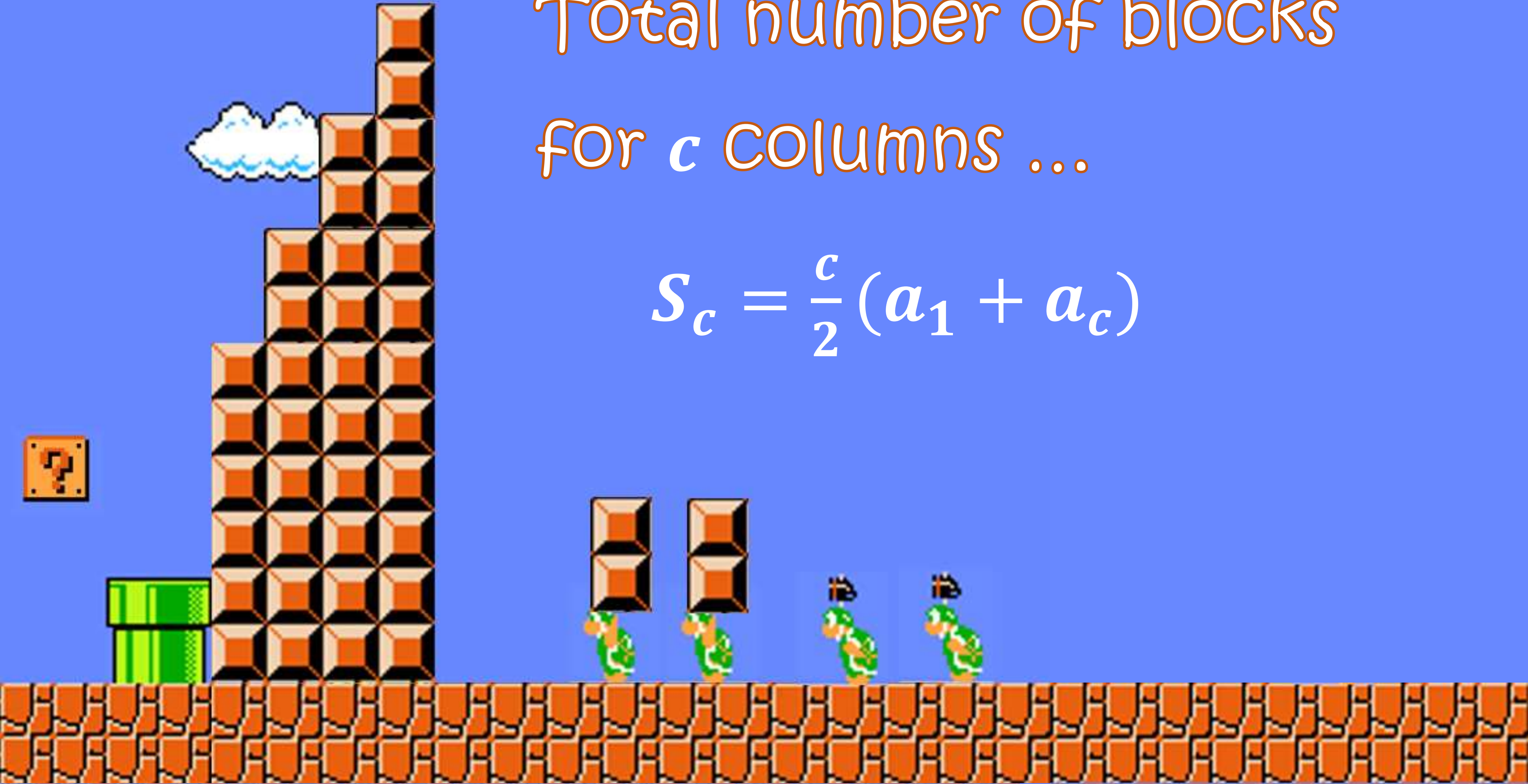
$$a_n = 2n + 4$$

Linear



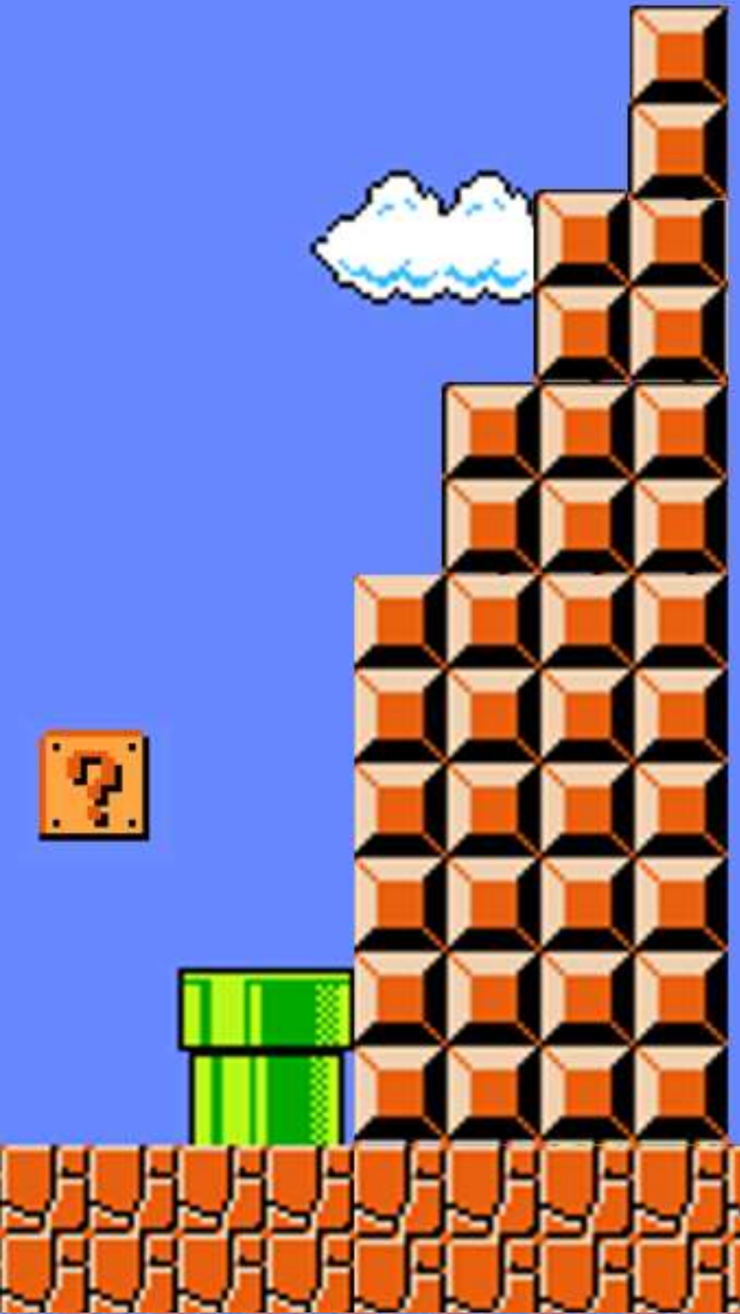
Total number of blocks for c columns ...

$$S_c = \frac{c}{2}(a_1 + a_c)$$



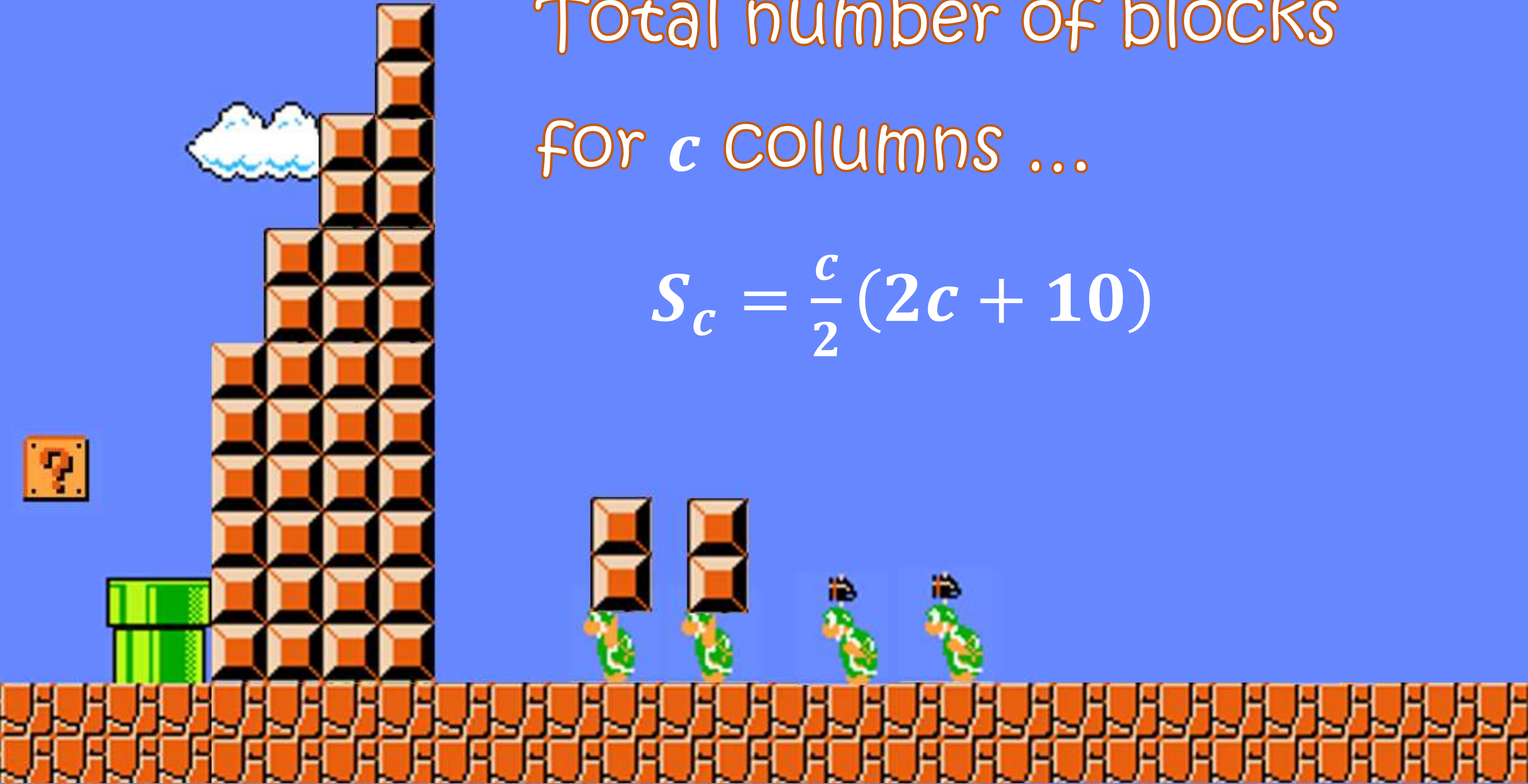
Total number of blocks for c columns …

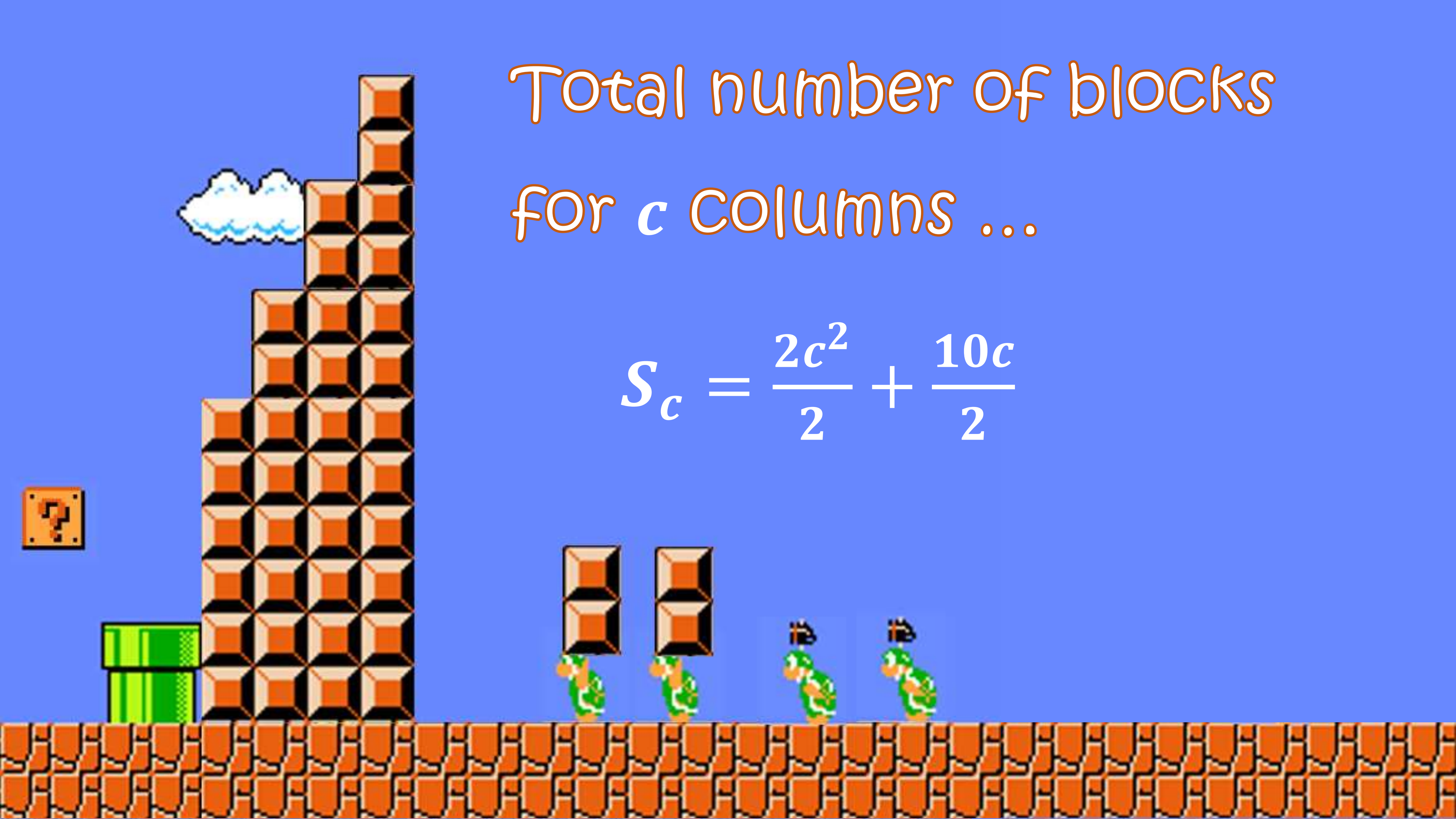
$$S_c = \frac{c}{2}(6 + (2c + 4))$$



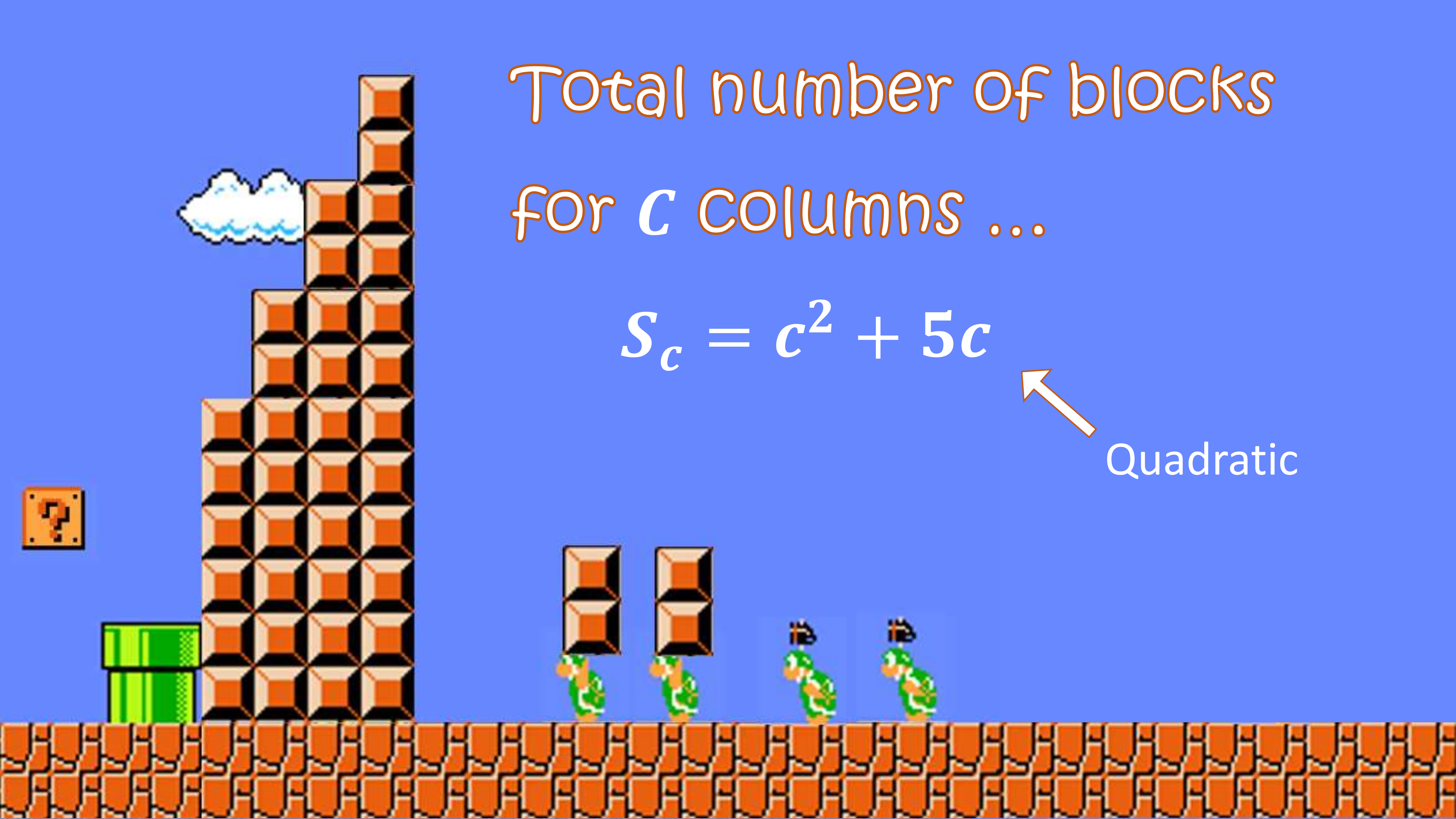
Total number of blocks for c columns ...

$$S_c = \frac{c}{2}(2c + 10)$$





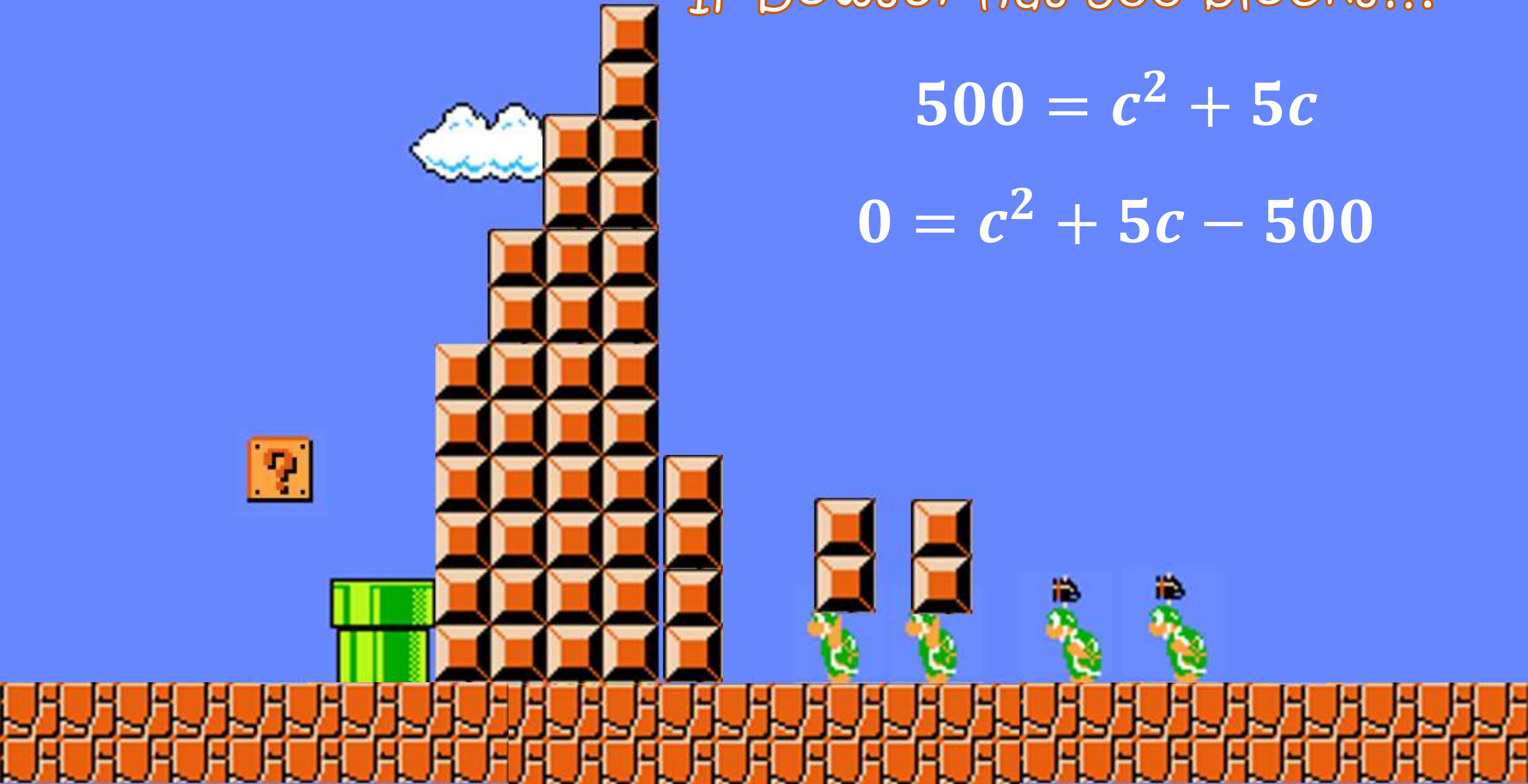
Total number of blocks
for c columns …
$$S_c = \frac{2c^2}{2} + \frac{10c}{2}$$


Total number of blocks
for c columns …
$S_c = c^2 + 5c$
Quadratic

If Bowser has 500 blocks…

$$500 = c^2 + 5c$$

$$0 = c^2 + 5c - 500$$




MARIO
102150
×25
WORLD
8-4
TIME
315
Bridge Complete

Say Bowser has 500 blocks. How many columns of wall will he be able to build? Show the work that leads to your answer.

$500 = x^2 + 5x$

columns of wall

$500 = 20^2 + 5(20)$

$= 400 + 100$

$= 500$

Created and Designed by

Liem'nNate hare now on sale at the student store!

Our investors.

Our Hoodies.

We are currently selling 200 hoodies per day at $40 per hoodie.

Our investors predict that for every price increase of $1 we will sell 2 fewer hoodies per day.

What price increase will generate the highest daily earnings?

200	40	8000
198	41	8118
196	42	8232
194	43	8342
192	44	844
190	45	8550
188	46	8648
186	47	8742
184	48	8832
182	49	8918
180	50	9000
178	51	9,078
176	52	9152
174	53	9222
172	54	9288
170	55	9350
168	56	9408
166	57	9462
164	58	9512
162	59	9558
160	60	9600
158	61	9638
156	62	9672
154	63	9702
152	64	9728
150	65	9750
148	66	9768
146	67	9782
144	68	9792
142	69	9798
140	70	9800
138	71	9798
136	72	~~9792~~
134	73	9782
132	74	
128	75	
126	76	
124	77	
122	78	
120	79	
118	80	

200	198	196	194	192	190
x	x	x	x	x	x
40	41	42	43	44	45
8000	8,118	8,316	8,514	8,712	8,910

198
+118
316

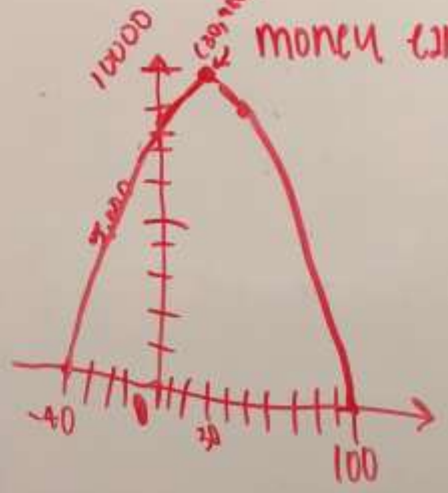
$a_n = 198$

$(40+D)(200-2d)$

198

$30 increase

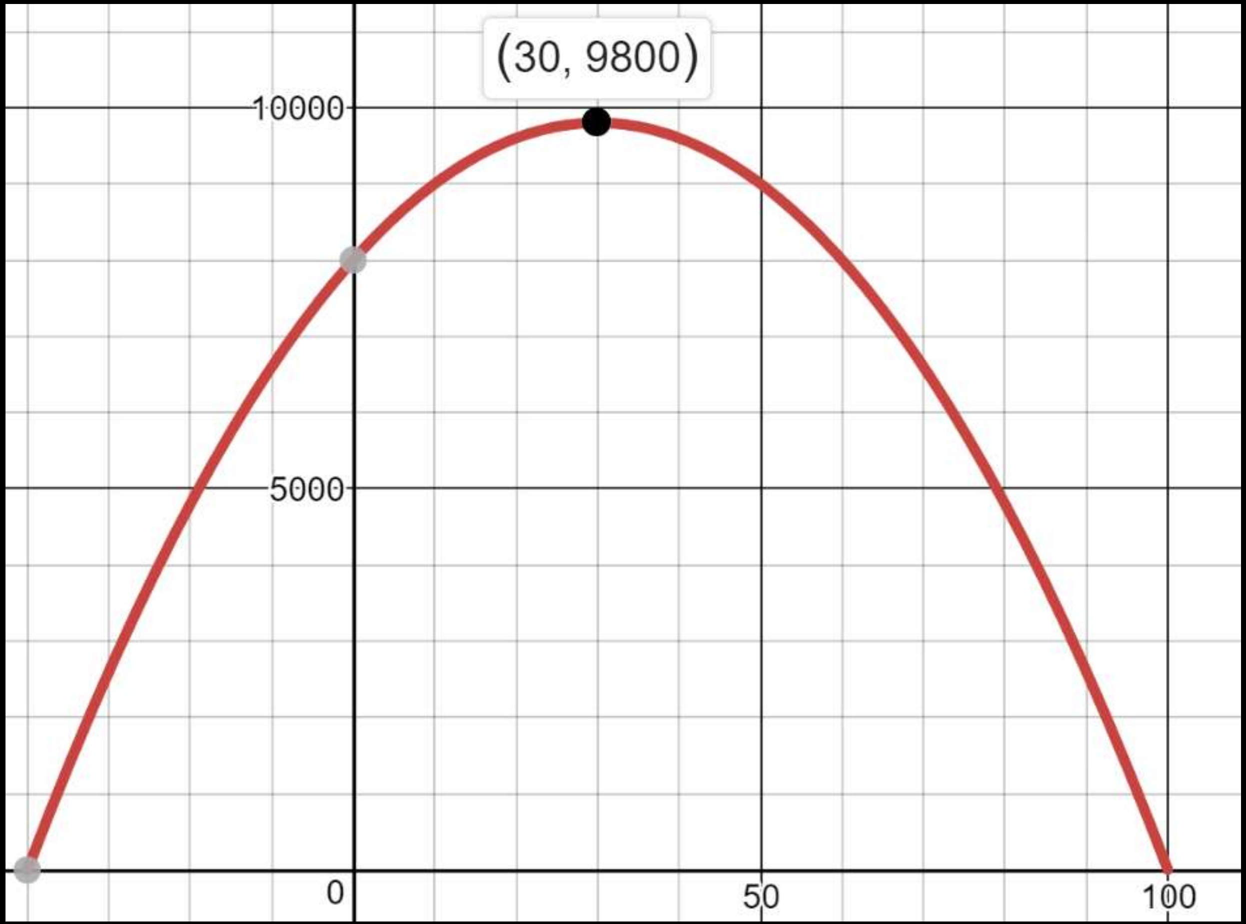
money earned = $9800



d is dollar increase

$$E(d) = (40 + d)(200 - 2d)$$

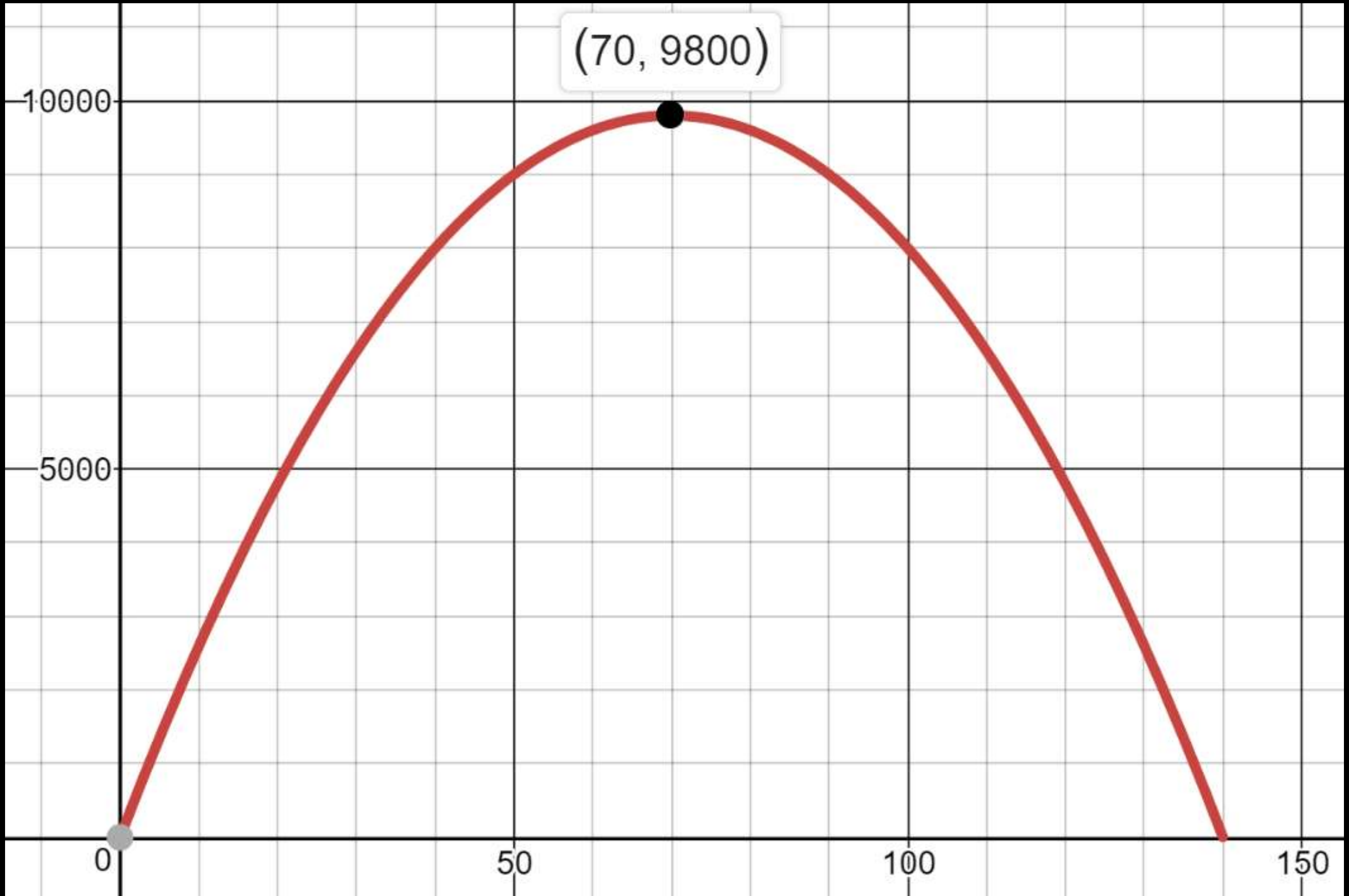
$$E(d) = -2d^2 + 120d + 8000$$



p is price per hoodie

$$E(p) = (p)(280 - 2d)$$

$$E(p) = -2p^2 + 280p$$



total earned

8000, 8118, 8232, 8342, 8448, 8550 ...

118, 114, 110, 106, 102, 98

1, 2, 3, 4, 5, 6

70 →

200	198	196	194
× 40	× 41	× 42	× 43
8000	8118	8232	8,342

192	190
× 44	× 45
8,448	8,550

70

30, 31, 30.5

$40 → 200
$41 → 198 −2
$42 → 196 −2
$43 → 194
$44 → 192
$45 → 190

8118
+ 114
8232

$a_n = 122 - 4n$

$0 = 122 - 4n$

$-122 \quad -122$

$-122 = -4n$

$\frac{-122}{-4} = \frac{-4n}{-4}$

$n = 30.5$

$2, −2

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"Sometimes Sets of Numbers are Related to Each Other"

Approximate Years of Education (After High School)	Median Yearly Income ($)
0*	34,736
1	38,532
2	41,184
4	57,252
6	68,952
8	85,228

2

$y_1 \sim mx_1 + b$

STATISTICS

$r^2 = 0.9607$

$r = 0.9801$

RESIDUALS

e_1

PARAMETERS

$m = 5567.46$ $b = -30945.9$

3

$y_1 \sim a(b)^{x_1}$

Log Mode

STATISTICS

$R^2 = 0.9949$

RESIDUALS

e_2

PARAMETERS

$a = 8976.55$ $b = 1.11955$

4

$y_1 \sim ax_1^2 + bx_1 + c$

STATISTICS

$R^2 = 0.9962$

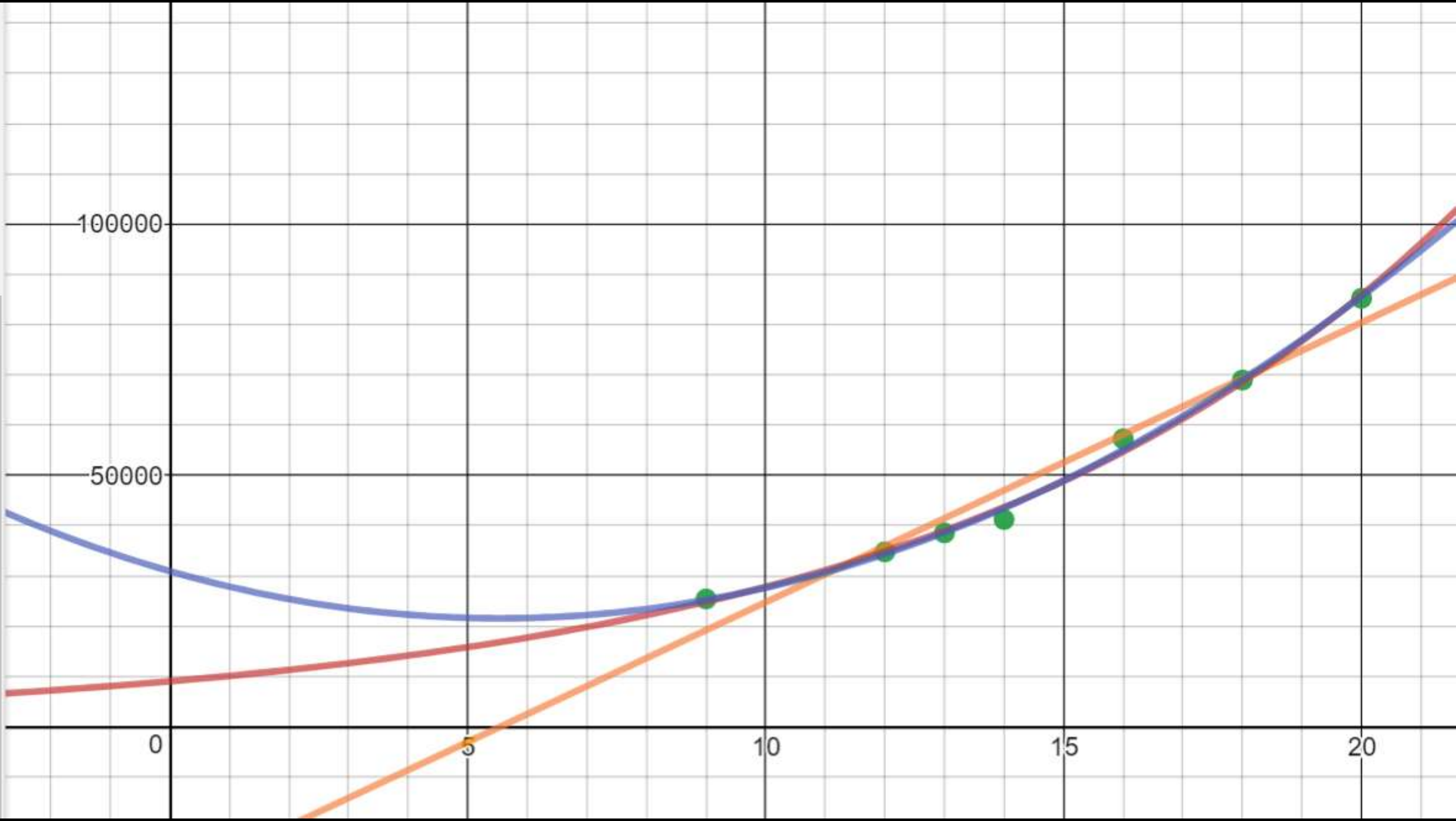
RESIDUALS

e_3 plot

PARAMETERS

$a = 306.29$ $b = -3392.32$

$c = 30914.4$



<u>Year 1: Algebra 1</u>

0) Numbers (Origins and Number Sense)
1) Relations & Functions
2) Linear Expressions & Functions (growing by a constant amount)
3) Arithmetic Sequences & Series
4) Exponential Expressions & Functions (growing by a constant factor)
5) Geometric Sequences & Series
6) Bridge to Quadratics (Arithmetic Series) & Quadratic Functions
7) Intro to Statistics
8) **Equations & Inequalities (including Absolute Value) & Systems**

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WORK IN PROGRESS

Year 2: The Bridge to Calculus (aka Turning Up the Math)

aka Pre Calculus

Year 2: Honors Advanced Math

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3) Operations on Functions (including Inverses and Composition)

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Quiz: Unit N

1) Assume $A = \frac{55}{12}$ and $B = \frac{14}{45}$. Evaluate and simplify each:

a) $A + B$

b) $A - B$

c) $A \cdot B$

d) $A \div B$

2) Say that C and D are complex numbers such that $C = 2 - 5i$ and $D = 8 + 3i$. Compute each:

a) $C + D$

b) $C - D$

c) $C \times D$

d) $\frac{C}{D}$

3) Give an example of each of the following:

a) An Integer that is not a Whole Number.

b) A Complex Number that is not a Real Number.

c) An Irrational Number.

d) A Rational Number that is not an Integer.

4) Between which two consecutive integers is… Explain how you know.

a) $\sqrt{15} + e$

b) $\pi + \frac{9}{4}$

5) Evaluate each:

a) $(3i)^5$

b) i^{100}

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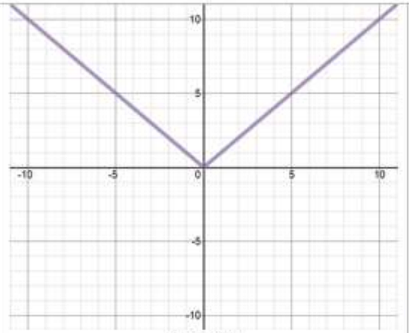
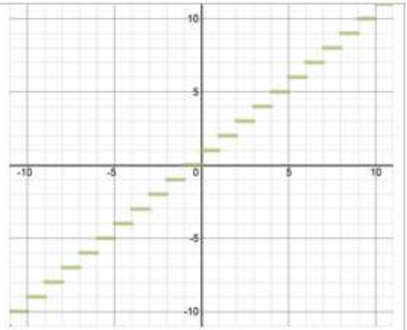
20 Functions

We grouped these 3 functions together because...

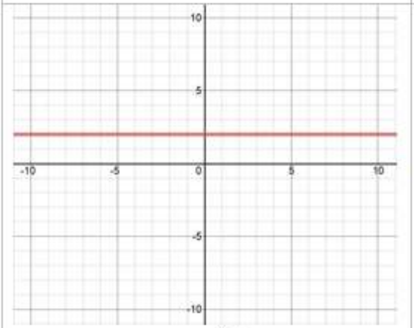
We grouped these 2 functions together because...



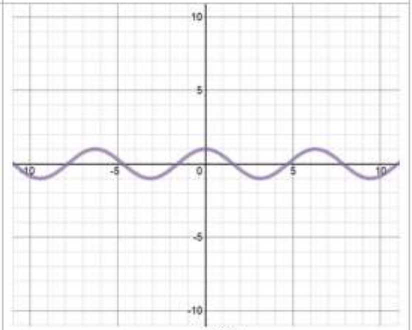
y = a(x)



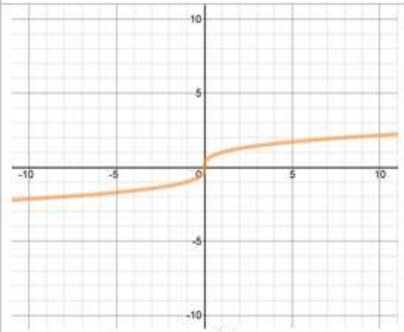
y = b(x)



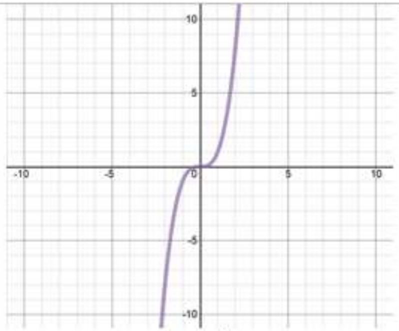
y = c(x)



y = d(x)

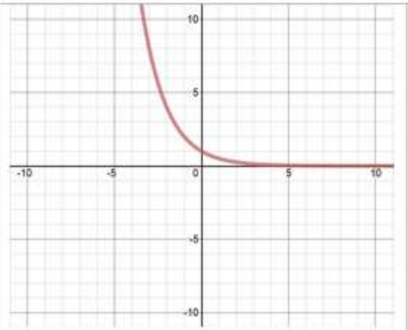


y = f(x)

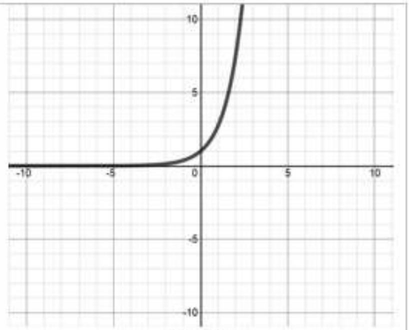


y = g(x)

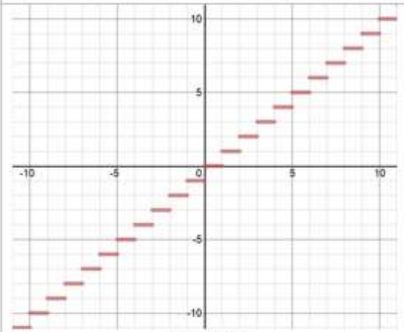
Created and designed by



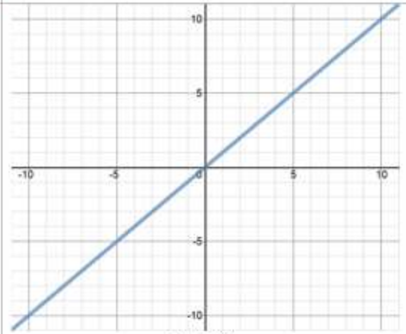
y = h(x)



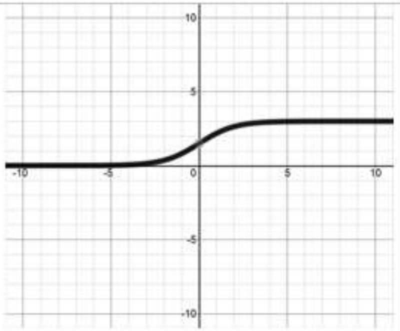
y = j(x)



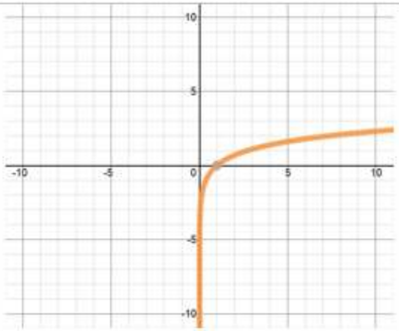
y = k(x)



y = l(x)



y = m(x)



y = n(x)

Created and designed by

The function j is a type of "exponential function." Answer the following questions about j.

1) What is the domain of j?

2) What is the range of j?

3) How many asymptotes does j have? How do you know?

4) Write the equation for each asymptote.

5) On which intervals is the function increasing? Decreasing?

6) Let's think about the ***intercepts*** of j.

   a) Does j have any roots? If so, what are they? If not, explain why not.

   b) Does j have a y-intercept? If so what is it? If not, explain why not.

8) As we input bigger *x-values* into the function j, what output does the function approach?

9) As we input smaller (negative) *x-values* into the function j, what output does the function approach?

20 Functions
Expert Edition

Sketch the graphs here	We grouped these functions together because vocab, vocab, vocab...	Write the Equations here

And so on...

Year 1: Algebra 1

0) Numbers (Origins and Number Sense)

1) Relations & Functions

Year 2: Honors Advanced Math

0) Numbers (Rational, Radical, and Complex)

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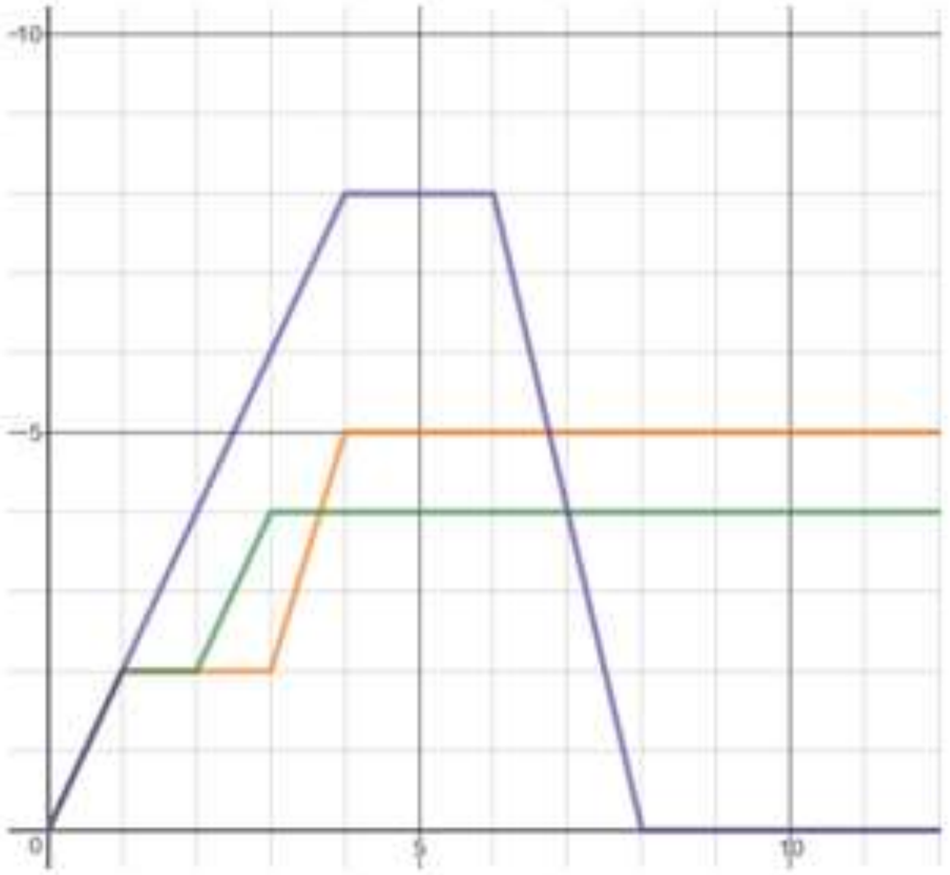
2.1 Shifting the Blame (Get a Clue)

Mrs. White has been murdered!

It happened at the Library at 4:30 PM on Saturday. She was found stabbed with a knife and hit with a candlestick in the …Mystery Section. The townspeople are furious! Detectives Tran and Detective Goza are on the case. They need your help!

GRAPH:



EQUATIONS:

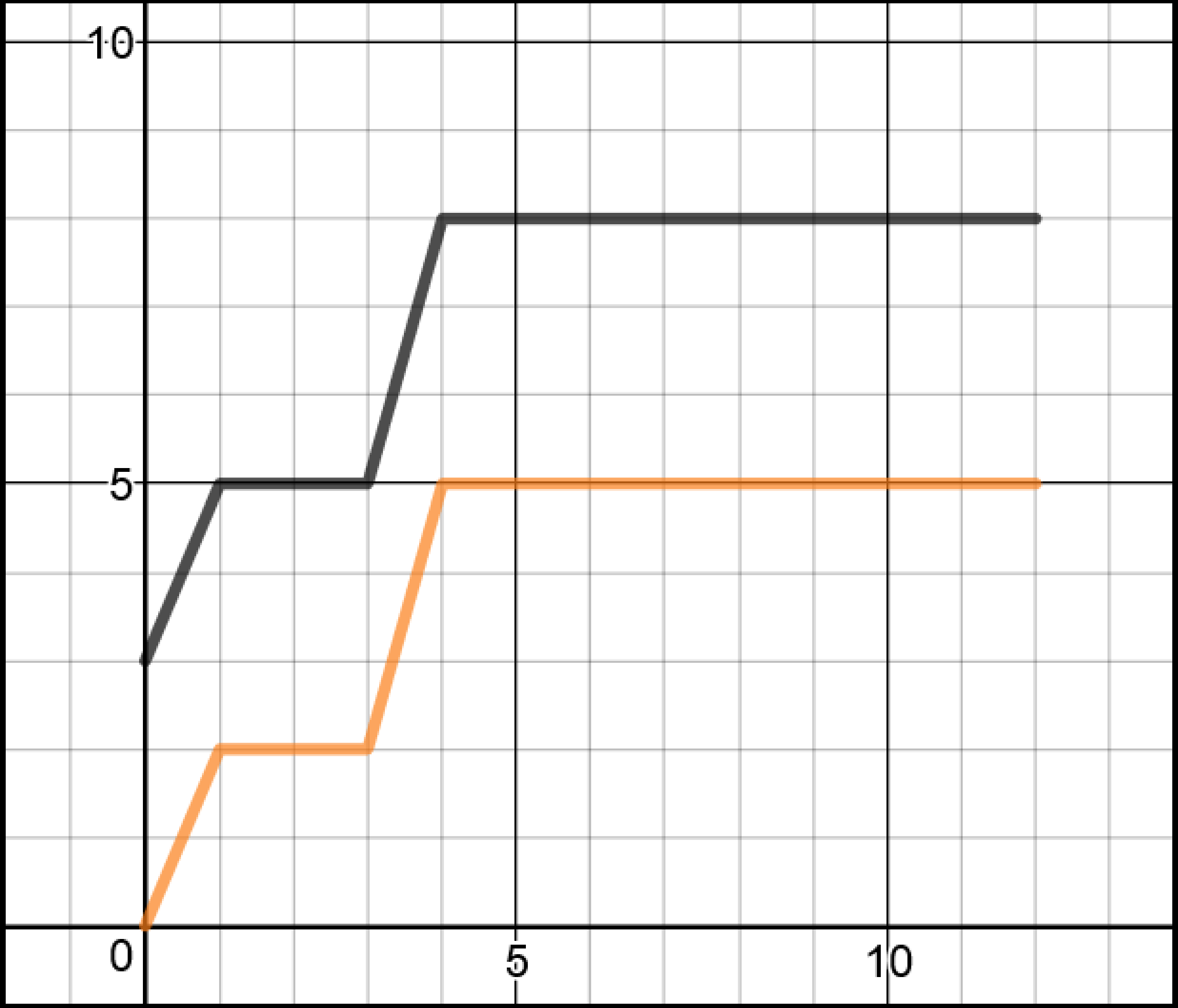
Mustard: $M(t)$ Green: $G(t)$ Plum: $P(t)$

$$M(t) = \begin{cases} 2t & 0 \le t < 1 \\ 2 & 1 \le t < 3 \\ 3t - 7 & 3 \le t < 4 \\ 5 & 4 \le t \le 12 \end{cases}$$

$$G(t) = \begin{cases} 2t & 0 \le t < 1 \\ 2 & 1 \le t < 2 \\ 2t - 2 & 2 \le t < 3 \\ 4 & 3 \le t \le 12 \end{cases}$$

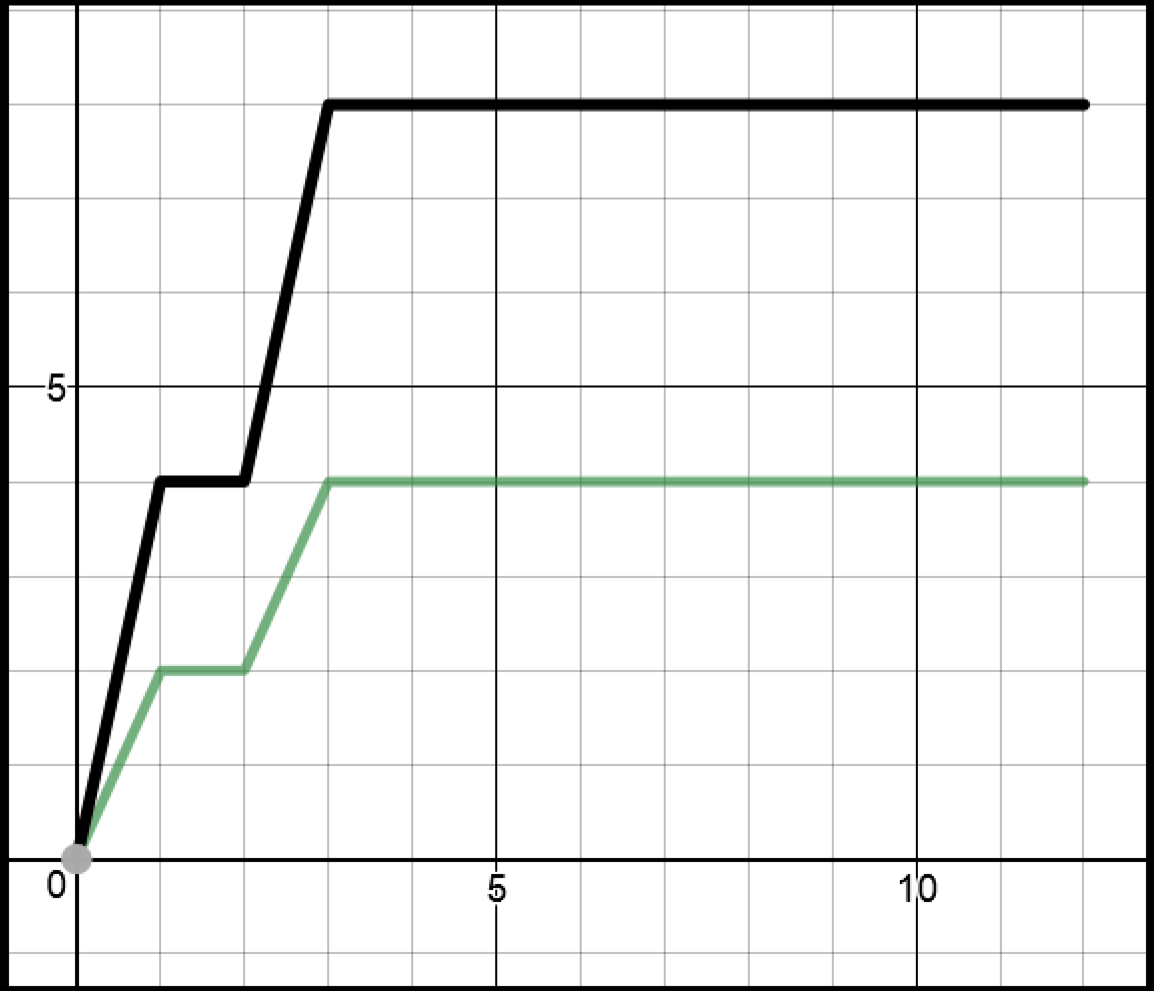
$$P(t) = \begin{cases} 2t & 0 \le t < 4 \\ 8 & 4 \le t < 6 \\ 32 - 4t & 6 \le t < 8 \\ 0 & 8 \le t \le 12 \end{cases}$$

CLUE 1: Colonel Mustard did not lie about the time intervals when he was walking or the rates at which he was walking, but he did lie about where he started. He actually started his day at his own house.



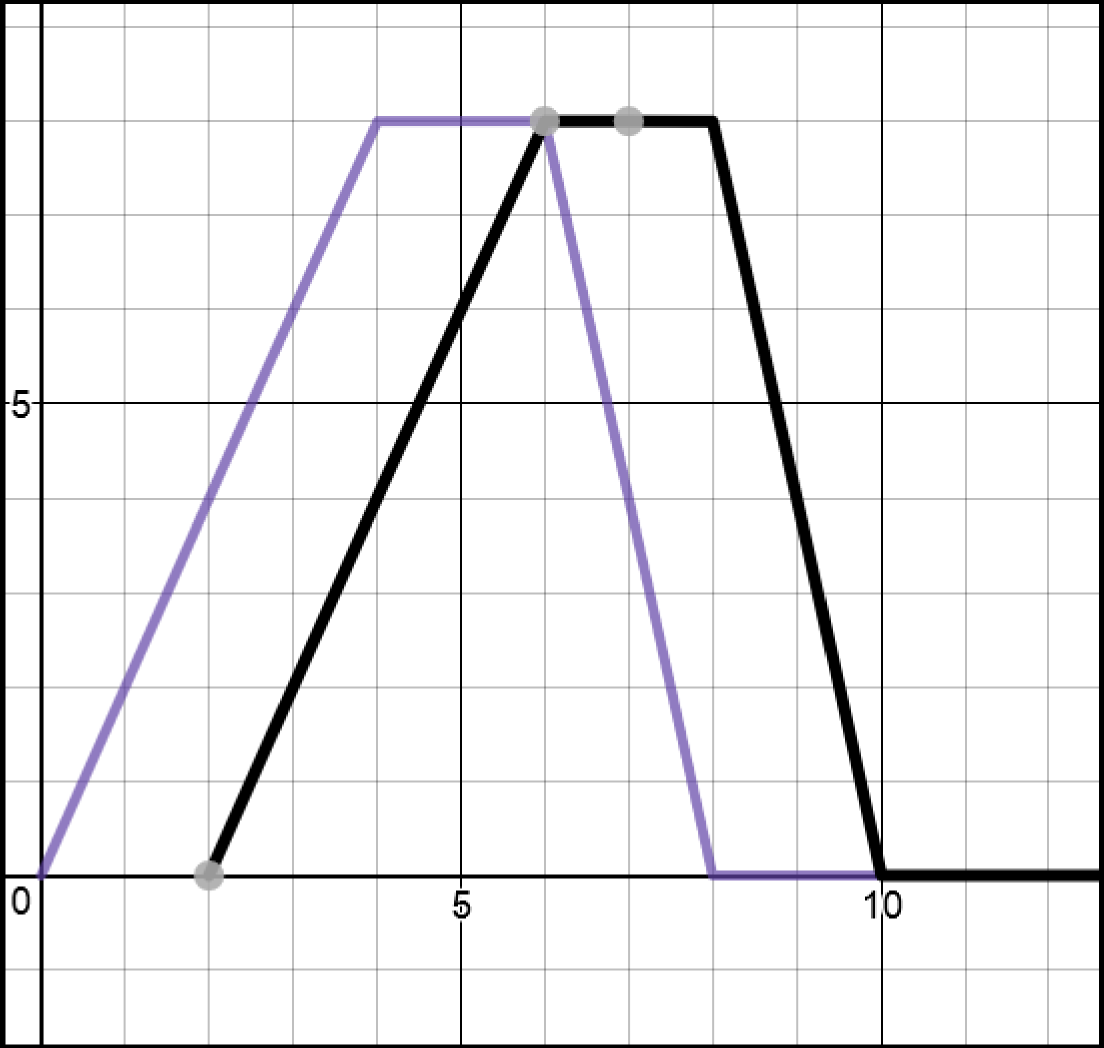
$$M_2(t) = M(t) + 3$$

CLUE 2: Mr. Green did not lie about the time intervals he was walking or where he started, but at all times when he was walking he moved twice as fast as he described in his story.



$$G_2(t) = 2G(t)$$

CLUE 3: Professor Plum did walk slowly to the library and quickly back, just as Mustard and Green described, but they lied about the times they were with him because he actually left his home 2 hours later than usual (at 2:00 PM).



$$P_2(t) = P(t-2)$$

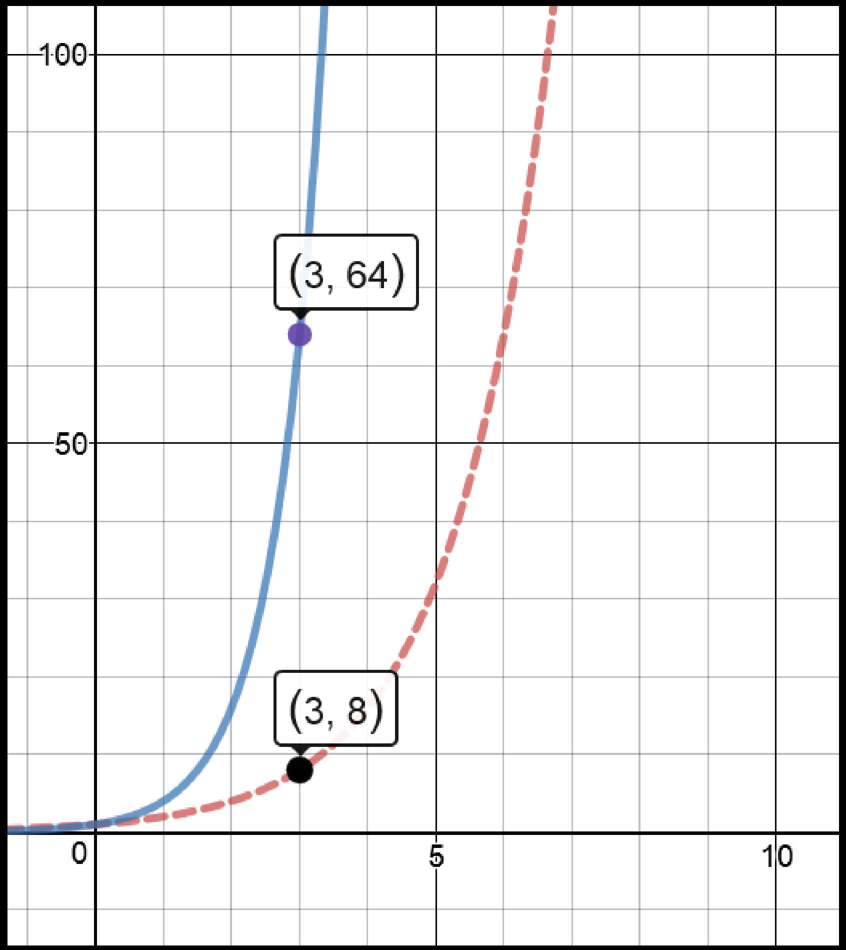
Rumor Has It

Apparently Mrs. Peacock was seen with Mr. Green and Colonel Mustard in her car on the evening of the murder!

Let's assume that the rumor was started by just one person at noon on Sunday. If each person who know the rumor tells one more person every hour…

$$R(t) = 2^t$$

If the rumor spreads twice as fast …

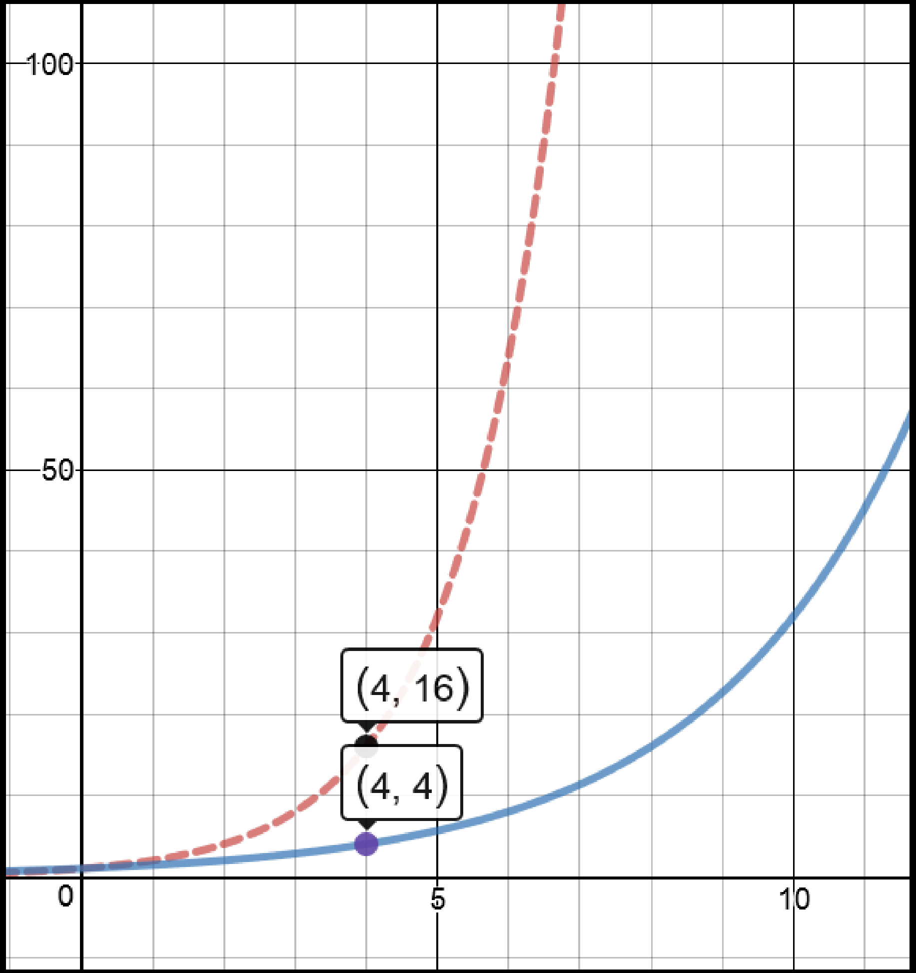


$$R_2(t) = 2^{2t}$$

or

$$R_2(t) = 2^{t/.5}$$

$R_2(t)$ is a horizontal **Shrink** of $R(t)$.



If the rumor spreads half as fast…

$$R_2(t) = 2^{.5t}$$

or

$$R_2(t) = 2^{t/2}$$

$R_2(t)$ is a horizontal **Stretch** of $R(t)$.

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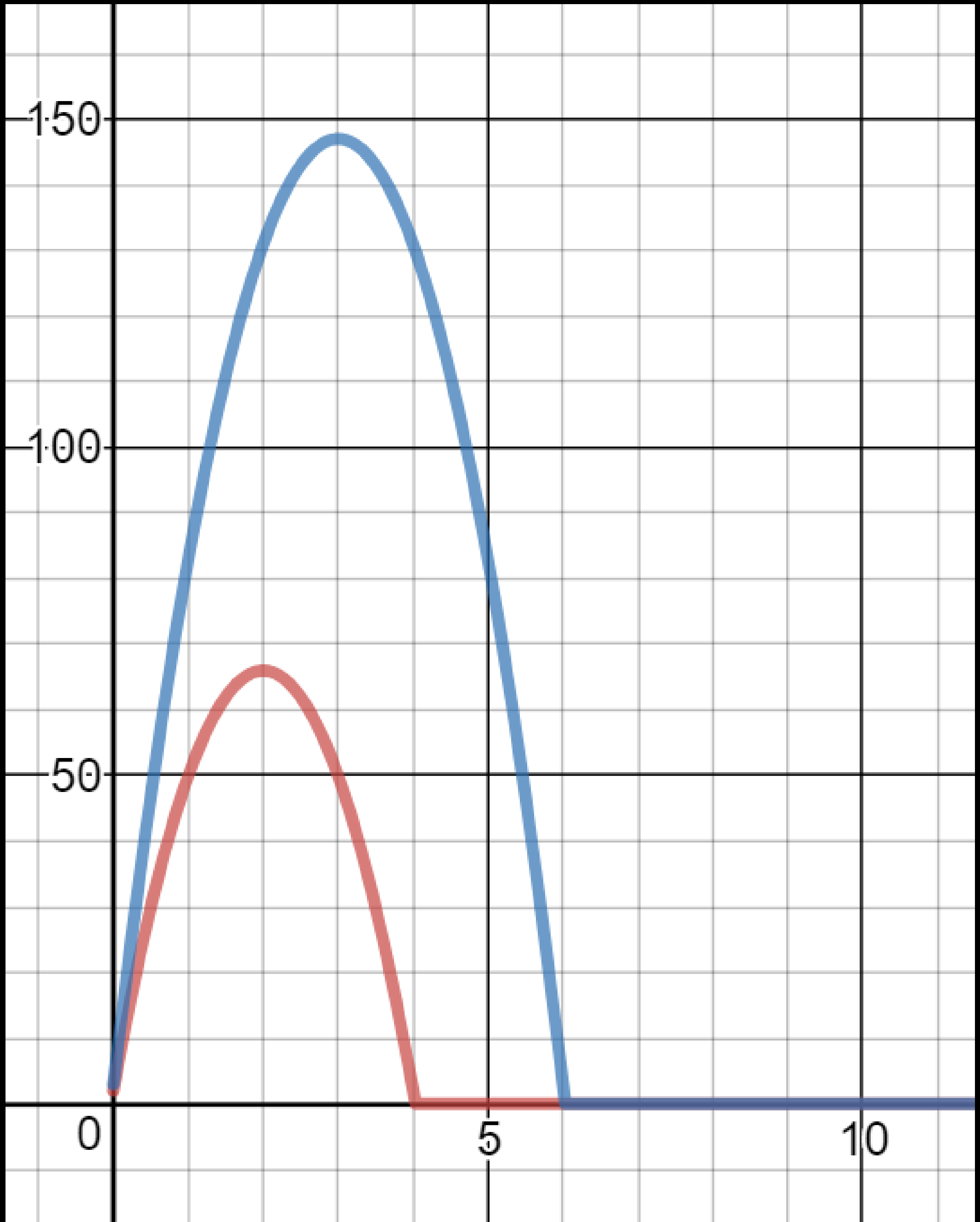
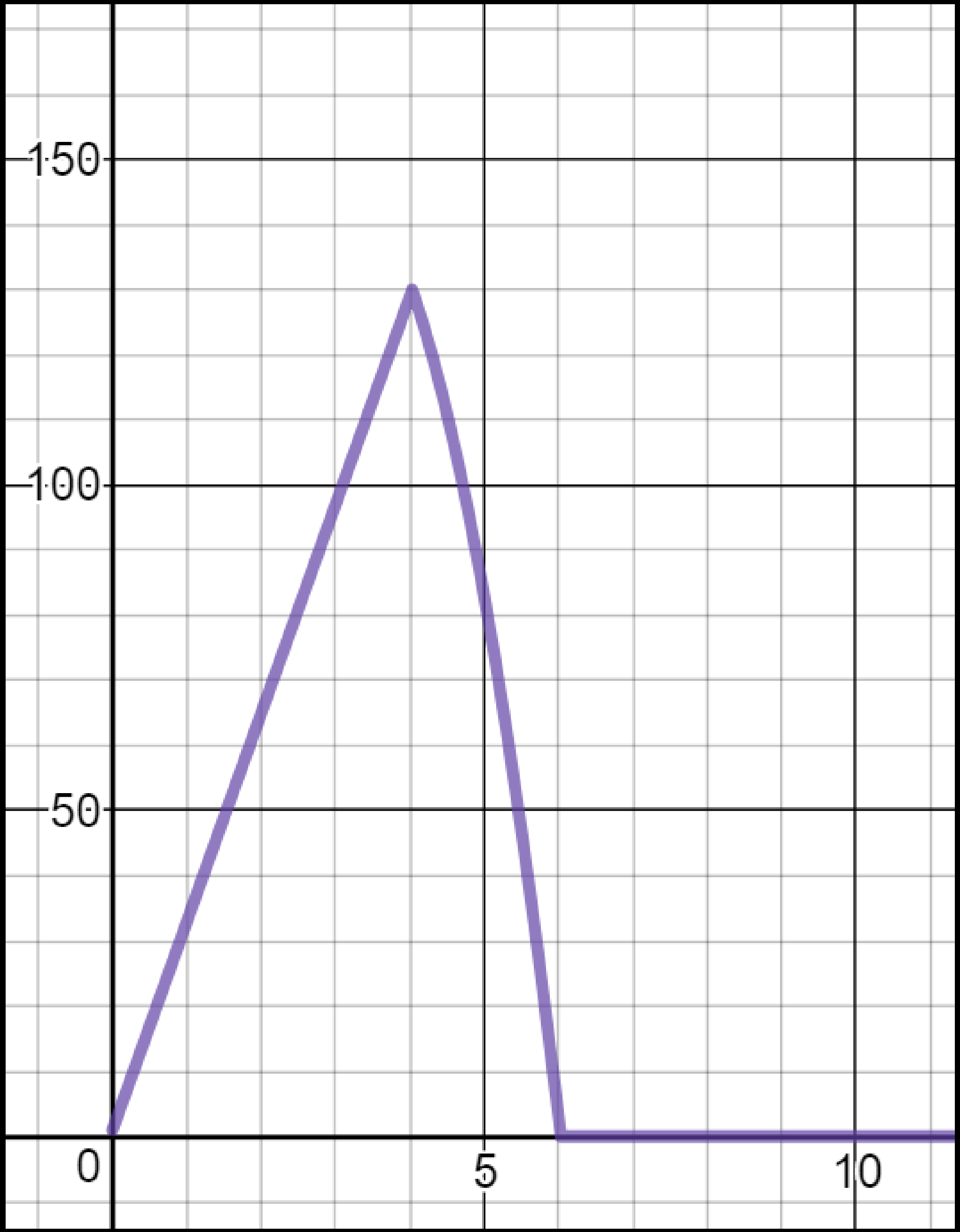
A Big League Difference

Mr. Goza challenged Yasiel Puig to a contest to see who could hit a baseball higher into the air. Mr. Goza went first. He placed a ball onto a tee that was 2 feet off the ground and hit the ball into the air with an initial vertical velocity of 64 ft/sec. Then Yasiel stepped up. He is taller than Mr. Goza, so he adjusted the tee to 3 feet and hit his ball giving it an initial vertical velocity of 96 feet/sec.



150
100
50
0
5
10


150
100
50
0
5
10

Breaking Bieber

Velocity (mph) v	Distance traveled during reaction time(feet) $R(v)$	Distance traveled after breaks are applied (feet) $D(v)$	Total Breaking Distance (feet) $B(v)$
0	0	0	
10	25	4	
20	50	16	
30	75	36	
40	100	64	
50	125	100	
60	150	144	
70	175	196	
80	200	256	
90	225	324	

$$B(v) = R(v) + D(v)$$

$$B(v) = 2.5t + .04t^2$$

$$v(t) = 10t$$

$$B(t) = B\big(v(t)\big) = 1.5(10t) + .04(10t)^2$$

$$B(t) = 4t^2 + 15t$$

5 seconds after the light turns green Selena Gomez (who is in the passenger seat) sees a bunny on the road 225 feet ahead of the car.

She screams out for Justin to stop!

Will Bieber be able to stop in time??

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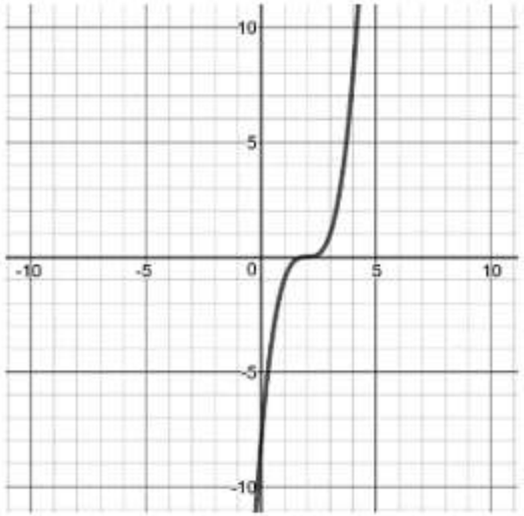
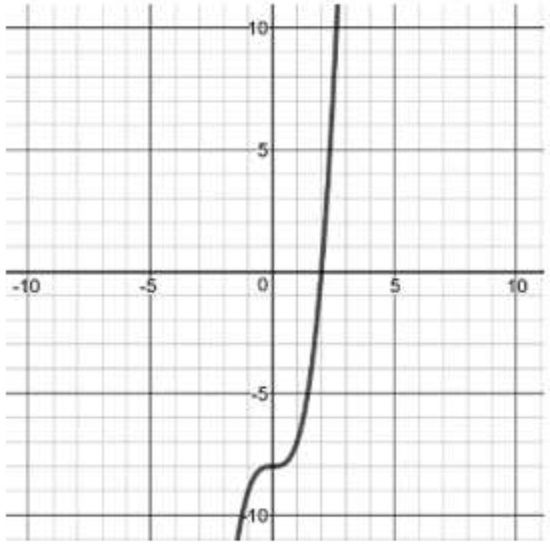
Given $f(x) = x^3 - 8$ and $g(x) = (x - 2)^3$…

a) Without graphing, show that $f(x) \neq g(x)$.

b) Which of the graph below is $f(x)$ and which is $g(x)$? Explain how you know.





________________ ________________

c) Without factoring, explain how you know that $f(x)$ has one real root and $g(x)$ has three.

d) Find all Complex Roots of $f(x)$.

Sketch the graph of **the** $f(x) = x^5 + 3x^4 - 8x^3 - 24x^2 + 16x + 48$.

(Hint: Factor by Grouping)

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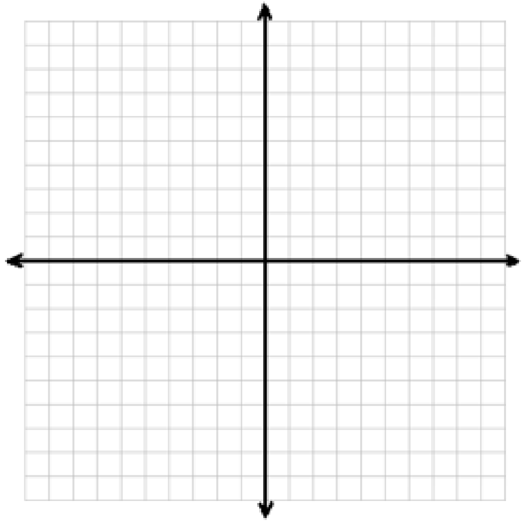
For 6 – 9 assume $g(x) = \frac{x-2}{x^2+4x-12}$.

6) Evaluate each:

a) $\lim_{x \to -5} g(x) =$

b) $\lim_{x \to 2} g(x) =$

7) Sketch the graph of $y = g(x)$ highlighting any important characteristics.



8) Explain how this function is related to the reciprocal function.

9) Evaluate each:

a) $\lim_{x \to -6^+} g(x) =$

b) $\lim_{x \to -6^-} g(x) =$

c) $\lim_{x \to -6} g(x) =$

Graph the function $f(x) = \frac{5(x+3)^5(x-5)^4(x-1)^2}{(x+1)(x-5)^7(x+3)^3}$.

Evaluate each:

a) $\lim_{x \to -3} f(x) =$

b) $\lim_{x \to \infty} f(x) =$

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Trig.

All of it.

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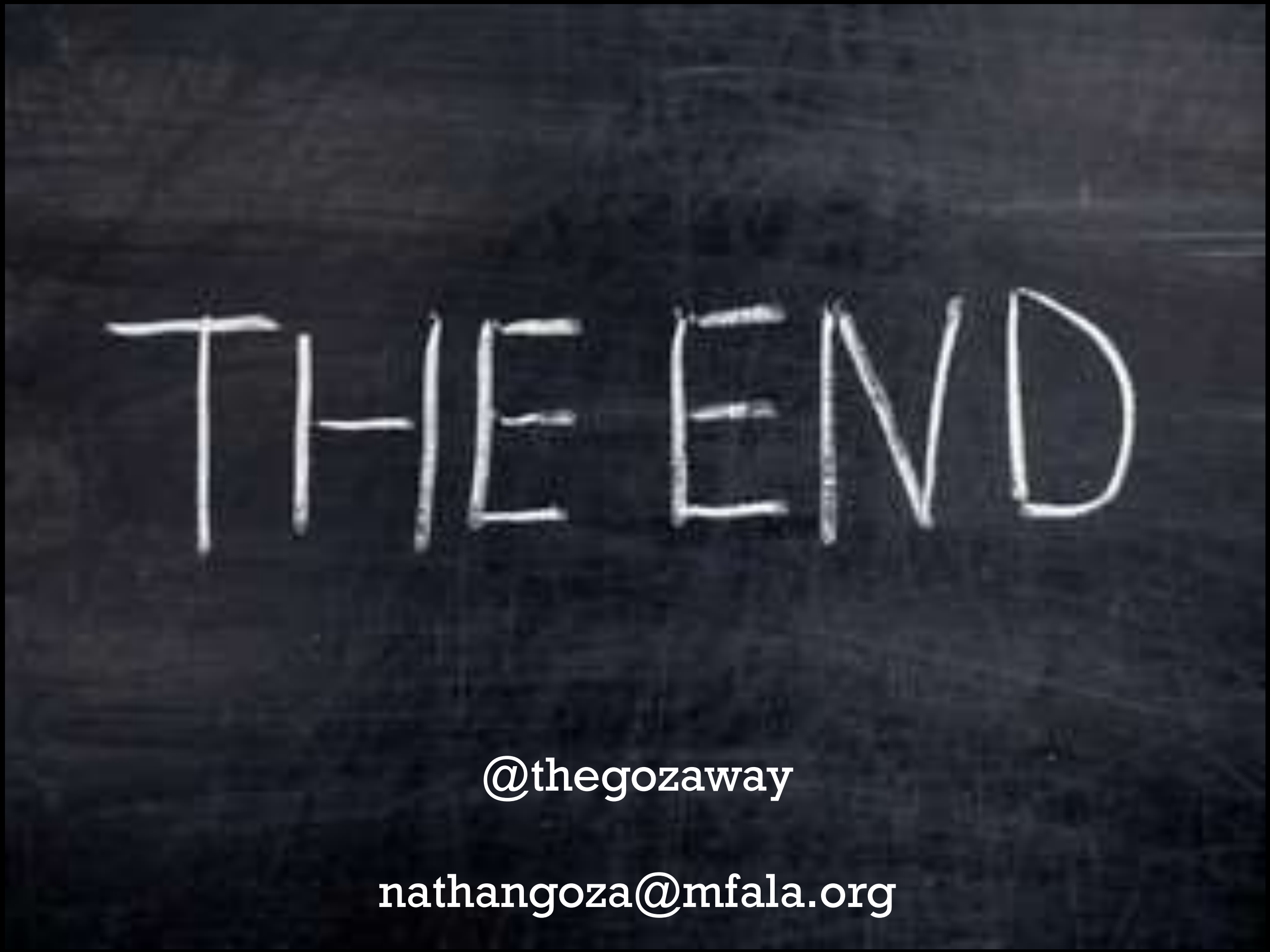
What's Missing??

Did we actually make it to …



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Thanks for coming. Thanks for staying.