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Developing Positive Math Identity

Mathematical Thinking Success Criteria aligned to Observable Student Actions in the Instructional Core

Core Component for Mathematical Thinking	Expected student actions in practice	Unacceptable student actions in practice
Description of Mathematical Thinking from research recommendations	What student actions demonstrate attainment of the core component/s? Include observable, evidence-based, best practices.	What student actions will NOT demonstrate attainment of the core component/s? Include observable, evidence-based counterexamples to best practices.
Specialising – trying special cases, looking at examples (Stacey, 2006)	• Use multiple forms of representations to make sense of and understand mathematics (NCTM, 2014, p. 29). • Consider the advantages or suitability of using various representations when solving problems (NCTM, 2014, p. 29).	
Generalising - looking for patterns and relationships	• Look closely to discern patterns or structure. Associate patterns with properties of operations and their relationships. (CCMP 7) • See complicated things, such as algebraic expressions, as single objects or as composed of several objects. (Younger children decompose and compose numbers.) (CCMP 7) • See repeated calculations and look for generalizations and shortcuts. (CCMP 8) • Understand the broader application of patterns and see the structure in similar situations. (CCMP 8) • Continually evaluate the reasonableness of their intermediate results (CCMP 8)	
Conjecturing – predicting relationships and results	• Are discussing with one another, making conjectures, planning a solution pathway, not jumping into a solution attempt or guessing at the direction to take (CCMP 1) • Make conjectures and explore the truth of their conjectures. (CCMP 3) • Recognize and use counterexamples. (CCMP 3) • Students' thinking becomes visible as they describe and	

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	justify their mathematical understanding and reasoning with drawings, diagrams, and other representations (NCTM, 2014, p. 29)	
Convincing – finding and communicating reasons why something is true.	• Reflect on and justify their reasoning, not simply providing an answer (NCTM, 2014, p. 41). • Recognize and explain flaws in arguments. (After listening or reading arguments of others, they respond by deciding whether or not they make sense. They ask useful questions to improve arguments.) Elementary students: construct arguments using concrete referents such as objects, drawings, diagrams, actions. (CCMP 3) • Justify and defend ALL conclusions and communicates them to others. (CCMP 3) • Are familiar with a variety of mathematics tools and use them when appropriate to explore and deepen their understanding of concepts (CCMP 6) • Put forth and defend his/her ideas/reasoning and peers build on each other's ideas (TRU Agency)	

References:

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: The National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards: Mathematics*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

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Stacey, K. (2006). *What is mathematical thinking and why is it important?* Proceedings of the APEC-Tsukuba International Conference 2007. Retrieved from http://www.criced.tsukuba.ac.jp/math/apec/apec2007/paper_pdf/Kaye%20Stacey.pdf

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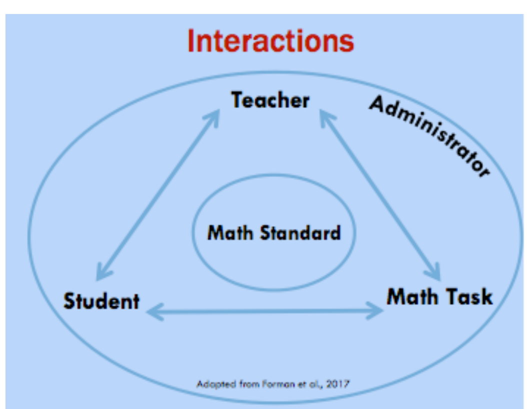
Ideate:

In thinking about the mathematical thinking success criteria and interactions, what would your "perfect generalising classroom" look like?



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Mathematical Thinking Script

Math Task for Clip 1:

"Jimmy says that x-4 is a factor of $f(x)= 3x^4+ 2x^3- 67x^2- 98x + 40$. Is he correct? Explain how you know."

Observation Clip 1:

Student 1 (S1): I was saying how when we learned trinomials what we did was we found the factors, the multiples of the middle number to see what numbers added together to equal that number. I'm not exactly sure if we can apply that to this.

Teacher (T): So, do you guys remember why we do that?

S1: In order to factor by grouping.

T: Yeah, in this you might have enough to be able to factor by grouping. The question is can you and what would your groups look like? There are a bunch of different ways you can try to group this.

S1: Can you put three in a group?

T: This is what you guys need to play with, so maybe split up and come up with a plan, but I don't know. Come up with a plan for what you think might happen. Try it and see what happens. But you gotta clue them in with what we're talking about.

S1: So in a trinomial you do that and can factor by grouping. Four sets of them. She suggested that you could factor by grouping in this one, so maybe three can be in a group, if there is a greater common factor in three of them. Maybe.

S2: So when we factor by grouping don't we do it in order like $3x^4$ and $2x^3$ would be together? And the -67 and -98.

S1: Yeah.

S2: Okay so 3 and 2 don't have anything in common?

S1: Well they have x^2 no actually x^3 as a common factor.

S2: Oh.

S1: So you can have 3x + 2 and then if this was to work, then that would also equal 3x+2.

S2: So 3x + 2.

S1: So factor it by x^3 so then…

S2: $67x^2 – 98x$. Um and then get x cubed in front of the parenthesis. Can we get a similar or exact same answer when we factor the negative 67x squared?

S1: No, no I am saying if we group them. Just group 3x by 4 plus $2x^3$.

S2: Just those 2?

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S1: Just those 2 and then you'd move x^3 in front of the parentheses. Then you'd get 3x + 2. We need to see if you can get a similar or the exact same factor when you do $-67x^2$.

S2: Oh, I get what you were saying.

Sample Coding for Observation Clip 1

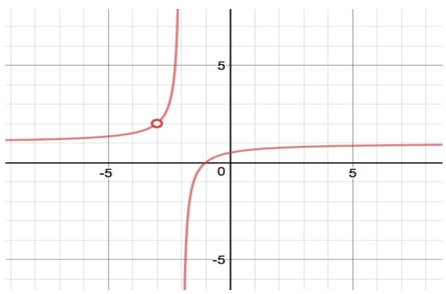
Student Success Criteria (Generalising): See complicated things, such as algebraic expressions, as single objects or as composed of several objects. (CCMP 7)

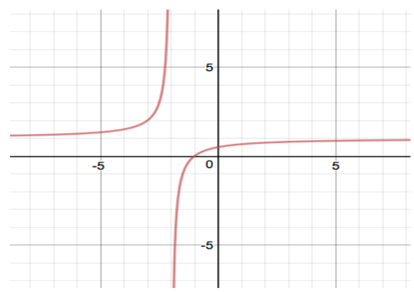
Interaction(s) between peer-task-teacher: S1 explains to S2 how factoring by grouping works with 4 terms and how it might work in this situation, but groups might look different.

Math Task for Clip 2:

"Notice, Note, Wonderings…

$$f(x) = \frac{x^2 + 4x + 3}{x^2 + 5x + 6}$$





Observation Clip 2:

Student 3 (S3): So he was doing this equation, so f(x) = x^2+ 4x +3 over x^2+ 5x + 6, so when you factor that you get x+1, x+3 over x+3 and x+2.

S4: So this graph and this graph are the same in everyway except for the fact that this 3, -3 rather.

S3: It is undefined.

S4: Undefined and you can find that with this (points at x+3) we forgot it when we canceled it. And so the -2 is undefined because that is the middle of thing and the…

S3: And-1 is undefined.

S4: -1 is not undefined because -1 is not in the denominator.

S5: Oh yeah, so…

S4: Well x+ 1 is not in the denominator.

S5: Because we divided by 0.

S3: What does it mean that it is in the numerator? How is that different?

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S4: Its just part of the numerator.

S3: Is it still 0, but 0 over…

S4: Well no because when you plug in 1 you get.

S5: Plug in -1.

S3: If you plug in -1.

S5: You get 0.

S3: You get 0 for that part, times.

S4: Over something.

S3: Over something, which is still zero.

S5: It’s 0 but if you have 0 in the denominator you…

S4: But that is just it is zero. *points at x-intercept on the graph at (-1,0)*

S3: Oh yeah.

S3: So does this equation belong to this one or this one.

S4: To this one.

S3: Because of the 3.

S4: Because in this one the 3 is defined.

S3: Oh I see.

S5: Ohhh.

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Implementation recommendations:

Generalising: looking for patterns and relationships (MP 7 & 8)

- Teach Accountable Talk to make student thinking visible (Hattie, 2017)
- Build understanding of multiple representations/ forms
- Support students to find patterns/structures in representations
- Facilitate students' general description of the pattern/structure
- Encourage students to make connections to find a relationship
- Ask students to create general rule that works for all cases

Implementation recommendations:

Making thinking visible through Accountable Talk

ACCOUNTABLE TALK MOVES

Move	Examples
Press for clarification and explanation	• Could you describe what you mean? • Can you provide an example that supports your claim? • Can you tell me more about your thinking about . . . ?
Require justification of proposals and challenges	• Where did you find that information? • How did you know that? • How does that support your claim?
Recognize and challenge misconception	• I don't agree because . . . • Have you considered an alternative such as . . . ? • I think that there is a misconception here, specifically . . .
Require evidence for claims and arguments	• Can you give me an example? • Where did you find that information? • How does this evidence support your claim?
Interpret and use each other's statements	• David suggested . . . • What I heard Marla say was . . . • I was thinking about Jackson's idea and I think . . .

(Hattie, 2017)