Making Mistakes Intentional.

Analyzing the Nature of Student Understanding with Low-Entry, High-Ceiling Problems

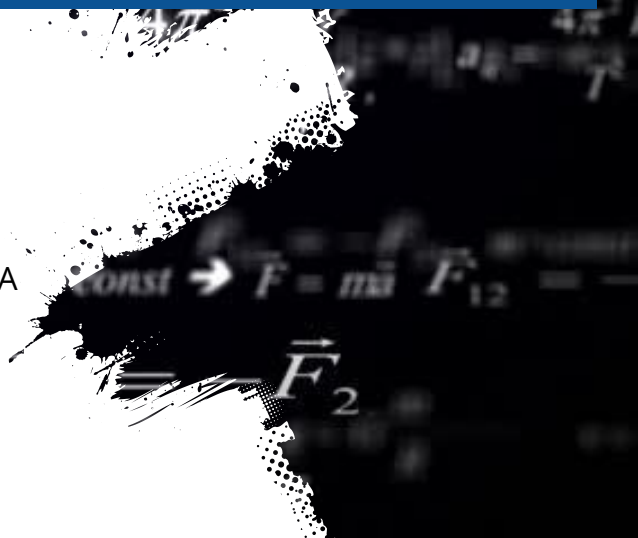
Peter Morris
@1pmorris

Carroll School, Waltham, MA
Head of Lower School
Mathematics

Jen McAleer
@jennifuhs4

Carroll School, Lincoln, MA
Head of Middle School
Mathematics

http://msmaths.weebly.com/NCTM18.html



- Math induces anxiety
- Performance culture - answers
- Learning culture - process
- Listening to students


If a euro is worth $1.50, five euros is worth what?
A: Thirty quarters
B: Fifty dimes
C: Seventy nickels
D: Ninety pennies

Math-associated anxiety

- 25% of 4-year US college students
- 80% of community college students
- Nearly 50% of first and second graders

Nervous About Numbers 2014, RACHEL ADELSON https://www.psychologicalscience.org/observer/nervous-about-numbers

Study of 2nd and 3rd graders

Group of typical learners

- Not generally anxious
- Average working memory
- Average intelligence
- Average reading ability

Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science, 23*(5), 492–501.

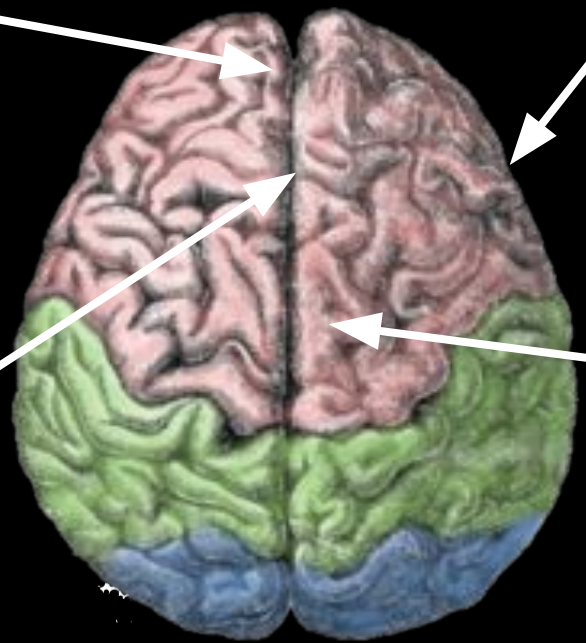
Math anxiety and the brain

Reduced activity in the prefrontal cortex

WM, **attention**, and **number reasoning**

Heightened activity in the hippocampus

Forms new **memories**



Heightened activity in the right amygdala

Associated with **fear**

Stronger connection between amygdala and prefrontal cortex

Regulates **negative emotions**

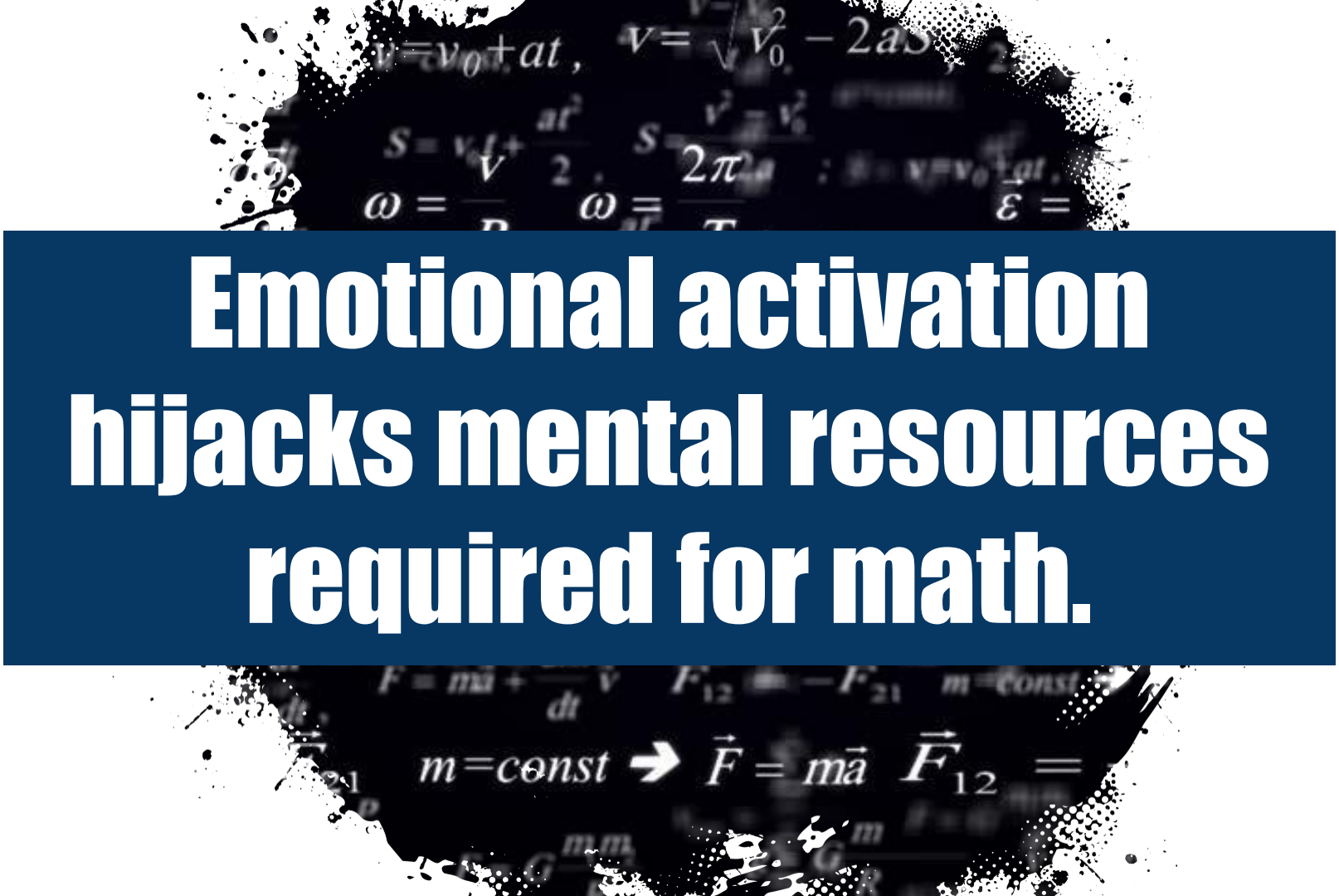
Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science, 23*(5), 492–501.

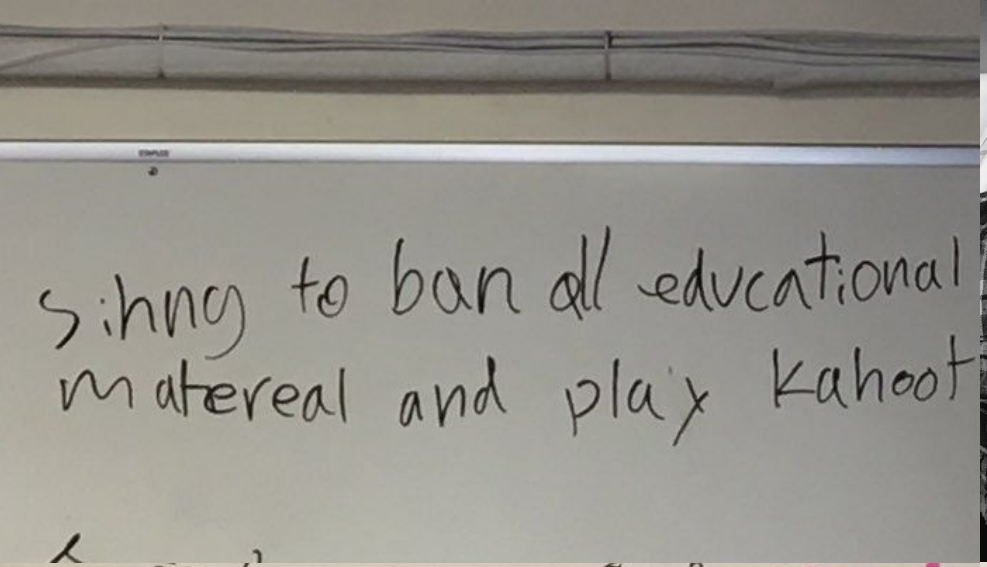
Reduced working memory

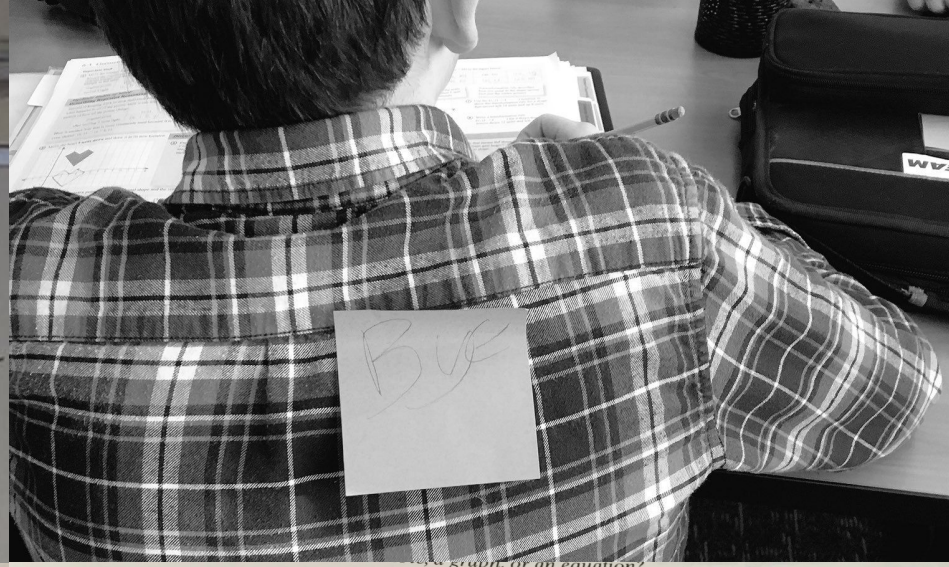
Reduced attention

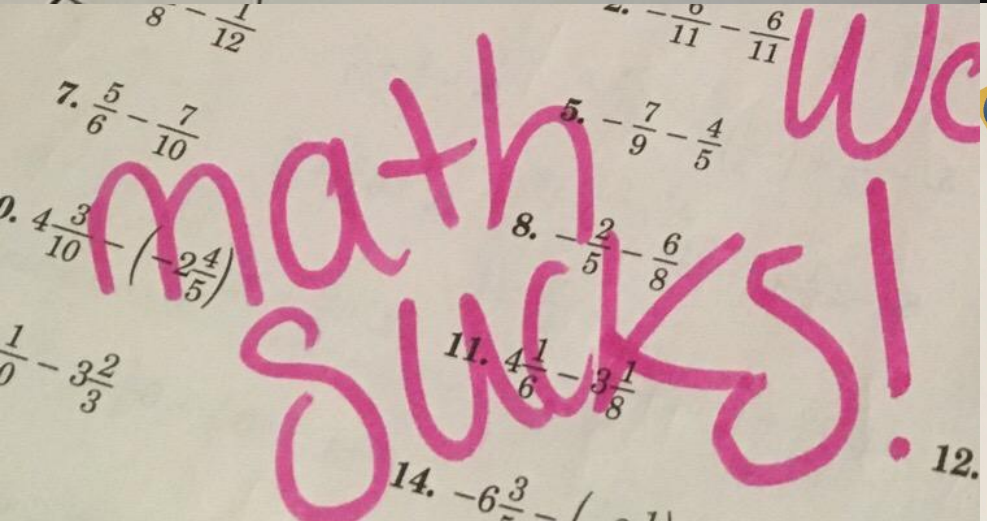
Reduced reasoning ability

Heightened negative emotions (fear!)


Emotional activation
hijacks mental resources
required for math.







graph, or an equation?

yay!

1.1 Walking Marathons

ok? never heard of that!

Ms. Chang's class decides to participate in a walkathon. Each participant must find sponsors to pledge a certain amount of money for each kilometer the participant walks. Leanne suggests that they determine their walking rates in meters per second so they can make predictions.

Do you know what your walking rate is?

omg rly?! srsly?!

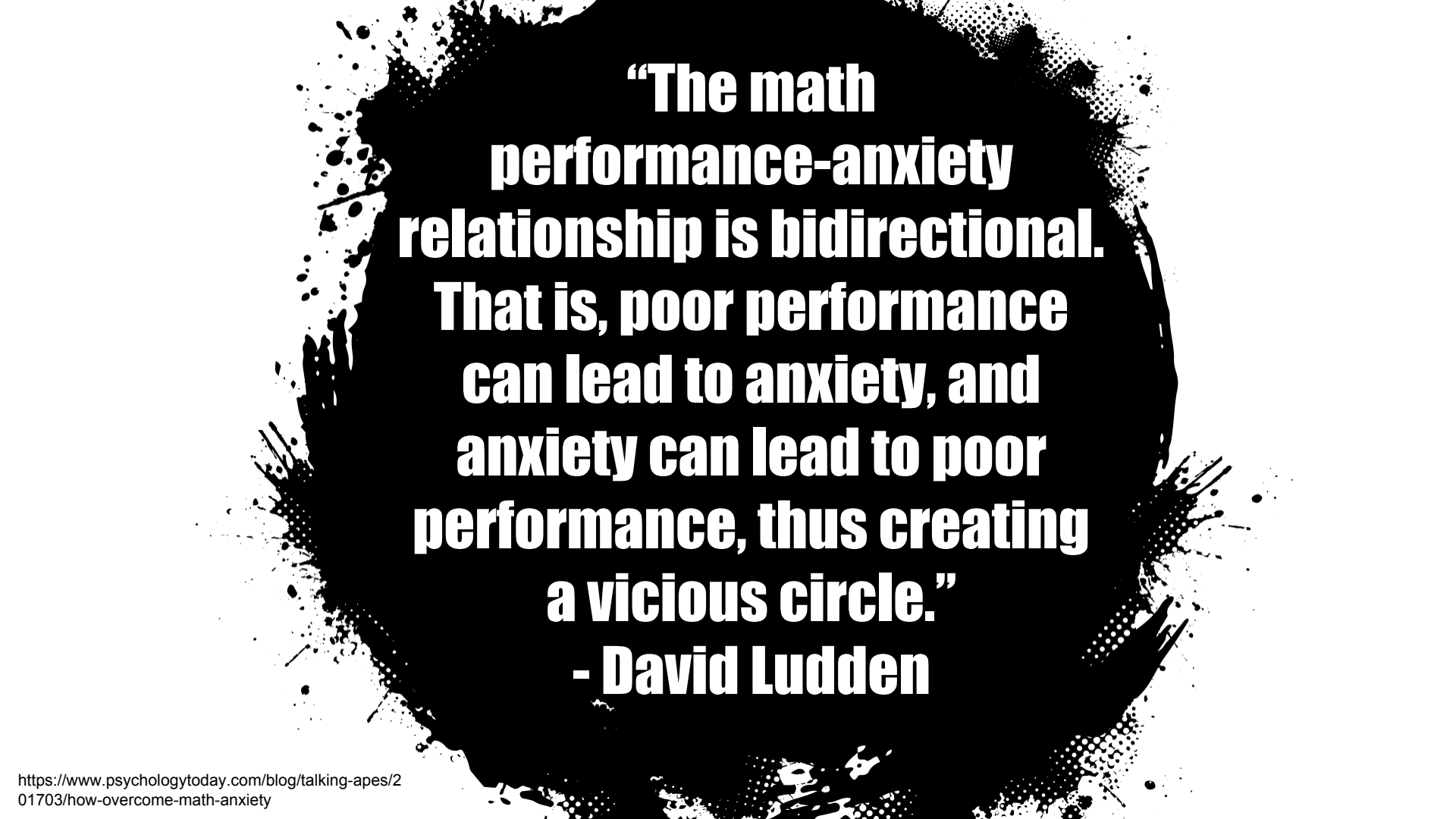
Working memory processing is critical to both arithmetic and math problem solving

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review, 12*(2), 243–248.

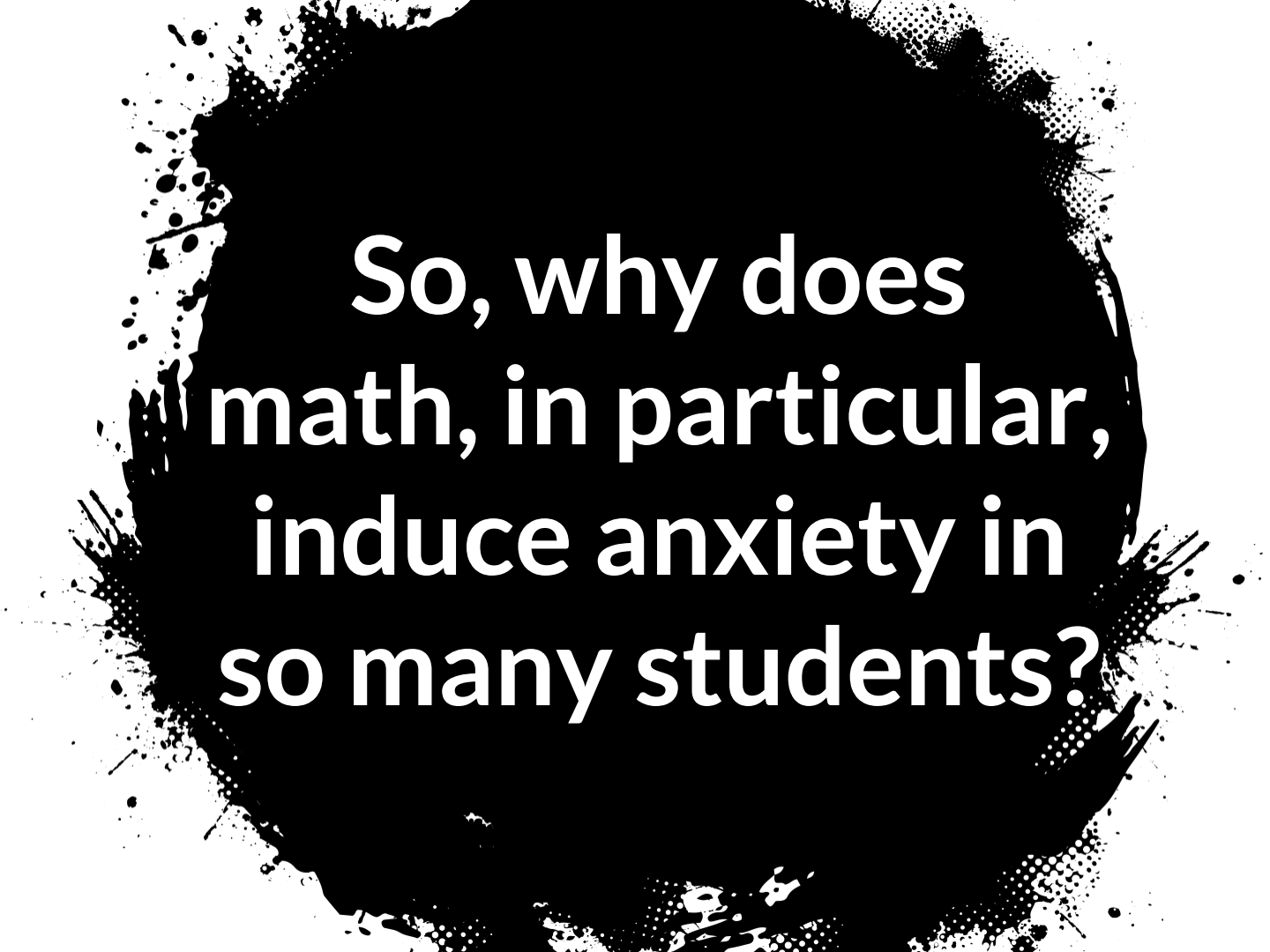
For students with already compromised attention and working memory, the effects of math anxiety are compounded

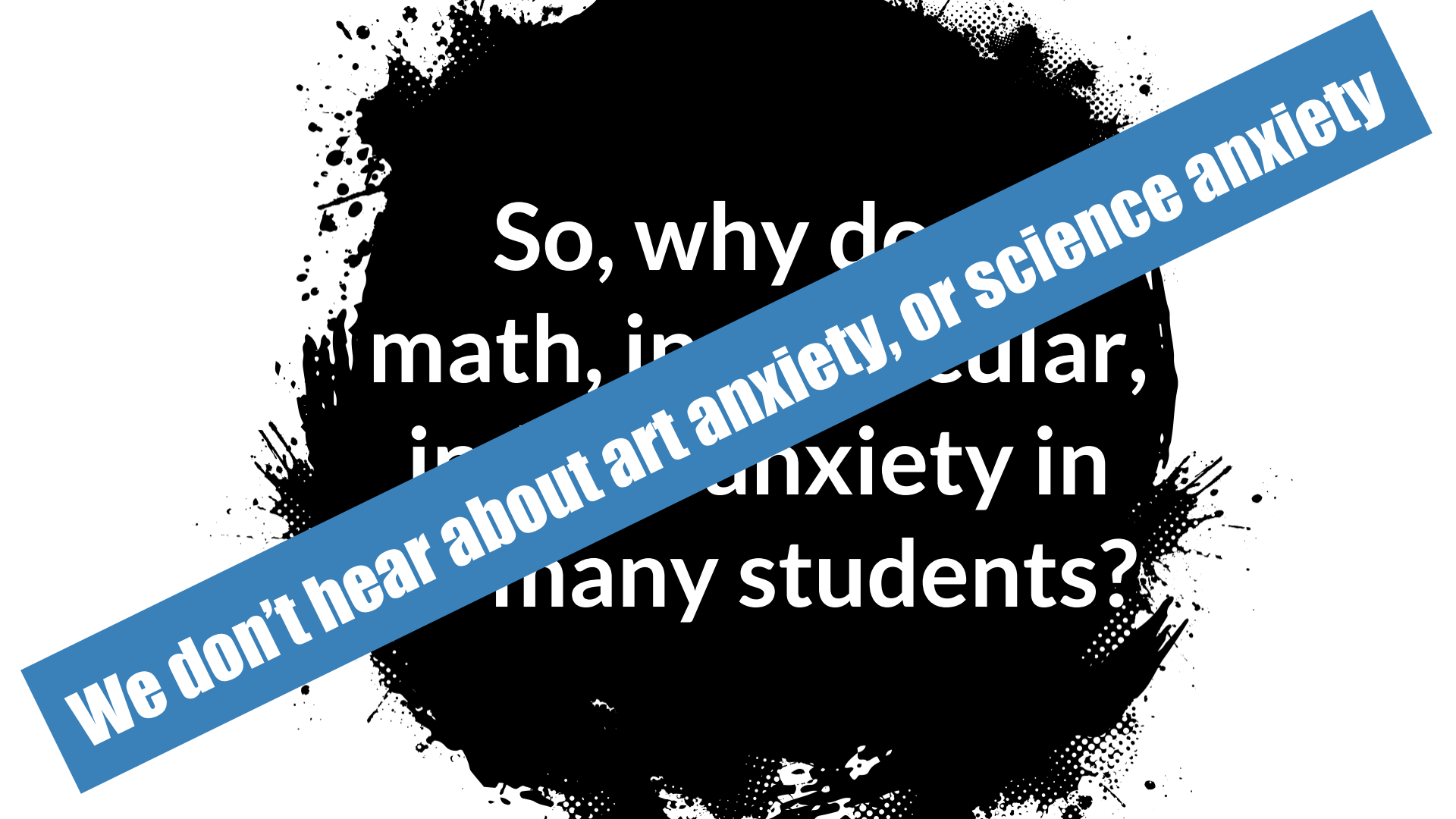
Students who are anxious about math avoid doing it

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review, 12*(2), 243–248.



https://www.psychologytoday.com/blog/talking-apes/201703/how-overcome-math-anxiety


So, why does
math, in particular,
induce anxiety in
so many students?

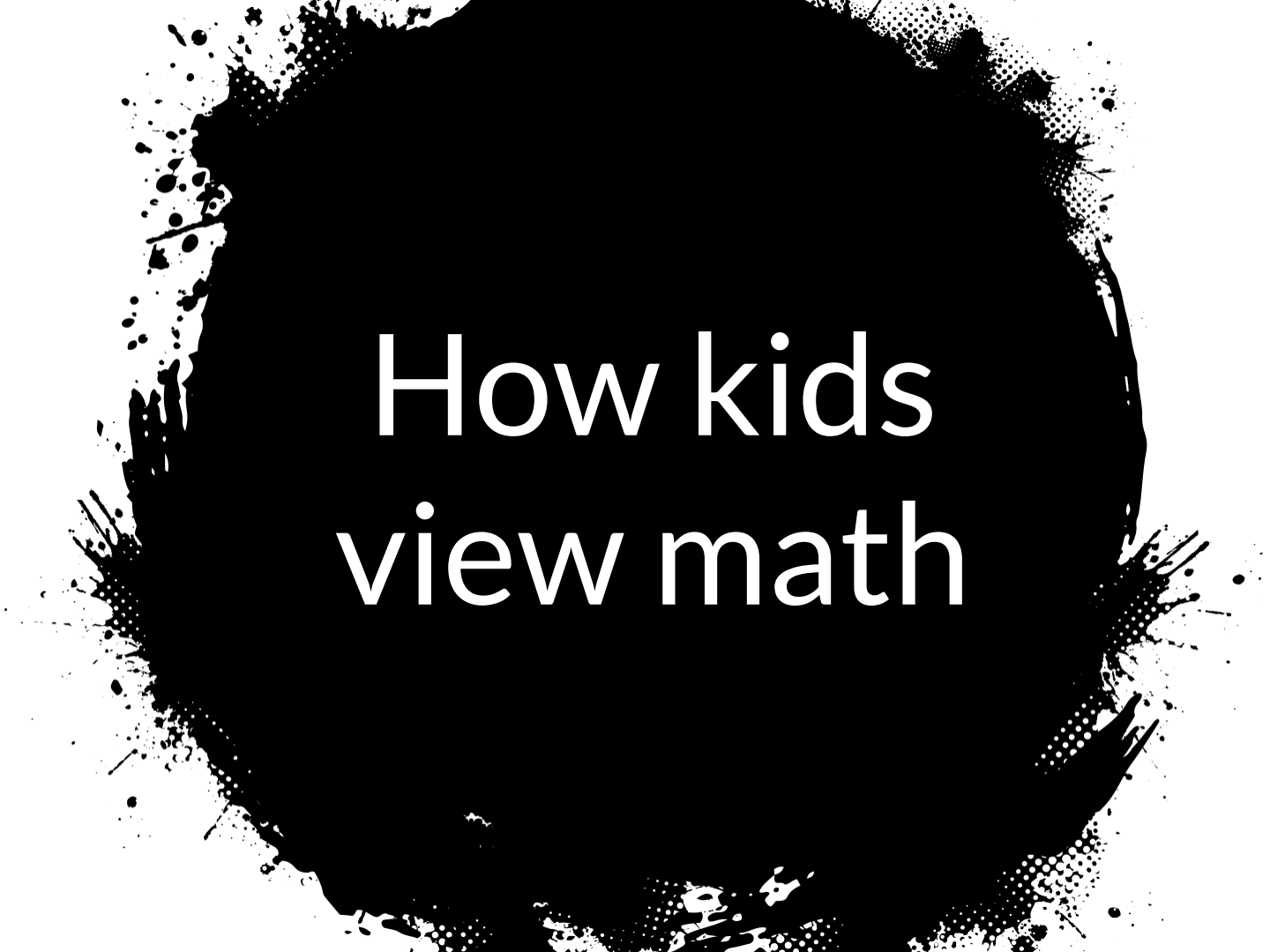

So, why do
math, in ecular,
in anxiety in
many students?
We don't hear about art anxiety, or science anxiety

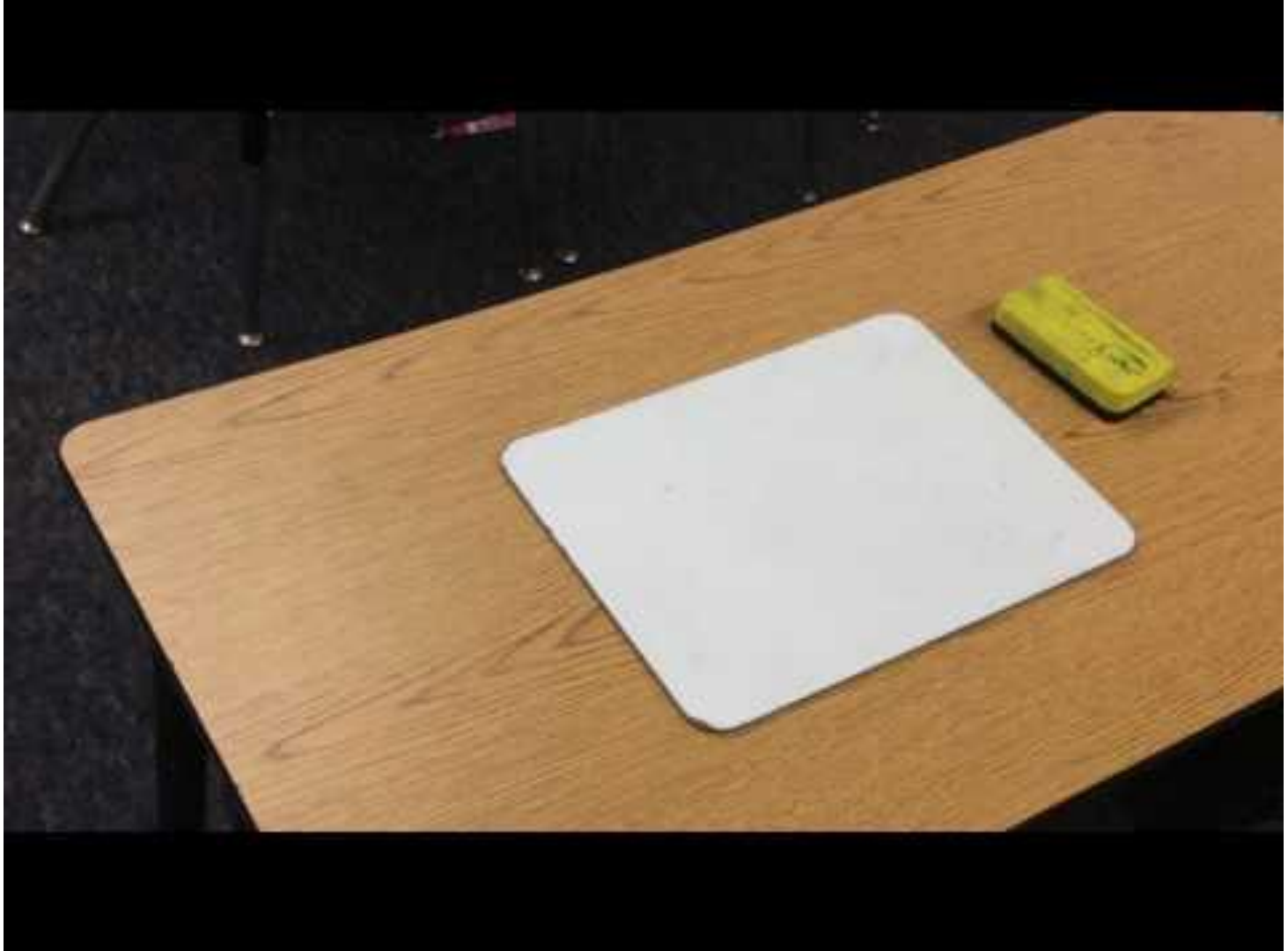

If a euro is worth $1.50, five euros is worth what?

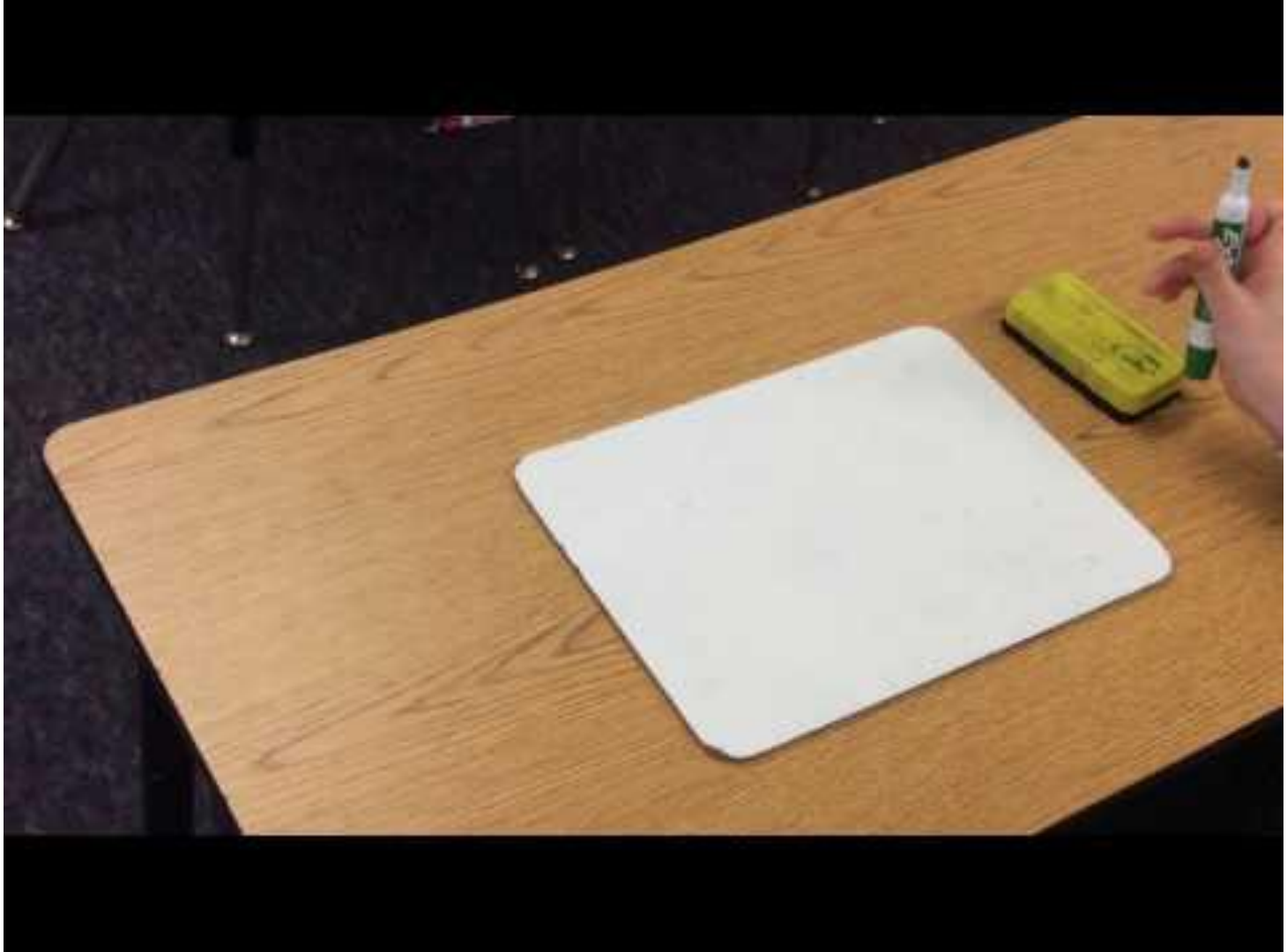
Factors

- Environmental (classroom experiences)
- Personal (confidence, self-esteem)
- Cognitive (working memory, number sense)

Nervous About Numbers 2014, RACHEL ADELSON https://www.psychologicalscience.org/observer/nervous-about-numbers


How kids
view math





-you are fast
-you get answers quickly
-you get the right answer
-you have it memorized

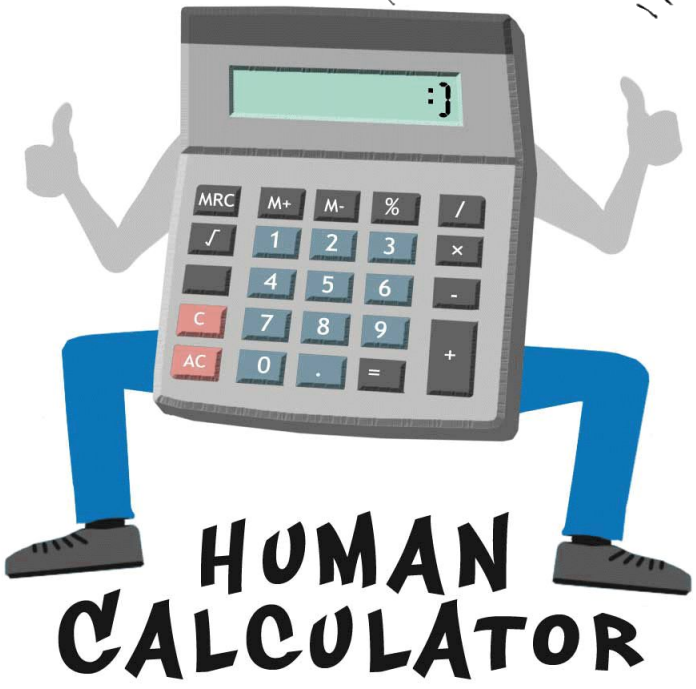
Performance Culture

"Most students asked what they think their role is in math classrooms say it is to answer questions correctly."

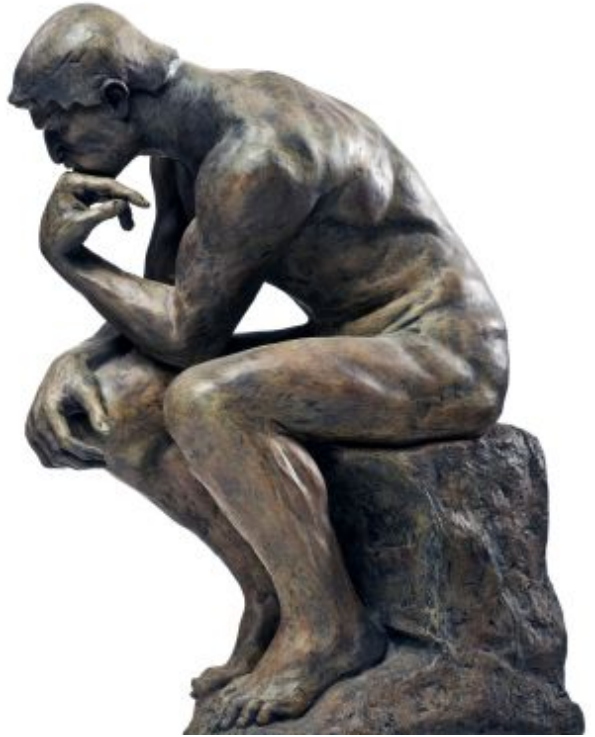
-Jo Boaler

https://www.heinemann.com/blog/the-mathematics-of-hope-moving-from-performance-to-learning-in-mathematics-classrooms/

What do we value?



OR



What types of experiences ought we provide to students who struggle with math?

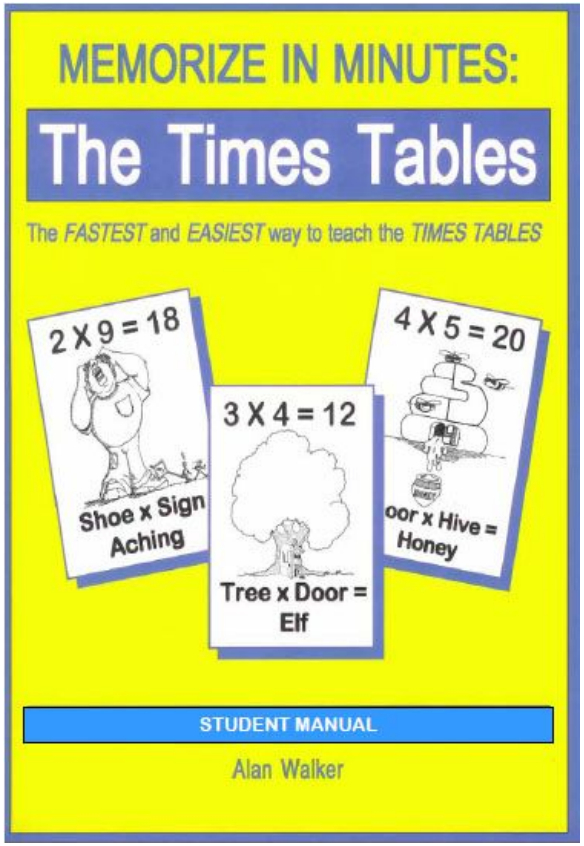
Give them more opportunities to build intuition and understanding?

OR

Compensation strategies (memorization, usually) that bypass understanding?

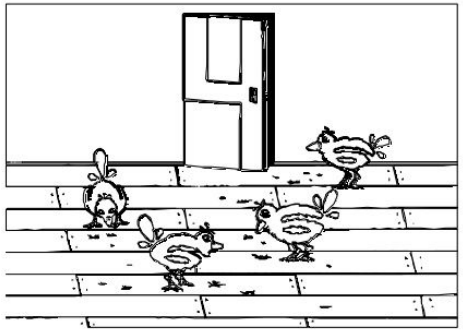
3 x 7 = 21

Tree x Surfin' = Denty Sun

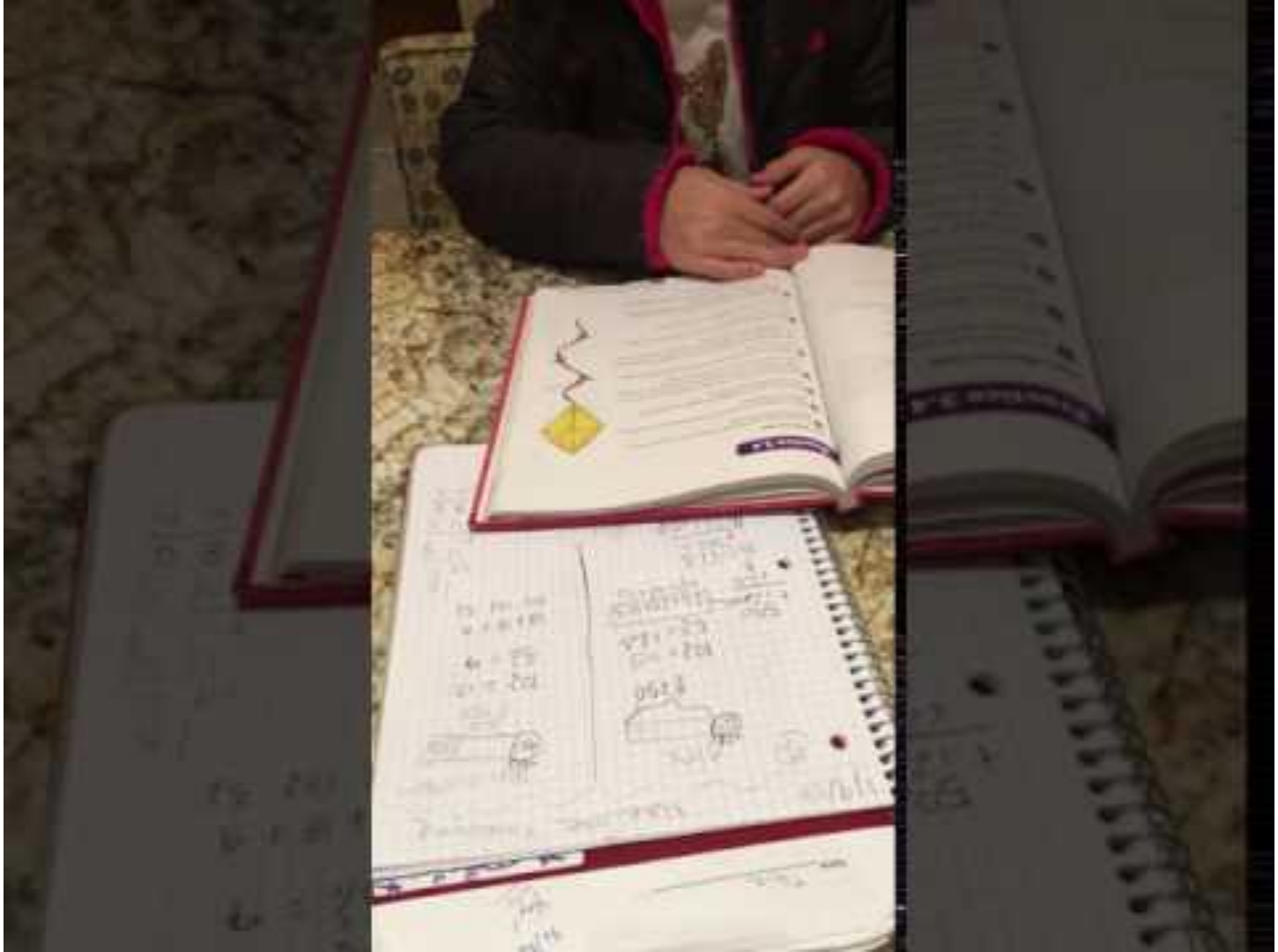


Memorize in Minutes: The Times Tables

4 x 6 = 24



Door x Chick = Denty Floor



But oh, what a sticky floor!
Let me in, I'm thirsty
I have no water. I'm thirsty too!
32
MEMORIZE IN
The Time
The FASTEST and EASIEST way
2 X 9 = 18
Shoe x Bean Aching
3 X 4 = 12
Tree x Door = Elf
Door x Hive = Honey
TEACHING MANUAL
Alan Walker

Learning Culture

"When students think they're in class to learn — to explore ideas and think freely — they understand more and achieve at higher levels than when they think the point is to get questions right."

-Jo Boaler

http://time.com/4970465/how-to-improve-math-class/

Emphasizing process

"With less of an emphasis on right or wrong and more of an emphasis on process, teachers can help alleviate students' anxiety about math."

Furner, Joseph M., Berman, Barbara T., "Math anxiety: Overcoming a major obstacle to the improvement of student math performance", *Childhood Education*, Spring 2003

Valuing Mistakes

Being “mistake friendly” has been shown to have a greater positive impact on student effort than both classroom and personal achievement goals.

Steuer, G., Rosentritt-Brunn, G., & Dresel, M. (2013). Dealing with errors in mathematics classrooms: Structure and relevance of perceived error climate. *Contemporary Educational Psychology*, 38(3), 196-210.

-you are curious
-you take risks
-you figure it out by persevering
-you justify your thinking
-you understand


If a euro is worth $1.50, five euros is worth what?
A: Thirty quarters
B: Fifty dimes
C: Seventy nickels
D: Ninety pennies

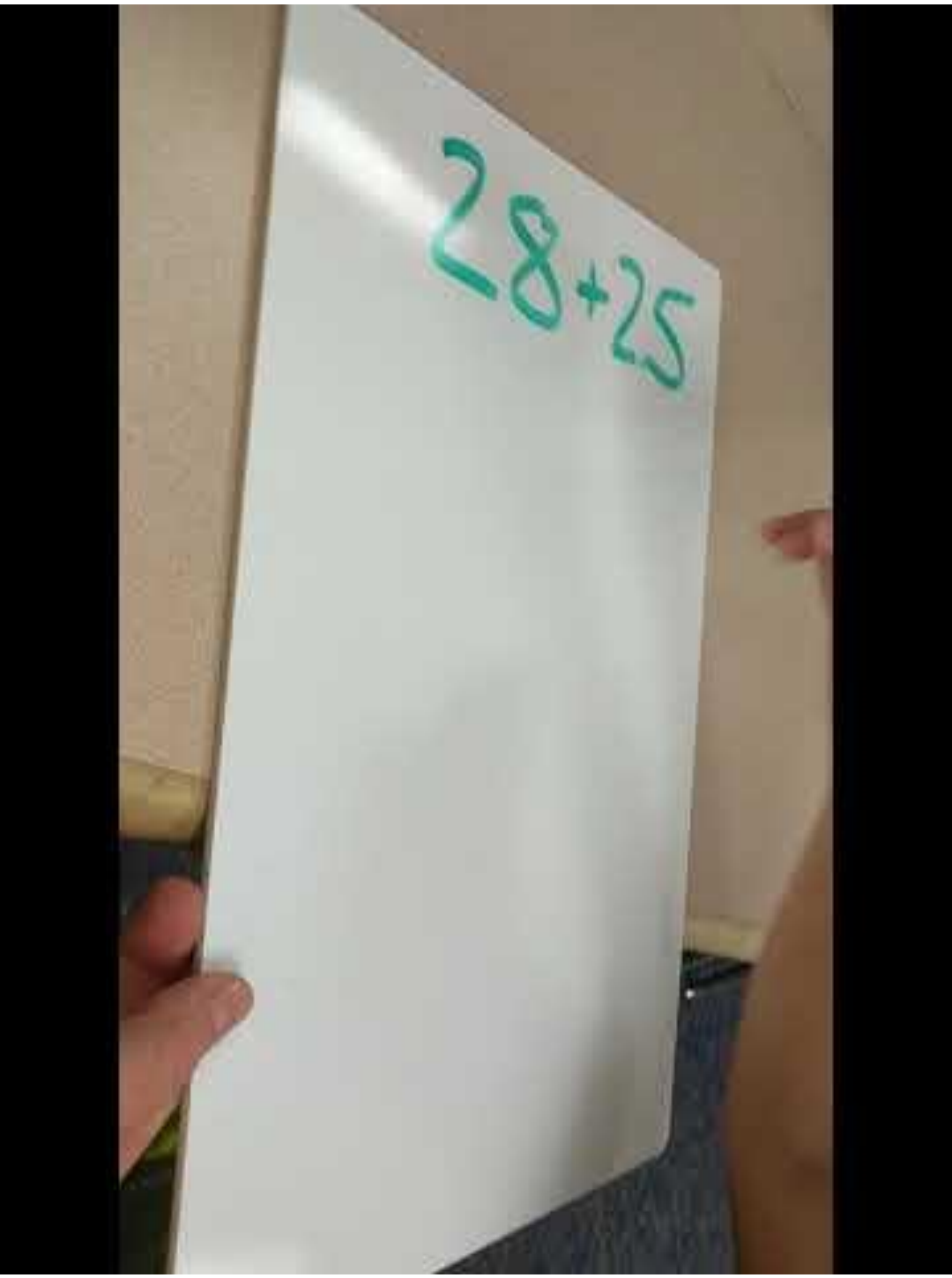
Process Focused

Not all correct answers are equal


28 + 25
Grade 3

28+25

28×25
20+20=40

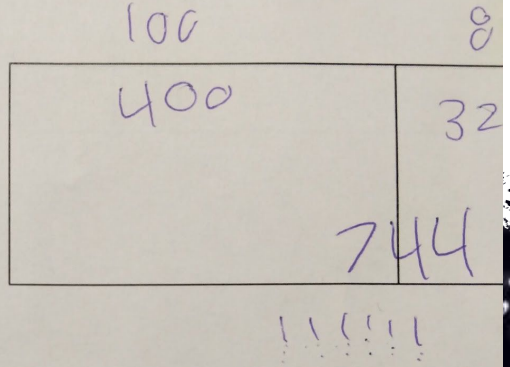

28+25

All 3 students got the right answer!

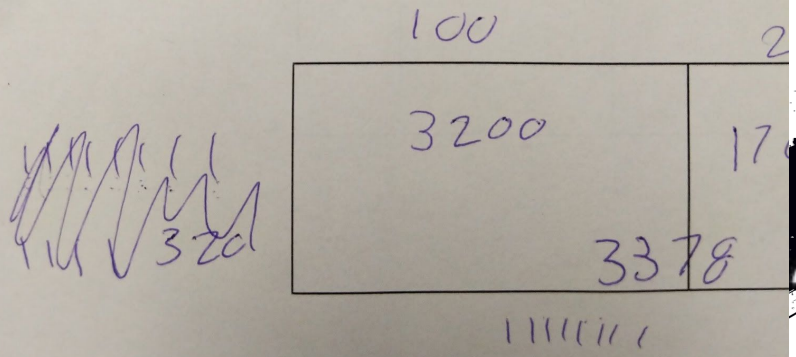
But are their answers all the same?

Solve. Use the boxes to organize your work.

1) 4 x 186



2) 8 x 421



3) 3 x 724

Grade 5

Megan Schmidt
@Veganmathbeagle

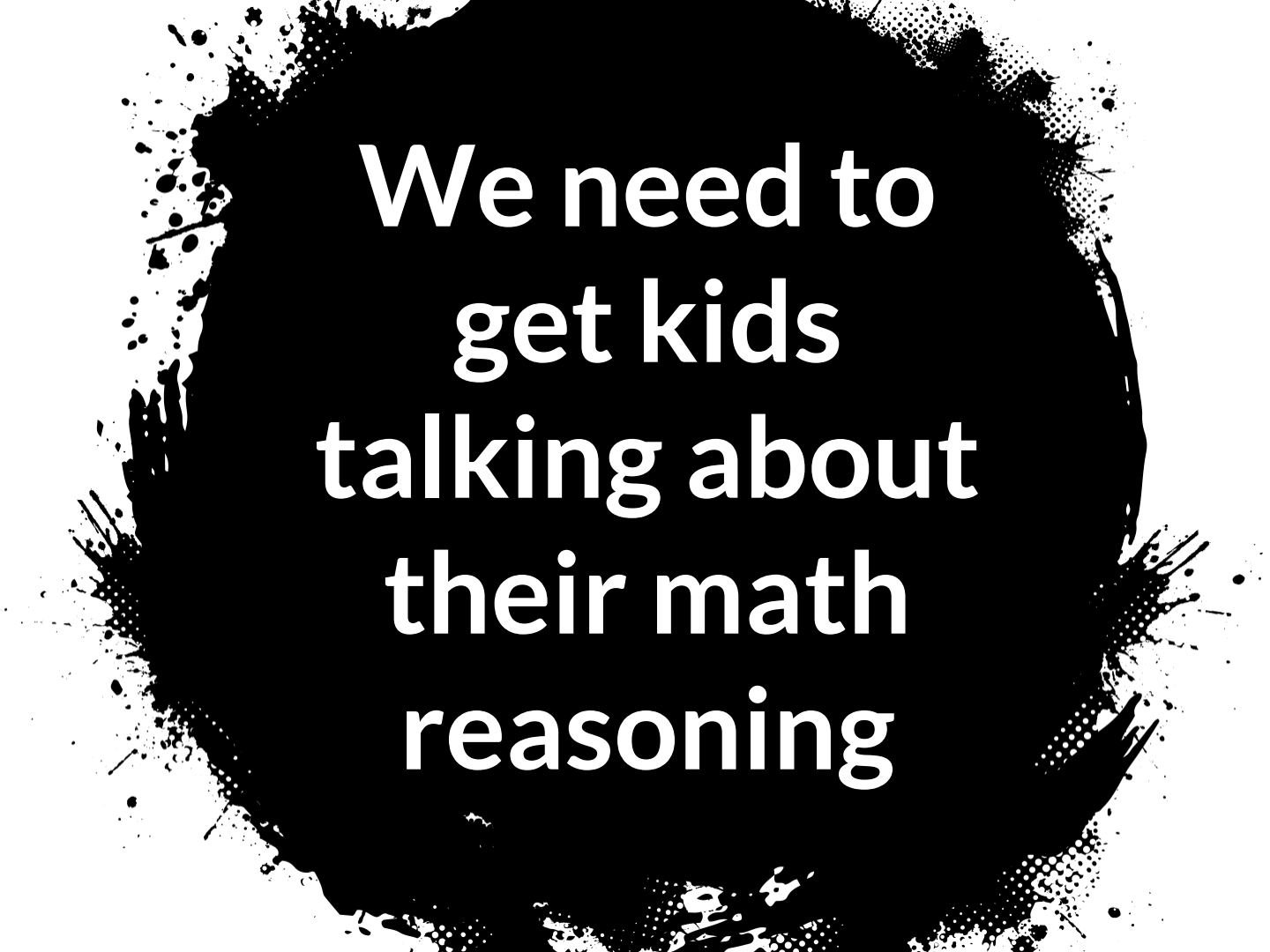
Following

I feel like we demonize standardized tests as a necessary evil, but uphold our multiple choice finals like they are a great demonstration of learning.

5:53 AM - 21 Nov 2017

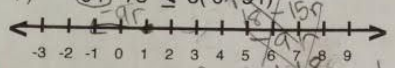
2 Retweets **6** Likes

2 6

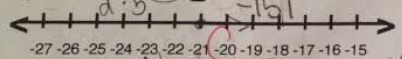

We need to get kids talking about their math reasoning

Solve and Graph the Inequalities

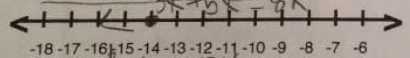
1) $9r - 78 \leq 3(6 + 5r)$



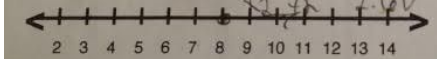
2) $2d - 4 + 5d \geq -151$



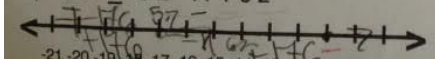
3) $-128 \leq 3x + 5x$



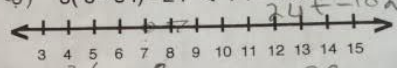
4) $9v - 150 \geq 2(3 - 6v) - 5v$



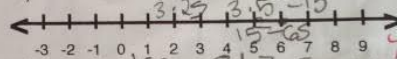
5) $-176 \geq 5z - 11 + 6z$



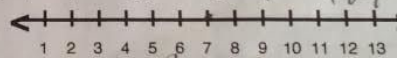
6) $3(6 - 5f) - 2f < 7f - 102$



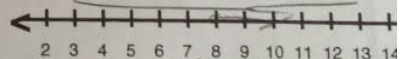
7) $10s - 81 > 3(5 - 2s)$



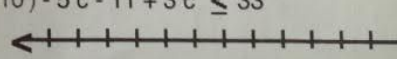
8) $3(6 - 2g) > 9g - 72$

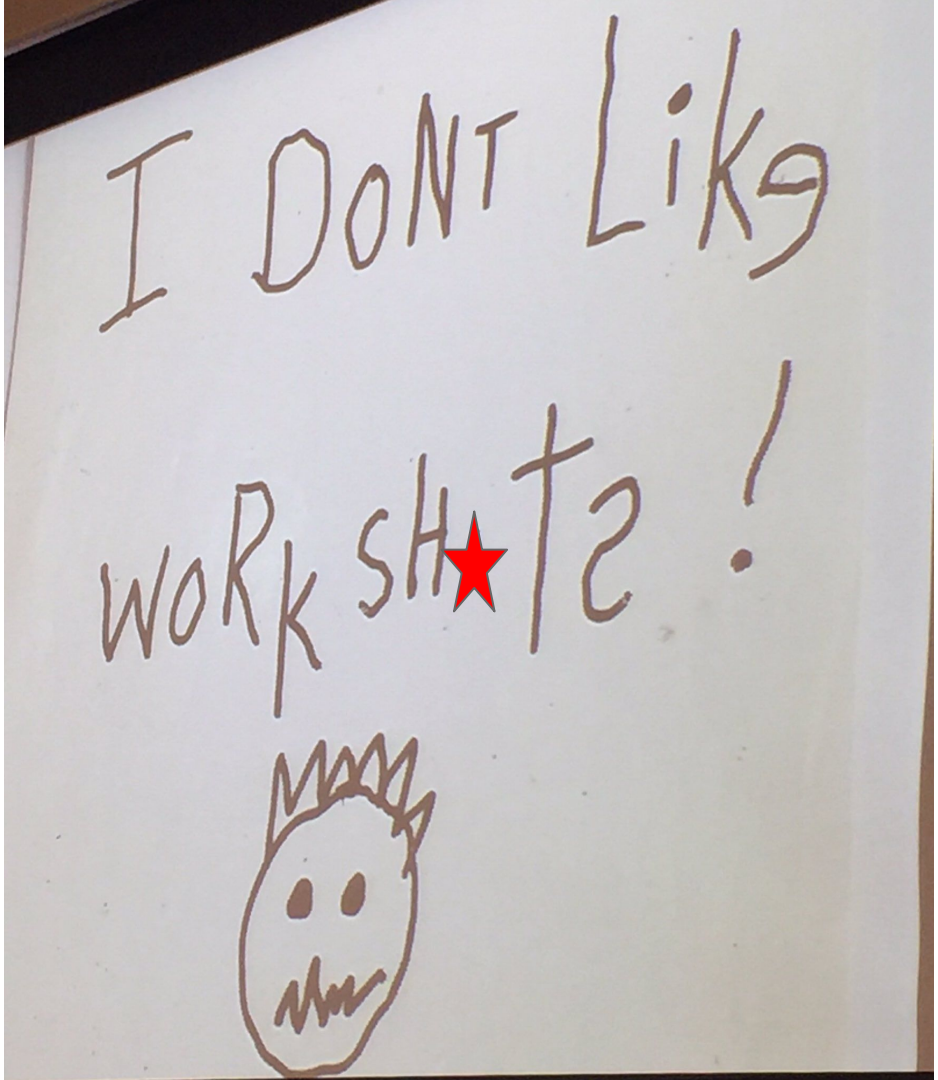


9) $2(5 - 6y) > 9y - 137$

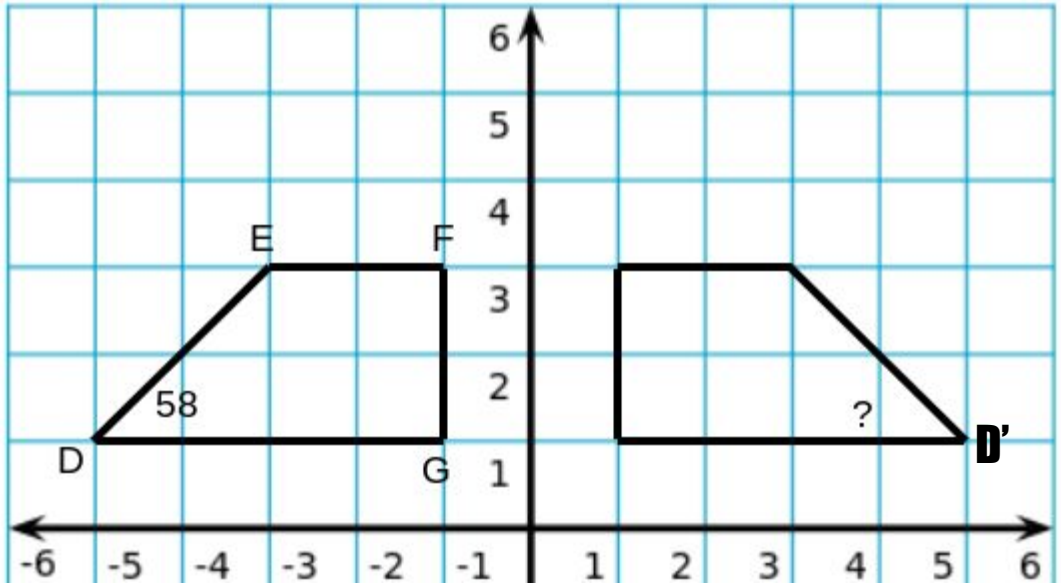


10) $-5c - 11 + 3c \leq 33$



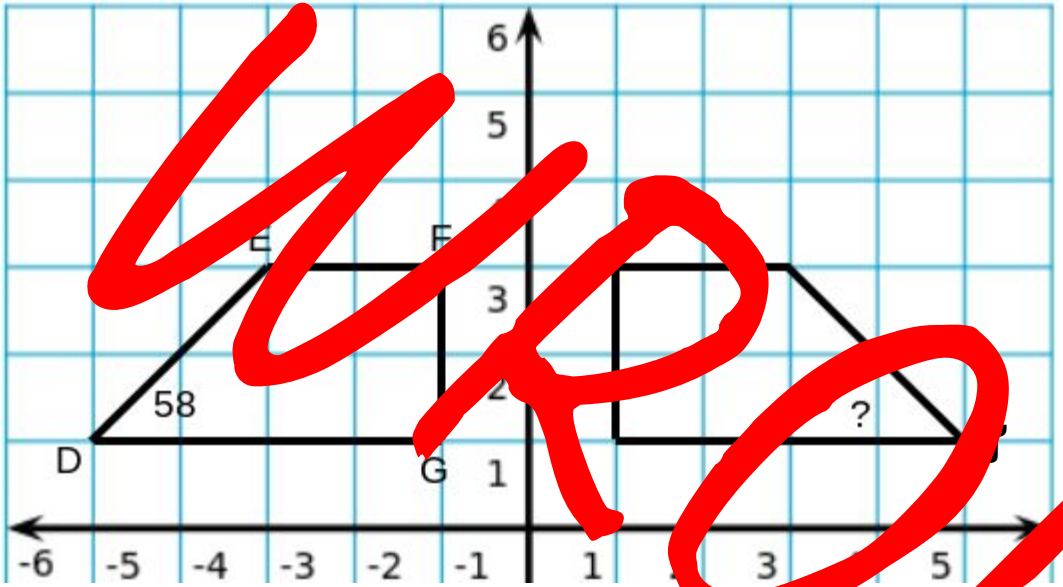

I DoNT Likg
woRk sHtz !

Shape DEFG is reflected over the y-axis what is the measure of D'?



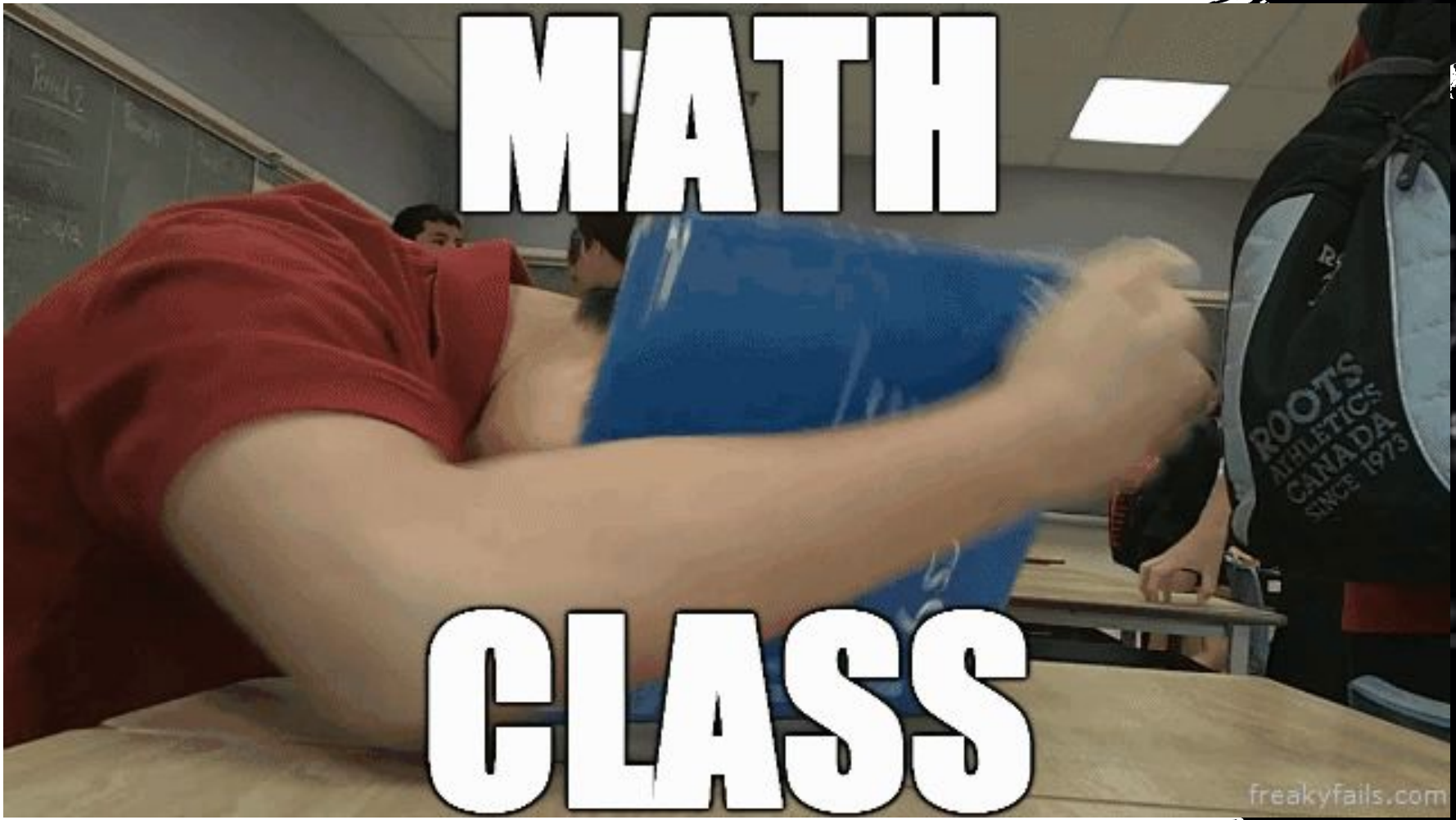
Student answer: 45°

Shape DEFG is reflected over the y-axis what is the measure of D'?



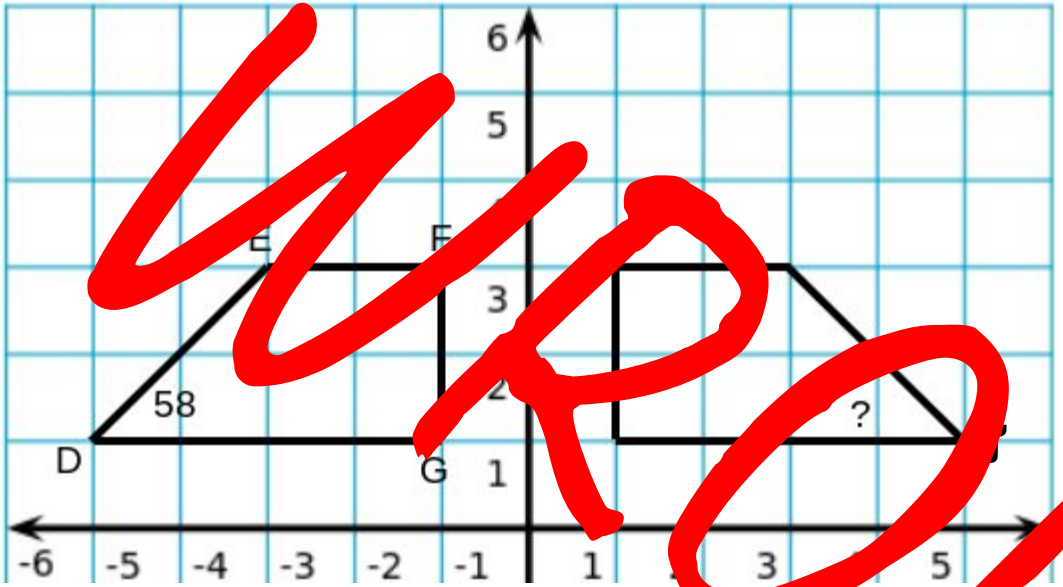
WRONG

Student answer: 45°


MATH
CLASS
ROOTS
ATHLETICS
CANADA
SINCE 1973
freakyfails.com

Students see their errors as an indication of a lack of “smartness”

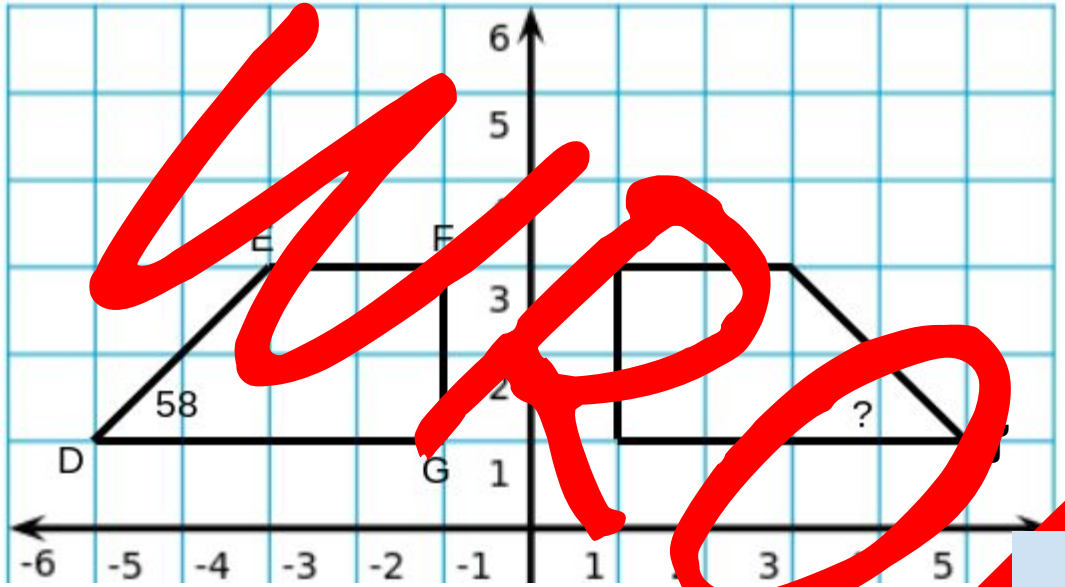
Shape DEFG is reflected over the y-axis what is the measure of D'?



WRONG

Student answer: 45°

Shape DEFG is reflected over the y-axis what is the measure of D'?

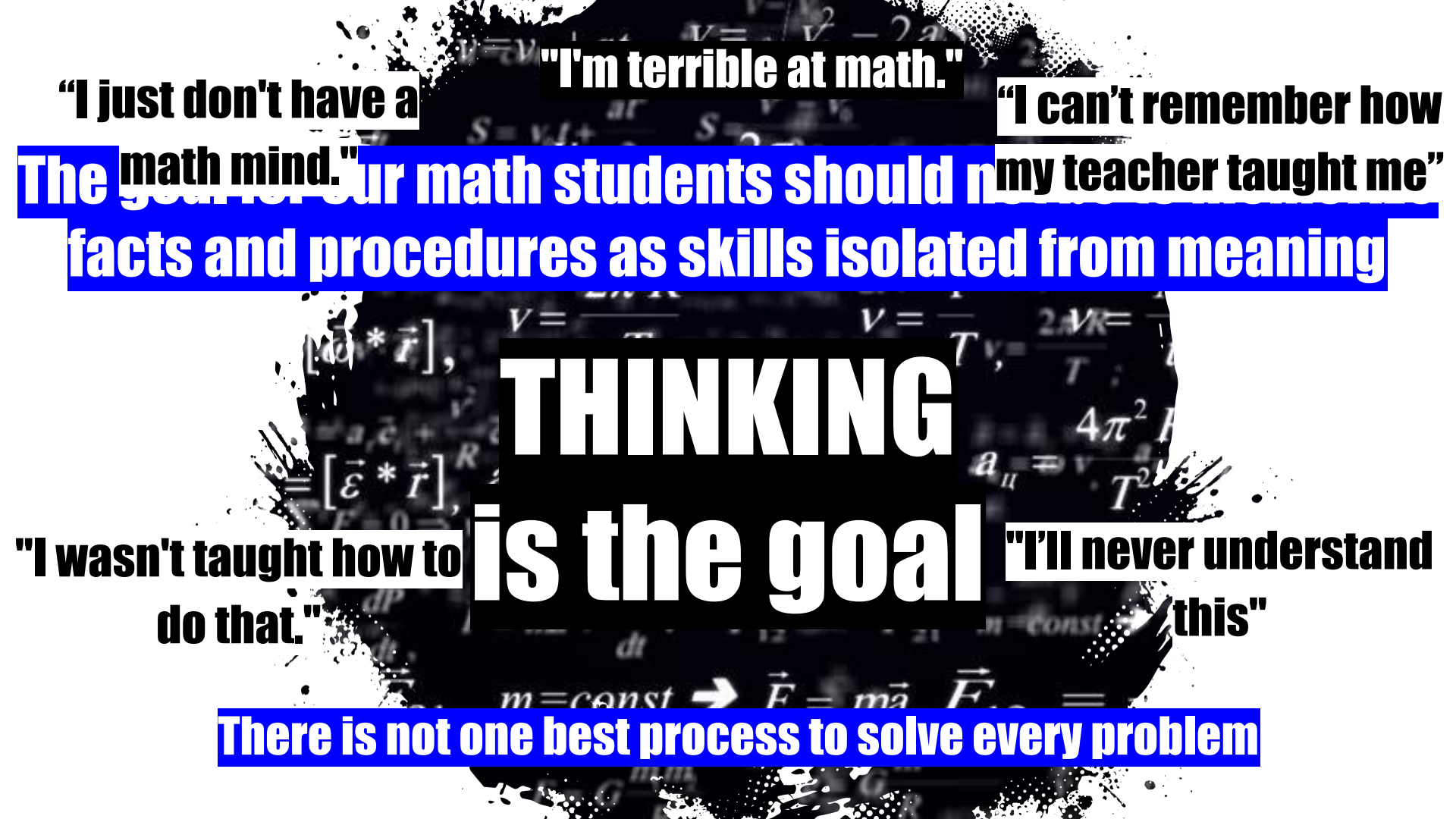


WRONG

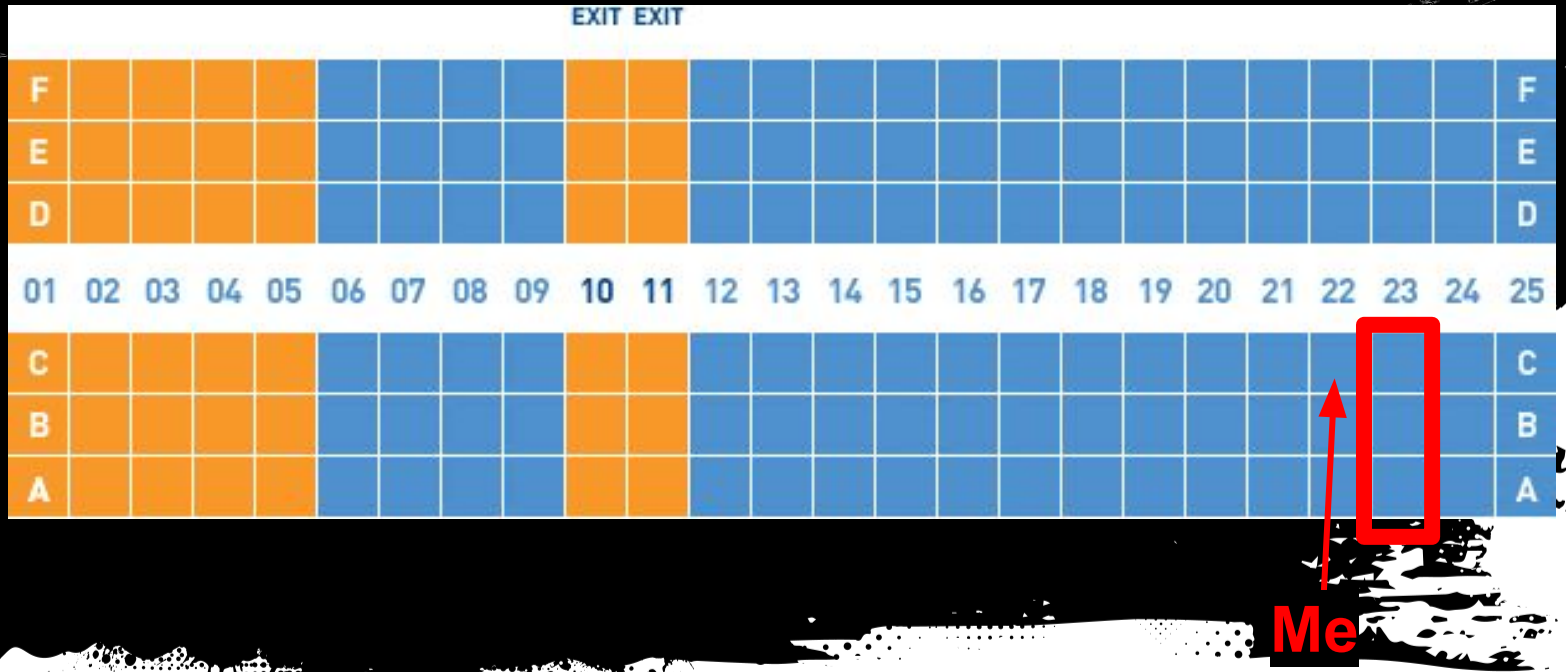
What's the error?

What's the thinking?

Student answer: 45°


"I'm terrible at math."
“I just don't have a math mind."
“I can’t remember how my teacher taught me”
The goal for our math students should not be memorizing facts and procedures as skills isolated from meaning
THINKING is the goal
"I wasn't taught how to do that."
"I’ll never understand this"
There is not one best process to solve every problem

Story of a 5 hour trip on a plane



Story of a 5 hour trip on a plane

How many people have to exit the plane before us?

How many people have to exit the plane before us?

800!

That does not make sense. Look...

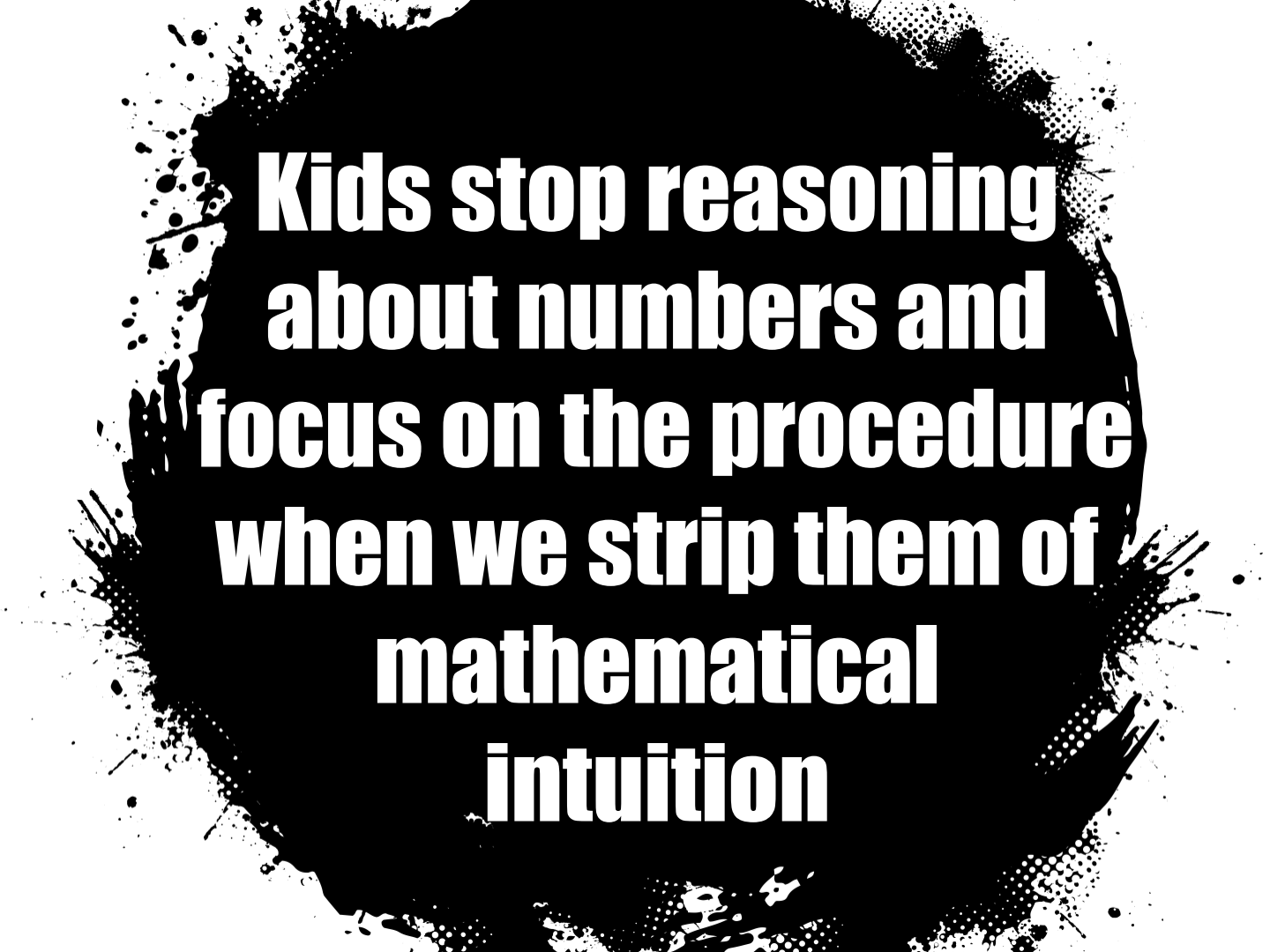
300?

Hun... do the math. We are in row 23, which means there are 22 rows in front of us. Each row has 6 people. So 6 x 22...

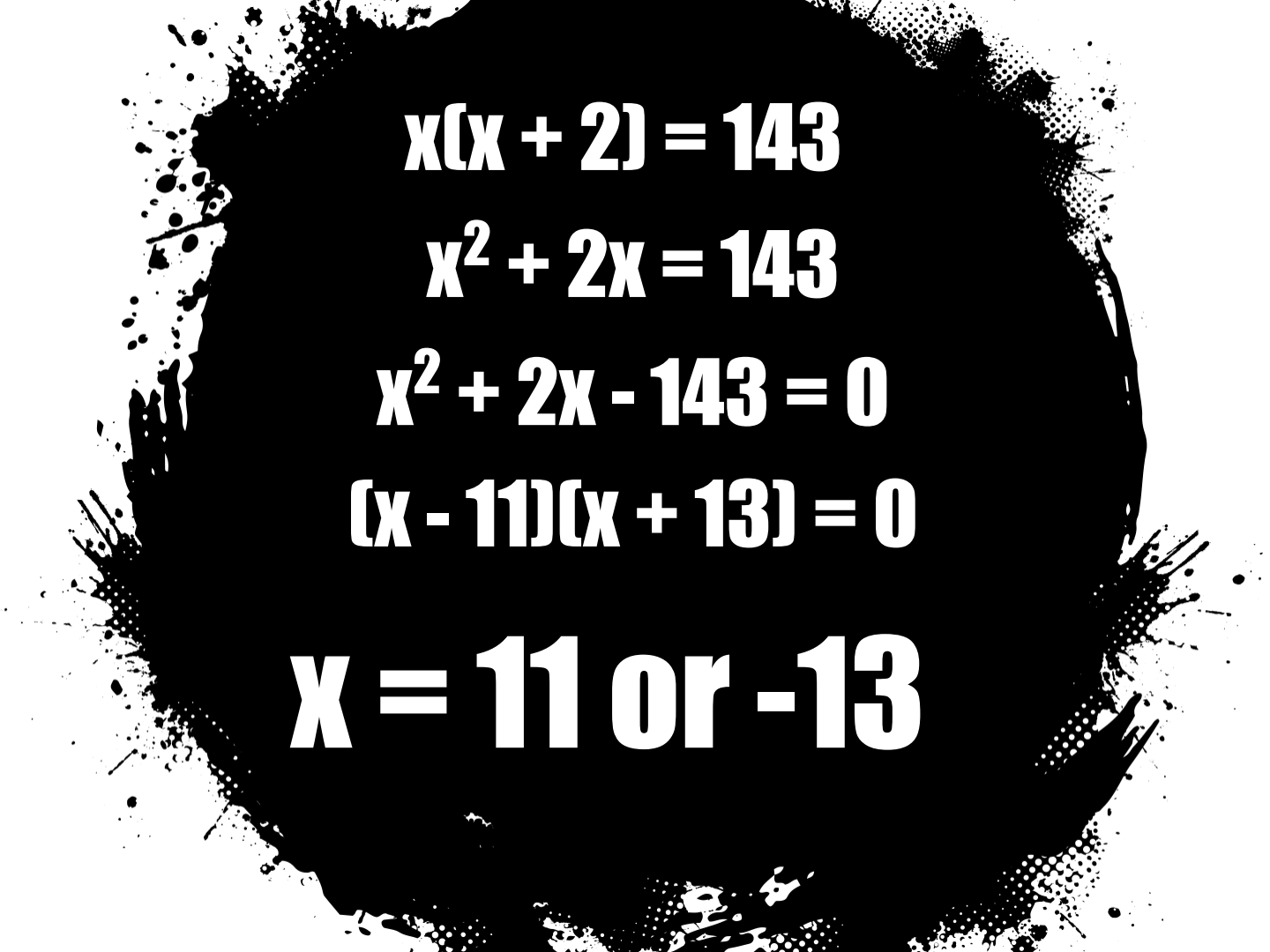
100!

6 times 22, 2 times 6 is 12, carry the one. Then 6 times 2 is 12 again but add one so... 132. How didn't you get that?

Oh.. I must have forgot.


Kids stop reasoning about numbers and focus on the procedure when we strip them of mathematical intuition

Find two consecutive, positive, odd integers whose product is 143.


x(x + 2) = 143
x² + 2x = 143
x² + 2x - 143 = 0
(x - 11)(x + 13) = 0
x = 11 or -13

The approximate root of 143 is 12... I know it needs to be odd... ...so 11 and 13? Check!

I know 10 x 10 = 100
so I'll try 11 and
the next odd.
Check!

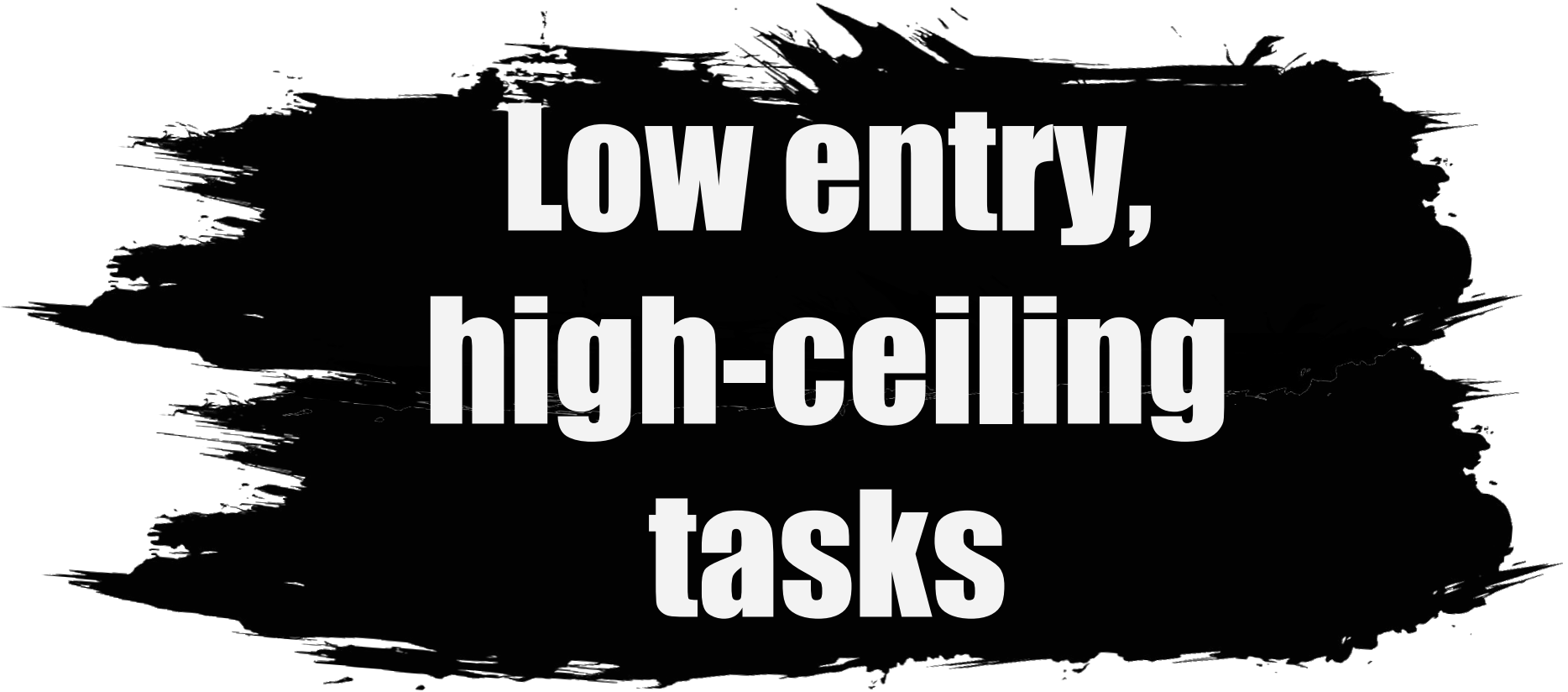
No need for the process with the numbers given

Find two consecutive, positive, odd integers whose product is 143. Write an expression to represent the situation given.


x(x + 2) = 143

Create the headache


SENSE MAKING
EFFICIENCY


Low entry,
high-ceiling
tasks

What do low-entry, high-ceiling tasks have?

- Has a low floor and a high ceiling.

  <u>Everyone</u> can start

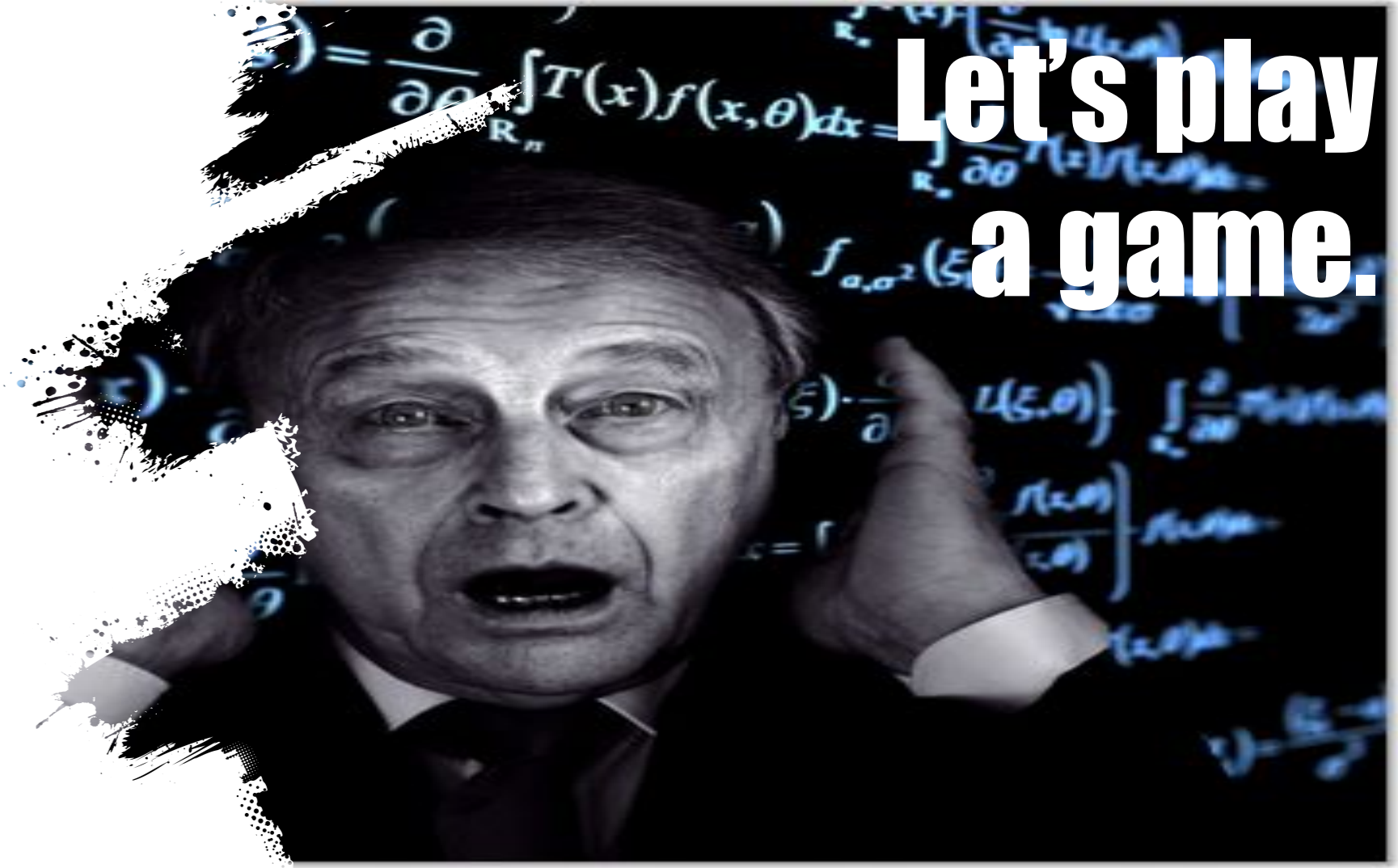
- Has multiple entry points
  - Student A starts by exploring numerically
  - Student B begins by investigating graphically
  - Student C jumps in by reasoning algebraically

- Integrates multiple topics.

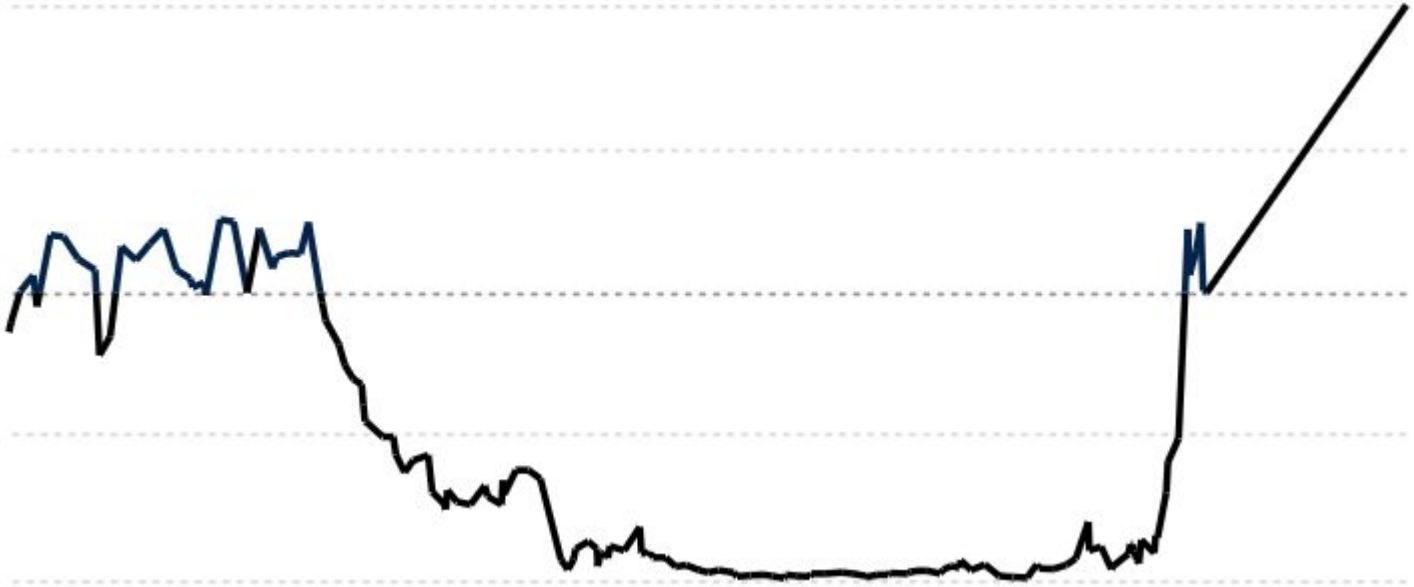
- Engages student interest, is mathematically/cognitively challenging.

Impacts of low-entry, high-ceiling tasks

- Persistence
- Flexible Thinking
- Student engagement
- Questioning and problem solving
- Application of past knowledge to new situations
- Clear and precise communication
- Teacher gains insight into the how a student approaches the task not just their result

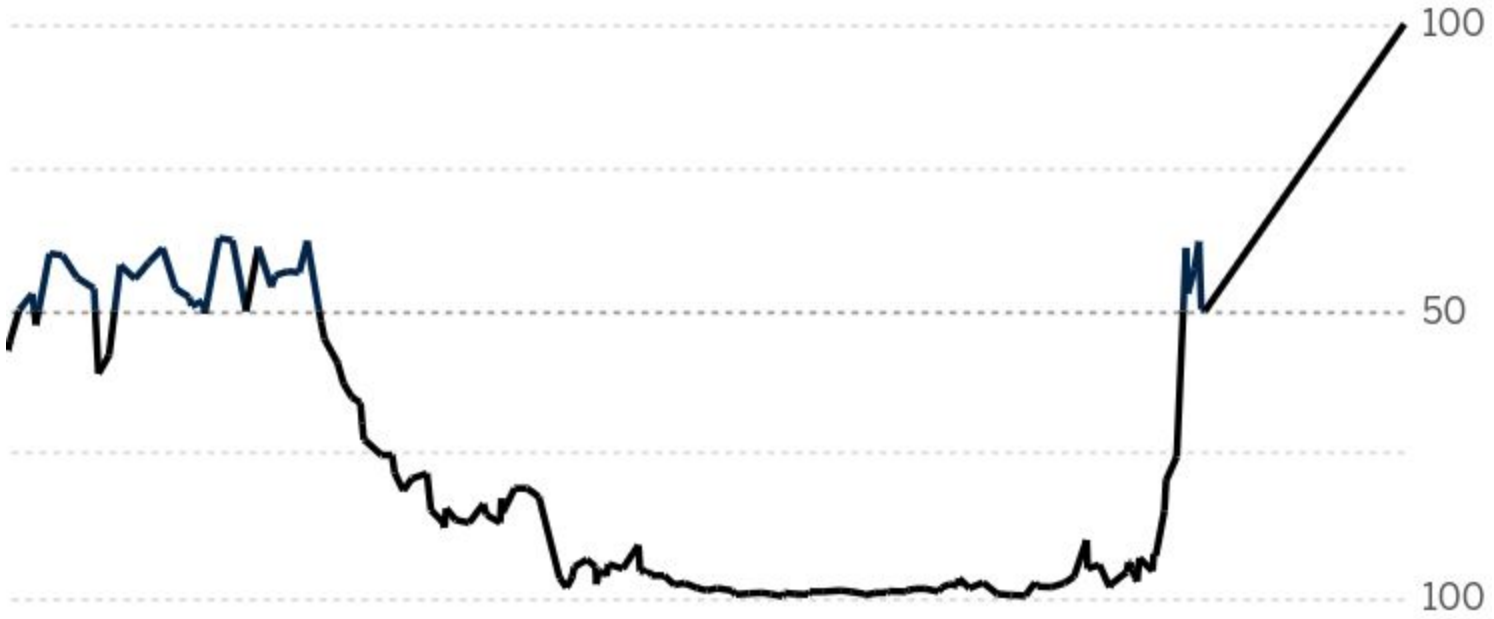

Let's play
a game.

What do you wonder?



What do you notice?

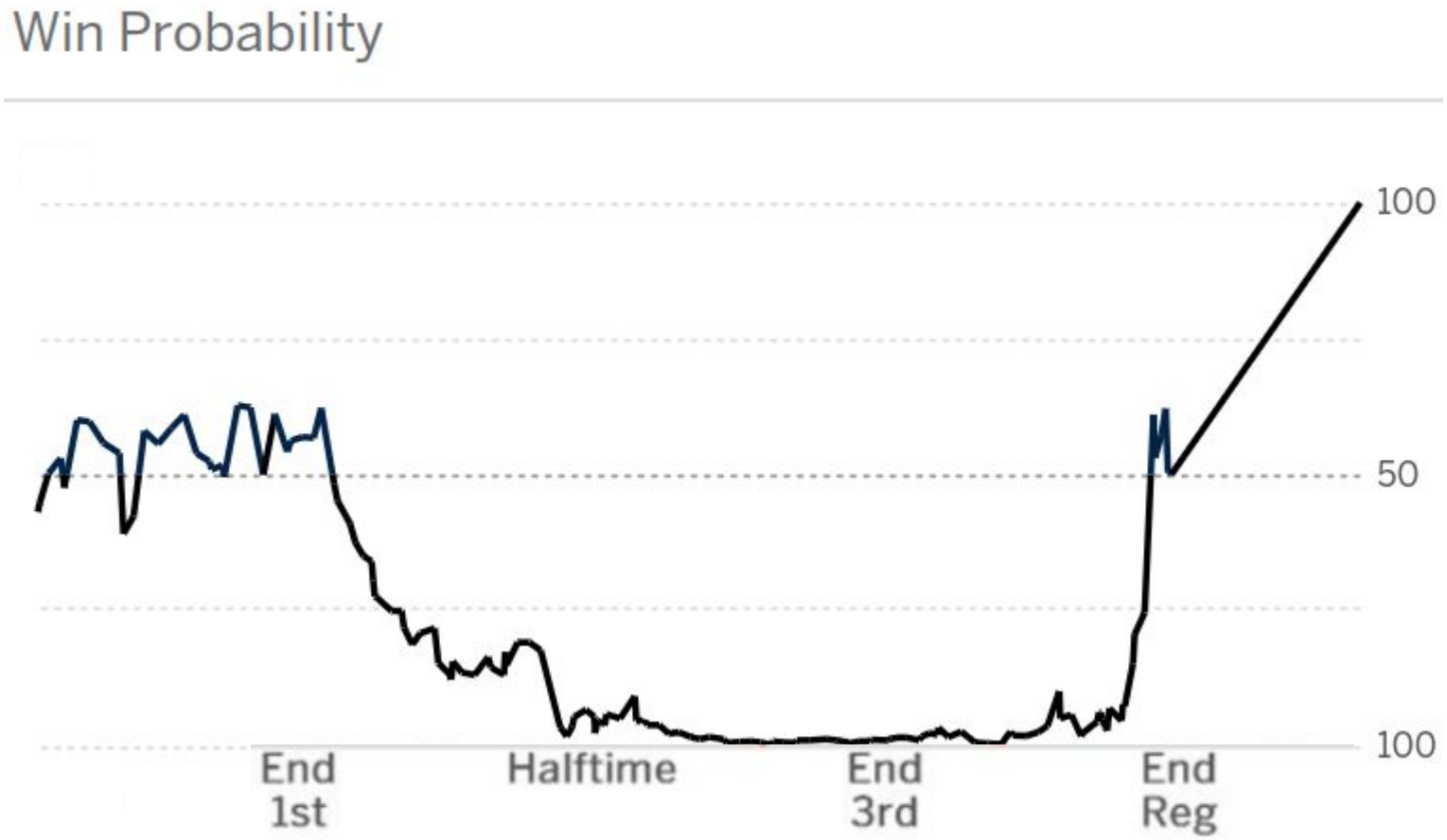
What do you notice? What do you wonder?



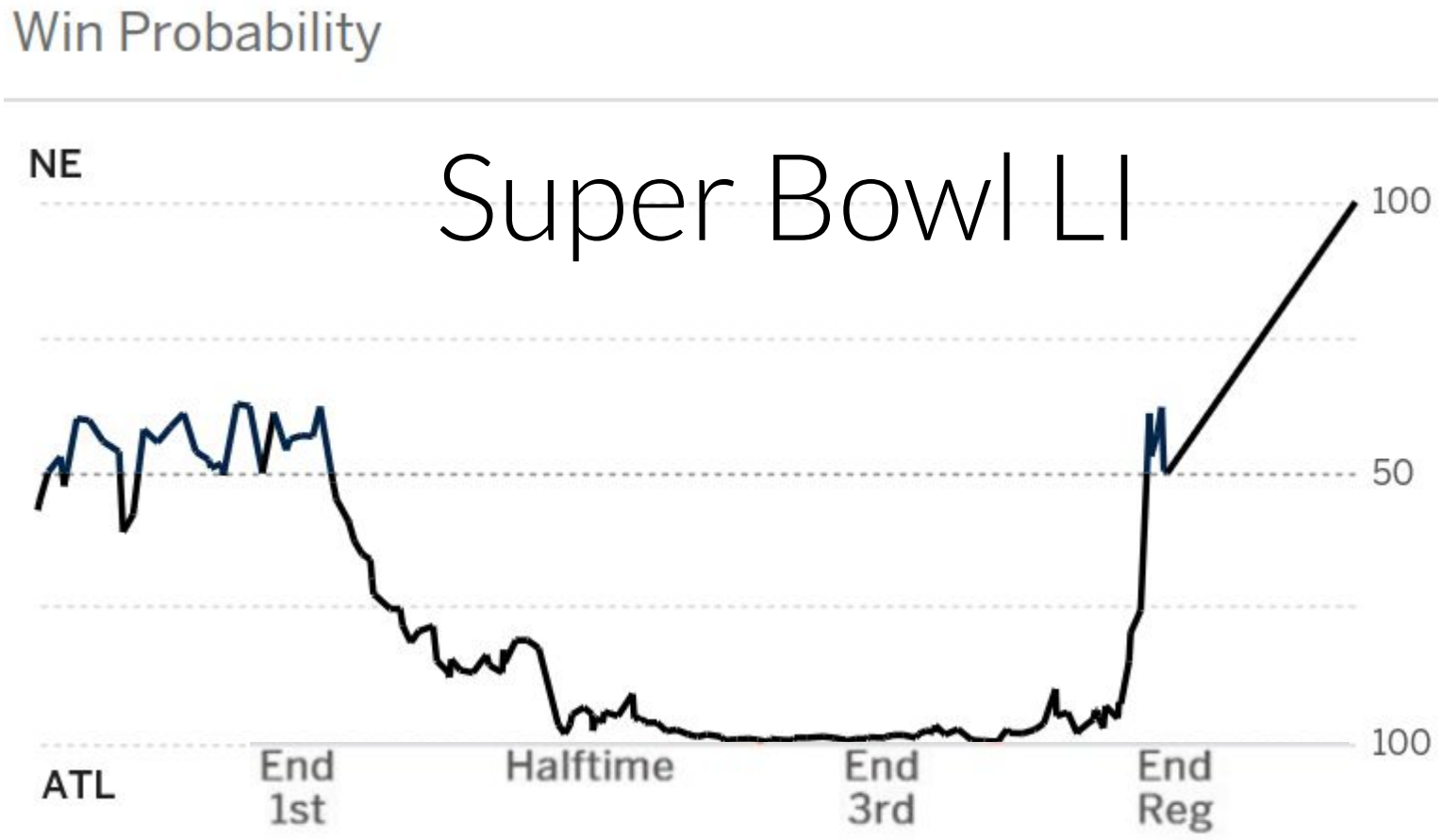
What do you know now?

What do you notice? What do you wonder?

What do you notice? What do you wonder?



What do you notice? What do you wonder?




Round 2

What do you notice? What do you wonder?

What do you notice? What do you wonder?

What do you notice? What do you wonder?

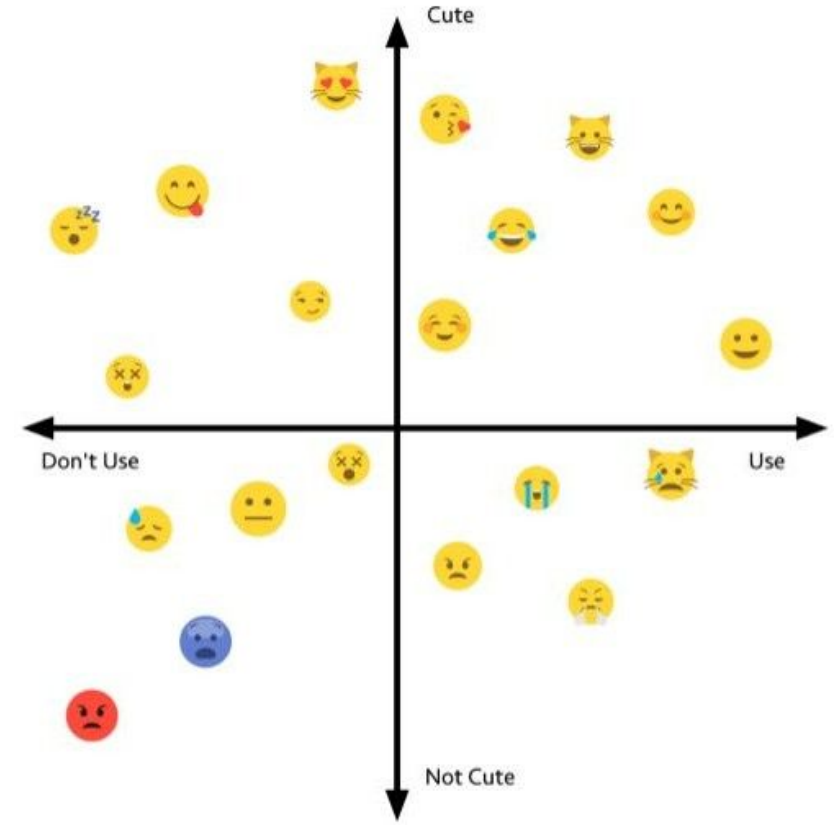
Peak Break-Up Times

According to Facebook status updates

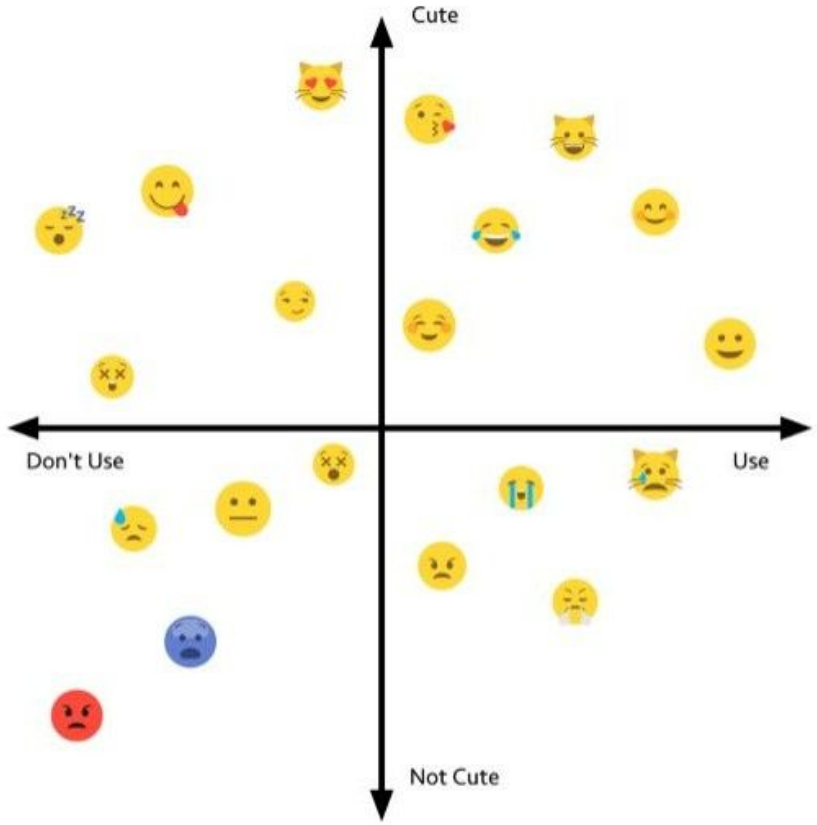
Notice and Wonder

What do you notice?

What do you wonder?

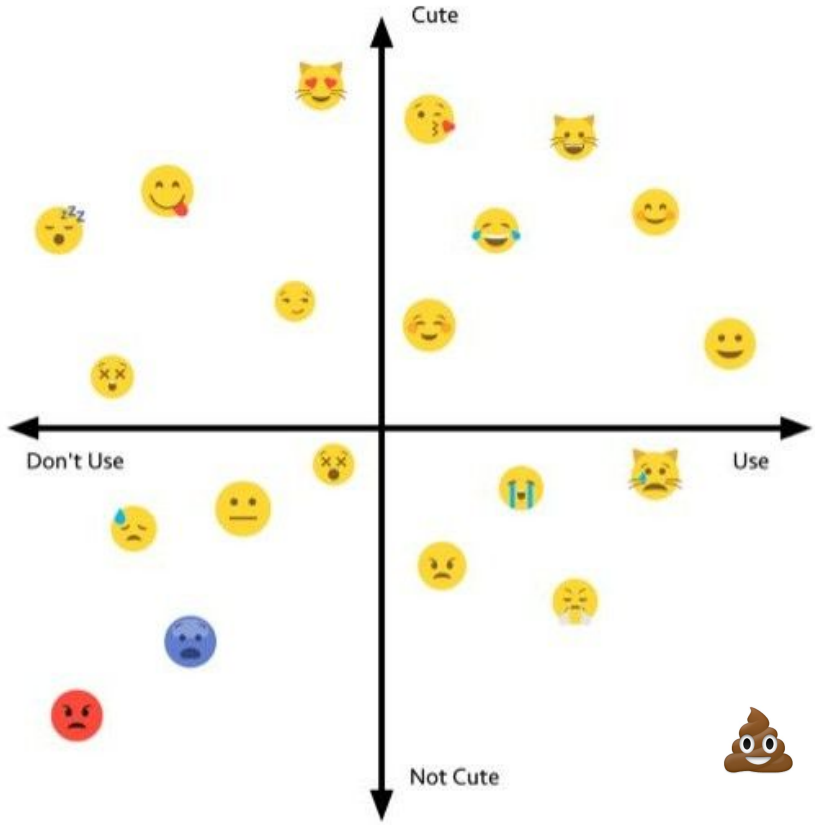


Where's the poop emoji?



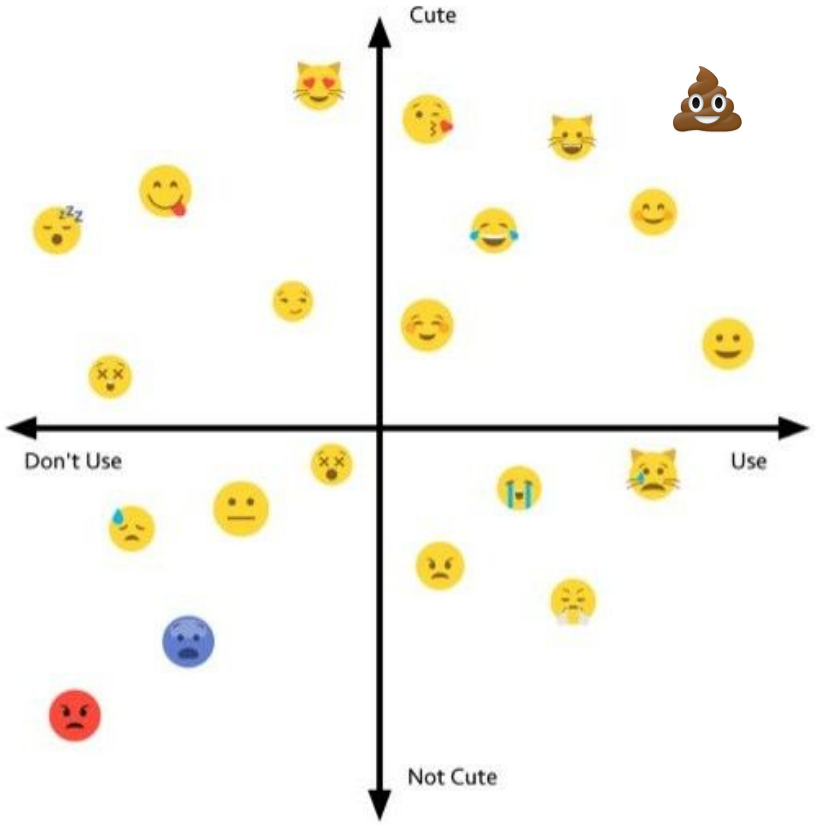
Jo Boaler

Where's the poop emoji?



Jo Boaler

Where's the poop emoji?



Jo Boaler

Estimation

Why is Estimation Important?

Longer than you think!

US federal guidelines dictate that the dashed lines separating traffic lanes or indicating where passing is allowed run 10 feet in length.

https://www.psychologicalscience.org/news/motr/lines-on-the-road-are-longer-than-you-think.html

How many beads in the small jar?

How many beads in the large jar?

estimation skills

How many beads are in the jar?

Eli – 95

Brendon – 10[illegible]

Matty – 105

Jake – 104

Hayes – 101

Wolf – 140

Mrs. Wilkin

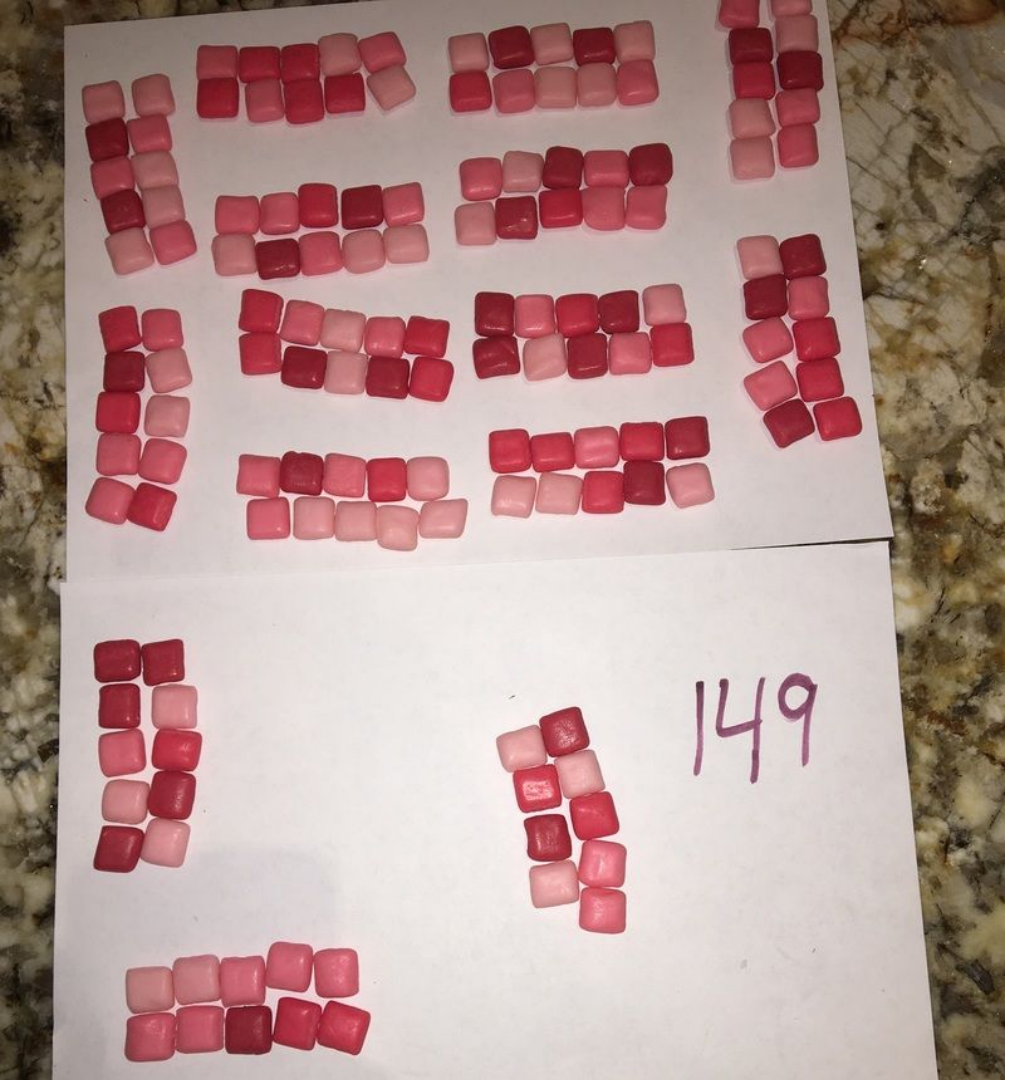
How many beads in the large jar?

Wanna make kids mad?

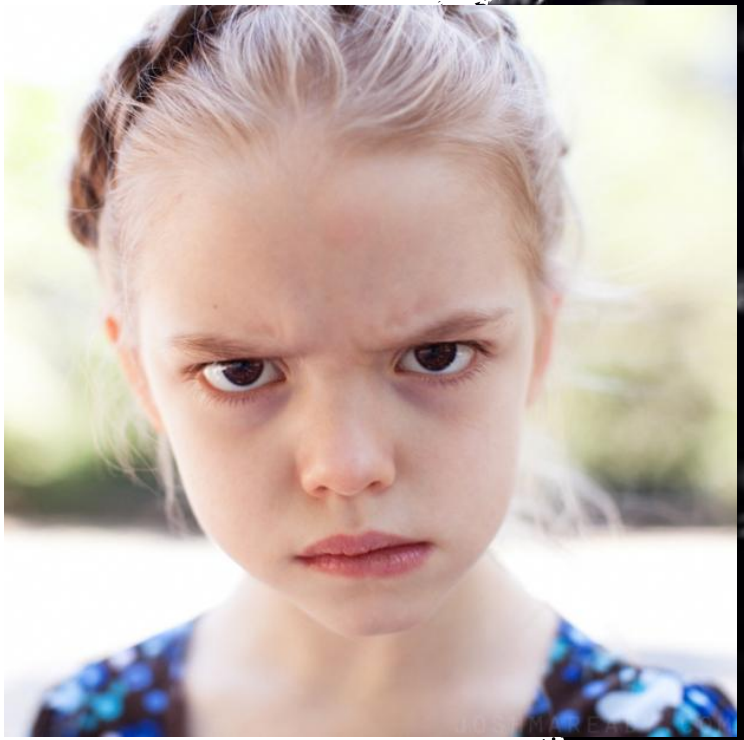

1/4 CUP


37


NEW
RESEALABLE!
FRESH PACK
FaveREDs
Starburst
FRUIT CHEWS
minis
unwrapped!

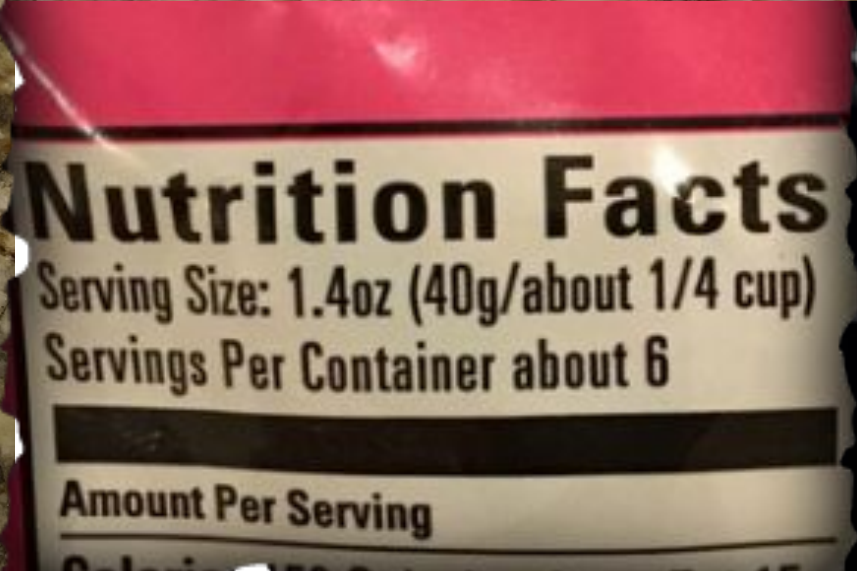


Hmm...




149


37
1/4 CUP


Nutrition Facts
Serving Size: 1.4oz (40g/about 1/4 cup)
Servings Per Container about 6
Amount Per Serving

It’s like taking candy from a baby

Dear Starburst...

The correct answer is A. Why might the test writers have chosen the other answers? Think about how they were trying to mislead you and common errors that people might perform.

B because the y-intercept is -2

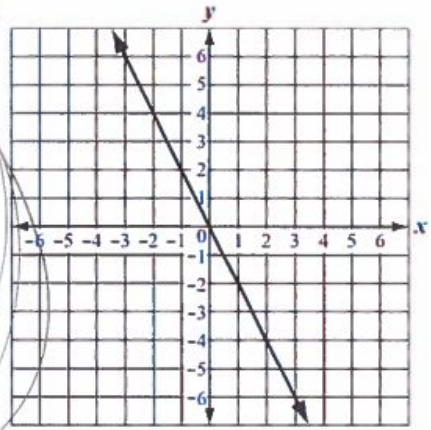
C because the slope is -2

D because the x-intercept is 2

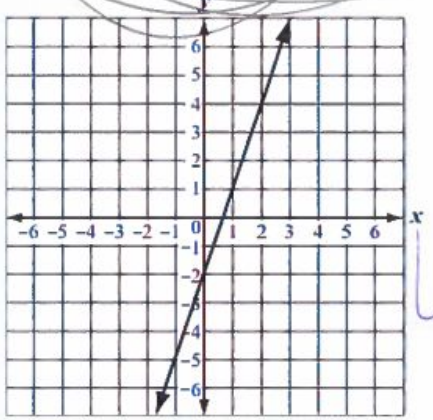
+1

s a line that has an **x-intercept** of -2?

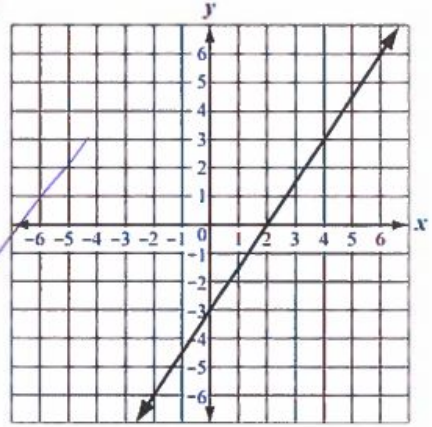
C.



B.

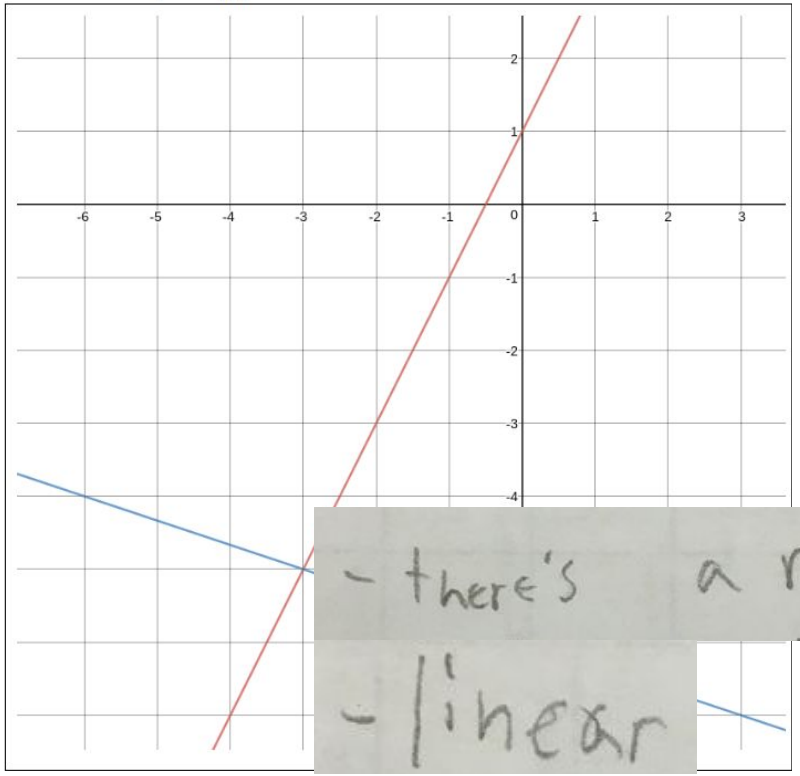


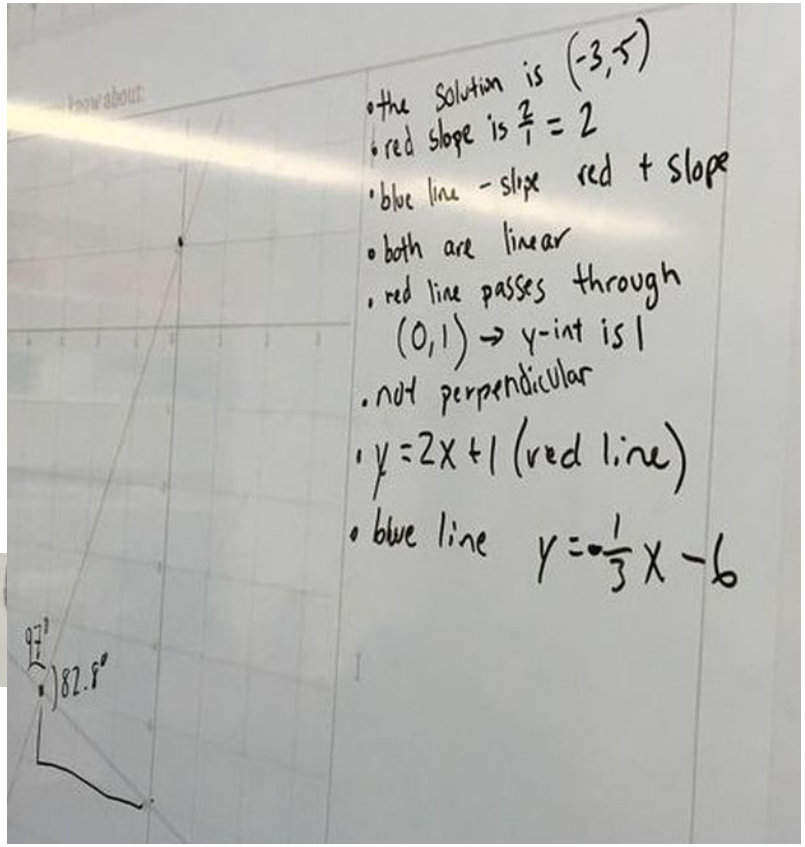
D.



Kids WILL surprise you

Tell me everything you know about:





Blue line equation y= -¼x-6
The lines are on a graph
Red line passes through (-1,-1)
Blue line passes through (-6, -4)
Blue line passes through (-4, -7)
Red line passes through (-2, -3)
Red line passes through (-3, -5)

These are linear equations
There are two lines
Both lines go on forever
Each line makes triangles in each grid square
Not 5 degrees
Not curved
Equations are to the power of 1 (no exponents)

Graph could be representing two different running rates of two different people

Blue is less steep than red
The red and blue lines do not make right angles as they intersect

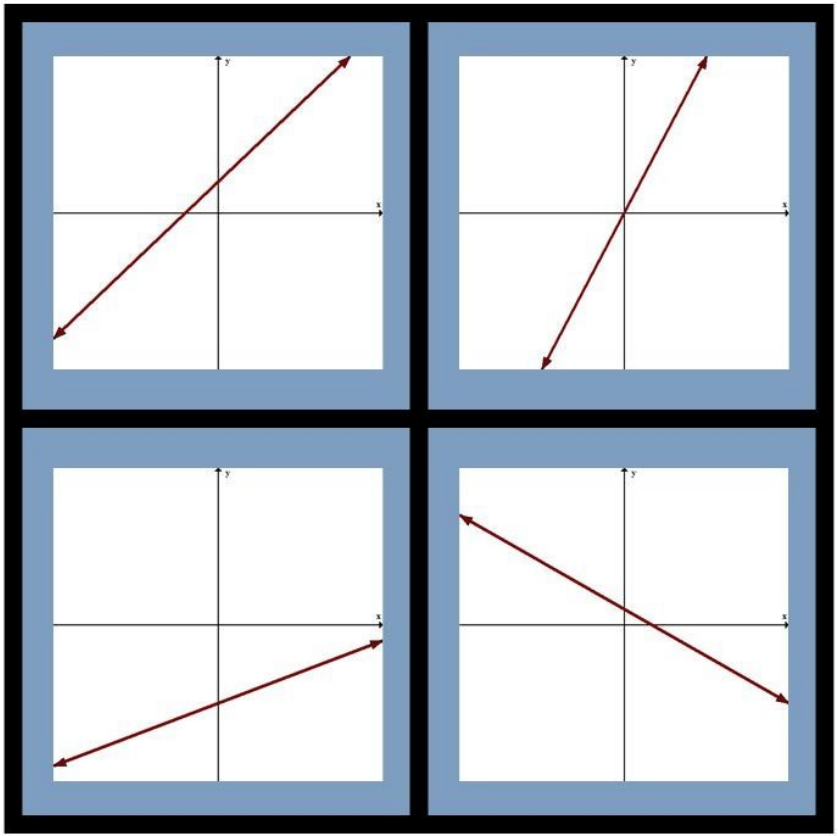
There is a x-axis and a y-axis
There are no undefined lines

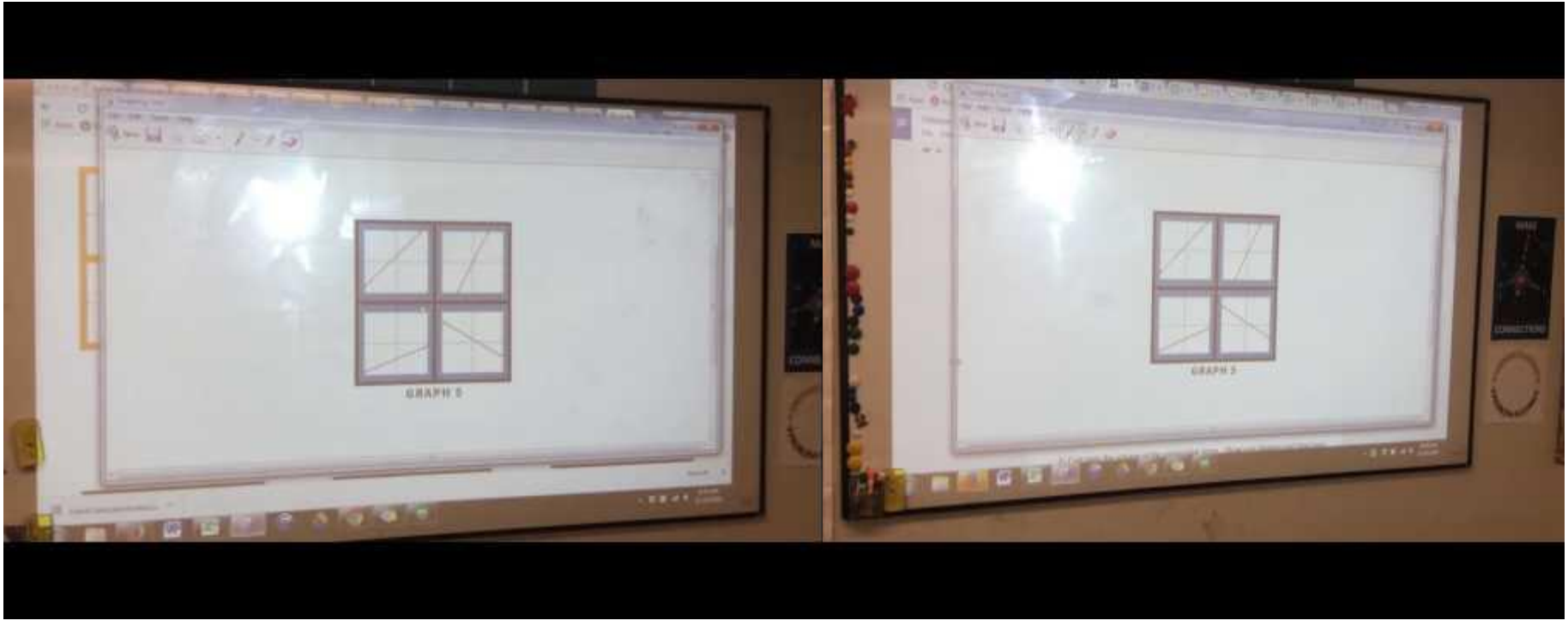
Red line y-intercept = 1
Blue line y-intercept = -6
Not parallel
One intersection point
One solution

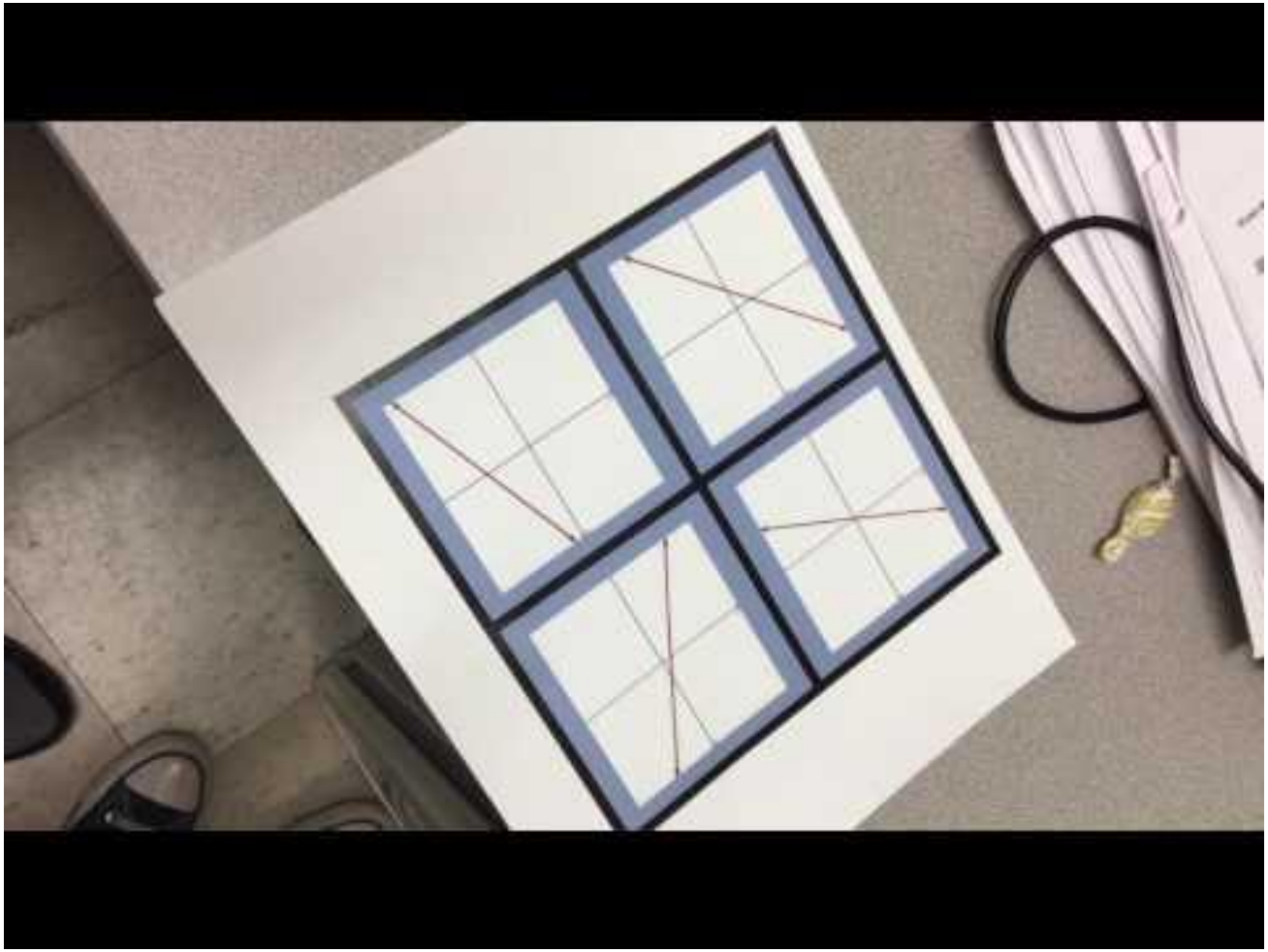
Intersection = (-3, -5)
Red line slope is positive
Blue line slope is negative
Not perpendicular
Lines are diagonal

Red line slope = 2
Red line is parallel to 2x+3
Red line equation is y=2x+1
Red line x-intercept is -0.5
Blue line slope = -⅓
Blue line is parallel to y=-⅓x-7
Blue line equation is y = -⅓x -6
Blue line x-intercept is -18

Which One Doesn't Belong? Why?







“Just like any other skill, you can learn to do math if you need to use it. Instead of telling our kids (and ourselves) that math is hard, we need to show them how relevant these skills are in the high-tech lives they’re living.” - David Ludden

https://www.psychologytoday.com/blog/talking-apes/201703/how-overcome-math-anxiety


If a euro is worth $1.50, five euros is worth what?
A: Thirty quarters
B: Fifty dimes
C: Seventy nickels
D: Ninety pennies

Making Mistakes Intentional.

Analyzing the Nature of Student Understanding with Low-Entry, High-Ceiling Problems

Peter Morris
@1pmorris

Carroll School, Waltham, MA
Head of Lower School
Mathematics

Jen McAleer
@jennifuhs4

Carroll School, Lincoln, MA
Head of Middle School
Mathematics

http://msmaths.weebly.com/NCTM18.html