Making Mistakes Intentional.

Analyzing the Nature of Student Understanding with Low-Entry, High-Ceiling Problems

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http://msmaths.weebly.com/NCTM18.html
• Math induces anxiety
• Performance culture - answers
• Learning culture - process
• Listening to students
If a euro is worth $1.50, five euros is worth what?

A: Thirty quarters
B: Fifty dimes
C: Seventy nickels
D: Ninety pennies
Math-associated anxiety

- 25% of 4-year US college students
- 80% of community college students
- Nearly 50% of first and second graders

Study of 2nd and 3rd graders

Group of typical learners

- Not generally anxious
- Average working memory
- Average intelligence
- Average reading ability

Math anxiety and the brain

- **Heightened activity in the right amygdala**
  - Associated with fear

- **Heightened activity in the hippocampus**
  - Forms new memories

- **Reduced activity in the prefrontal cortex**
  - WM, attention, and number reasoning
  - Regulates negative emotions

- **Stronger connection between amygdala and prefrontal cortex**

Reduced working memory
Reduced attention
Reduced reasoning ability
Heightened negative emotions (fear!)
Emotional activation hijacks mental resources required for math.
Emotional activation hijacks mental resources required for math.

sighing to ban all educational material and play kahoot!

math sucks!
Working memory processing is critical to both arithmetic and math problem solving.

For students with already compromised attention and working memory, the effects of math anxiety are compounded.
Students who are anxious about math avoid doing it.

“The math performance-anxiety relationship is bidirectional. That is, poor performance can lead to anxiety, and anxiety can lead to poor performance, thus creating a vicious circle.”

- David Ludden

https://www.psychologytoday.com/blog/talking-apes/201703/how-overcome-math-anxiety
So, why does math, in particular, induce anxiety in so many students?
So, why does math, in particular, induce anxiety in many students? We don’t hear about art anxiety, or science anxiety.
If a euro is worth $1.50, five euros is worth what?
Factors

Environmental (classroom experiences)

Personal (confidence, self-esteem)

Cognitive (working memory, number sense)
How kids view math
-you are fast
-you get answers quickly
-you get the right answer
-you have it memorized
“Most students asked what they think their role is in math classrooms say it is to answer questions correctly. ”

-Jo Boaler

What do we value?

OR

HUMAN CALCULATOR
What types of experiences ought we provide to students who struggle with math?

Give them more opportunities to build intuition and understanding?

OR

Compensation strategies (memorization, usually) that bypass understanding?
MEMORIZE IN MINUTES: The Times Tables

3 x 7 = 21

Tree x Surfin’ = Denty Sun

4 x 6 = 24

Door x Chick = Denty Floor
Learning Culture

“When students think they’re in class to learn — to explore ideas and think freely — they understand more and achieve at higher levels than when they think the point is to get questions right.”

-Jo Boaler

http://time.com/4970465/how-to-improve-math-class/
Emphasizing process

“With less of an emphasis on right or wrong and more of an emphasis on process, teachers can help alleviate students' anxiety about math.”

Furner, Joseph M., Berman, Barbara T., “Math anxiety: Overcoming a major obstacle to the improvement of student math performance”, Childhood Education, Spring 2003
Valuing Mistakes

Being “mistake friendly” has been shown to have a greater positive impact on student effort than both classroom and personal achievement goals.

- you are curious
- you take risks quickly
- you get the right answer
- you figure it out by persevering and justifying your thinking
- you have it memorized
- you understand
If a euro is worth $1.50, five euros is worth what?

A: Thirty quarters
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Process
Focused
Not all correct answers are equal
28 + 25

Grade 3
28 + 25 = 53
20 + 20 = 40
All 3 students got the right answer!

But are their answers all the same?
<table>
<thead>
<tr>
<th>Grade 5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve. Use the boxes to organize your work.</td>
<td></td>
</tr>
<tr>
<td>1) $4 \times 186$</td>
<td></td>
</tr>
<tr>
<td>$100$</td>
<td>$8$</td>
</tr>
<tr>
<td>$400$</td>
<td>$32$</td>
</tr>
<tr>
<td>$744$</td>
<td></td>
</tr>
<tr>
<td>2) $8 \times 421$</td>
<td></td>
</tr>
<tr>
<td>$100$</td>
<td>$2$</td>
</tr>
<tr>
<td>$3200$</td>
<td>$17$</td>
</tr>
<tr>
<td>$3378$</td>
<td></td>
</tr>
<tr>
<td>3) $3 \times 724$</td>
<td></td>
</tr>
</tbody>
</table>
I feel like we demonize standardized tests as a necessary evil, but uphold our multiple choice finals like they are a great demonstration of learning.
We need to get kids talking about their math reasoning.
Solve and Graph the Inequalities

1) \[ 9x - 7 < 3(6 + 5) \]
\[ \frac{c}{10} \]
2) \[ 2x + 4 + 5d \geq -15 \]
3) \[ -128 \leq 3x + 5x \]
4) \[ 9v - 150 \geq 2(3 - 6v) - 5v \]
5) \[ -176 \geq 5z - 11 + 6z \]
6) \[ 2x + 6v = -12v \]
7) \[ 10s - 81 > 3(5 - 2s) \]
8) \[ 3(6 - 2g) > 9g - 37 \]
9) \[ 2(5 - 6y) > 9y - 137 \]
10) \[ 5c - 11 + 3e \leq 33 \]
I DON'T LIKE WORKSH*T!
Shape DEFG is reflected over the y-axis what is the measure of D’?

Student answer: 45°
Shape DEFG is reflected over the y-axis what is the measure of D'?

Student answer: 45°
MATH CLASS
Students see their errors as an indication of a lack of “smartness”
Shape DEFG is reflected over the y-axis what is the measure of D'? 

Student answer: 45°
Shape DEFG is reflected over the y-axis. What is the measure of D'? What's the error? What's the thinking?

Student answer: 45°
The goal for our math students should not be to memorize facts and procedures as skills isolated from meaning. THINKING is the goal. There is not one best process to solve every problem. "I'm terrible at math." "I just don't have a math mind." "I wasn't taught how to do that." "I can't remember how my teacher taught me." "I'll never understand this."
Story of a 5 hour trip on a plane

Me
How many people have to exit the plane before us?
How many people have to exit the plane before us?

800!

That does not make sense. Look...

300?

Hun... do the math. We are in row 23, which means there are 22 rows in front of us. Each row has 6 people. So 6 x 22...

100!

6 times 22, 2 times 6 is 12, carry the one. Then 6 times 2 is 12 again but add one so... 132.

How didn’t you get that?

Oh.. I must have forgot.
Kids stop reasoning about numbers and focus on the procedure when we strip them of mathematical intuition.
Find two consecutive, positive, odd integers whose product is 143.
\[
x(x + 2) = 143
\]
\[
x^2 + 2x = 143
\]
\[
x^2 + 2x - 143 = 0
\]
\[
(x - 11)(x + 13) = 0
\]
\[
x = 11 \text{ or } -13
\]
The approximate root of 143 is 12... I know it needs to be odd... so 11 and 13? Check!
I know $10 \times 10 = 100$ so I’ll try 11 and the next odd. Check!
No need for the process with the numbers given
Find two consecutive, positive, odd integers whose product is 143. Write an expression to represent the situation given.
x(x + 2) = 143
Create the headache
Low entry, high-ceiling tasks
What do low-entry, high-ceiling tasks have?

- Has a low floor and a high ceiling. **Everyone** can start.

- Has multiple entry points
  - Student A starts by exploring numerically
  - Student B begins by investigating graphically
  - Student C jumps in by reasoning algebraically

- Integrates multiple topics.

- Engages student interest, is mathematically/cognitively challenging.
Impacts of low-entry, high-ceiling tasks

- Persistence
- Flexible Thinking
- Student engagement
- Questioning and problem solving
- Application of past knowledge to new situations
- Clear and precise communication
- Teacher gains insight into how a student approaches the task, not just their result
Let's play a game.
What do you wonder?

What do you notice?
What do you notice? What do you wonder?

What do you know now?
What do you notice? What do you wonder?

Win Probability

What do you think the story might be now?
What do you notice? What do you wonder?

Win Probability

[Graph showing win probability over different time periods: End 1st, Halftime, End 3rd, End Reg]
What do you notice? What do you wonder?

Win Probability

Super Bowl LI
Round 2
What do you notice? What do you wonder?
What do you notice? What do you wonder?
What do you notice? What do you wonder?
Peak Break-Up Times
According to Facebook status updates

- Spring Break "spring clean"
- Valentine's Day
- April Fool's Day
- Mondays
- Summer holiday
- 2 weeks before winter holidays
- Christmas "too cruel"
Notice and Wonder
What do you notice?
What do you wonder?
Where's the poop emoji?
Where’s the poop emoji?

Jo Boaler
Where's the poop emoji?
Estimation
Why is Estimation Important?
Longer than you think!

US federal guidelines dictate that the dashed lines separating traffic lanes or indicating where passing is allowed run **10 feet in length**.

US federal guidelines dictate that the dashed lines separating traffic lanes or indicating where passing is allowed run 10 feet in length. 

How many beads in the small jar?
How many beads in the large jar?
Estimation skills
How many beads are in the jar?

- Eli: 95
- Brendon: 102
- Matty: 105
- Jake: 104
- Hayes: 101
- Wolf: 140

Mrs. Wilkin
How many beads in the large jar?
Wanna make kids mad?
Hmm...
It’s like taking candy from a baby
Dear Starburst...
The correct answer is A. Why might the test writers have chosen the other answers? Think about how they were trying to mislead you and common errors that people might perform.

B. because the y-intercept is -2

C. because the slope is -2

D. because the x-intercept is 2
Kids WILL surprise you

Tell me everything you know about:

- there's a red linear
Blue line equation $y = -\frac{1}{3}x - 6$

The lines are on a graph
Red line passes through (-1, -1)
Blue line passes through (-6, -4)
Blue line passes through (-4, -7)
Red line passes through (-2, -3)
Red line passes through (-3, -5)

These are linear equations
There are two lines
Both lines go on forever
Each line makes triangles in each grid square
Not 5 degrees
Not curved
Equations are to the power of 1 (no exponents)

Graph could be representing two different running rates of two different people

Blue is less steep than red
The red and blue lines do not make right angles as they intersect

There is an x-axis and a y-axis
There are no undefined lines

Red line y-intercept = 1
Blue line y-intercept = -6
Not parallel
One intersection point
One solution

Intersection = (-3, -5)
Red line slope is 2
Blue line slope is negative
Not perpendicular
Lines are diagonal

Red line slope = 2
Red line is parallel to $2x + 3$
Red line equation is $y = 2x + 1$
Red line x-intercept is -0.5
Blue line slope = $-\frac{1}{3}$
Blue line is parallel to $y = -\frac{1}{3}x - 7$
Blue line equation is $y = -\frac{1}{3}x - 6$
Blue line x-intercept is -18
Which One Doesn’t Belong? Why?
“Just like any other skill, you can learn to do math if you need to use it. Instead of telling our kids (and ourselves) that math is hard, we need to show them how relevant these skills are in the high-tech lives they’re living.” - David Ludden

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