

Activities for Transitioning Students from Precalculus to AP[®] Calculus



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Is there a real number that is exactly two less than its reciprocal?

What strategies did you use to solve the problem?

What strategies would your students use to solve the problem?

What do you like about this problem?

What don't you like about this problem?

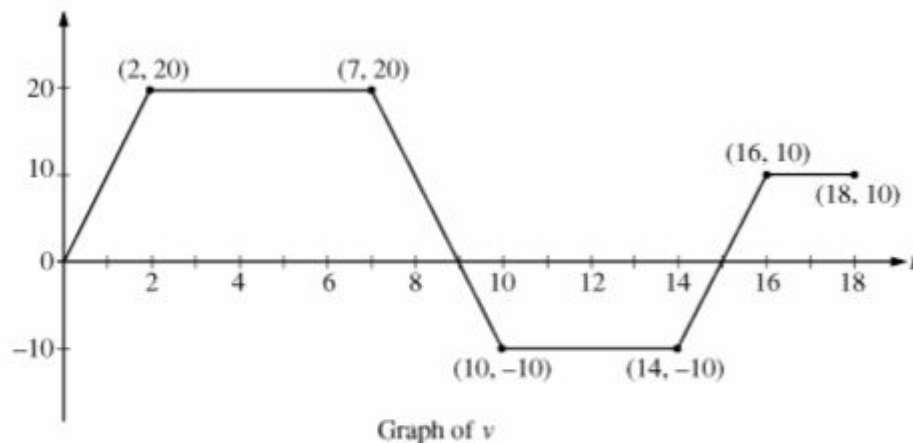
Mathematical Practices for AP[®] Calculus (MPACs)

1. Reasoning with definitions and theorems
2. Connecting concepts
3. Implementing algebraic/computational processes
4. Connecting multiple representations
 - a. Graphical
 - b. Numerical
 - c. Analytical
 - d. Verbal
5. Building notational fluency
6. Communicating

Squirrel

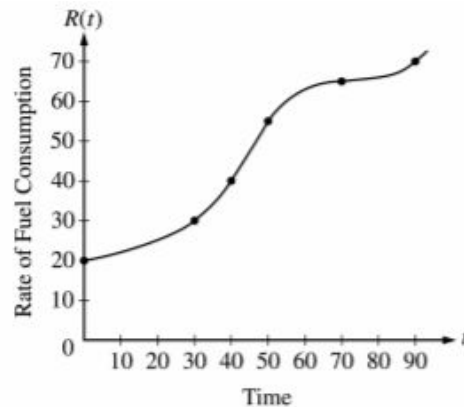


No calculator is allowed for these problems.



4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B . For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.
- At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.
 - At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A ? How far from building A is the squirrel at that time?
 - Find the total distance the squirrel travels during the time interval $0 \leq t \leq 18$.
 - Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

Fuel Consumption



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.
- Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
 - The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
 - Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
 - For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane.
Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

Basic Derivative Precalculus Question

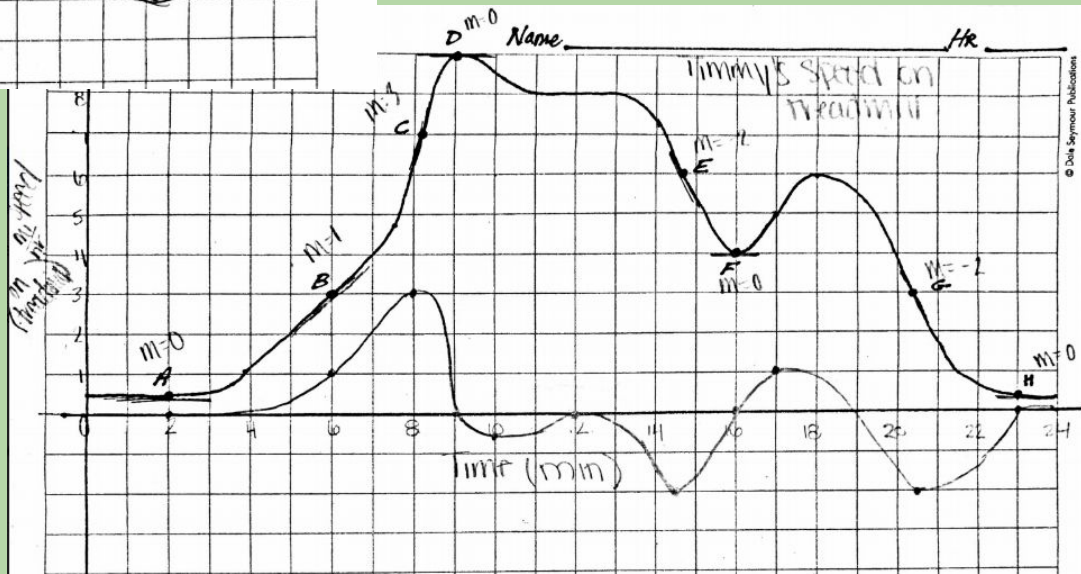
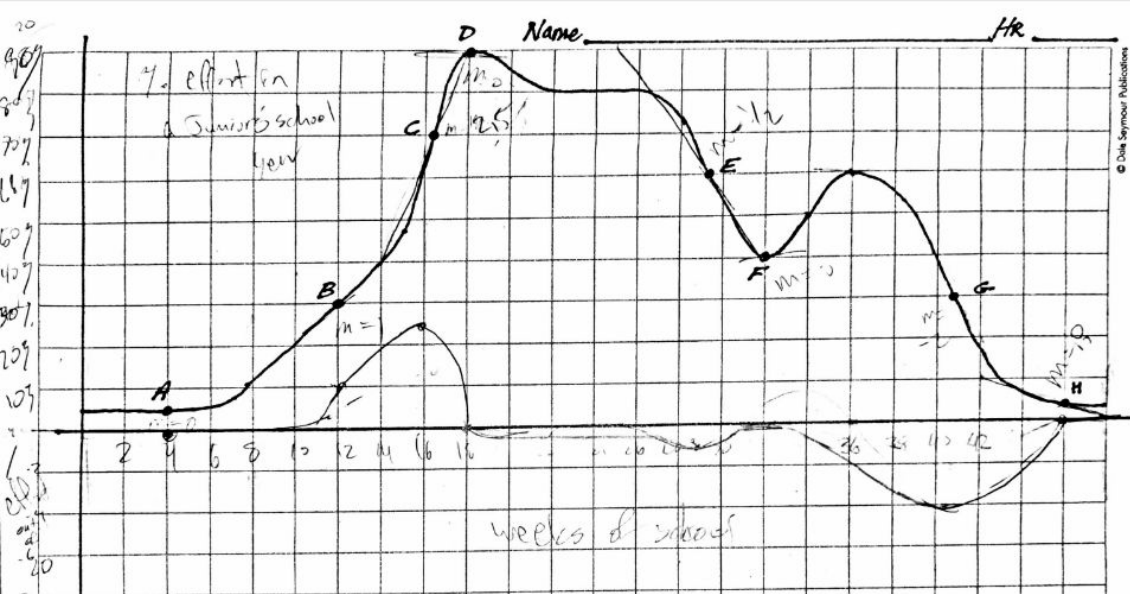
If $f(x) = x^3 - 5x$, what is $f'(2)$?

Verbal

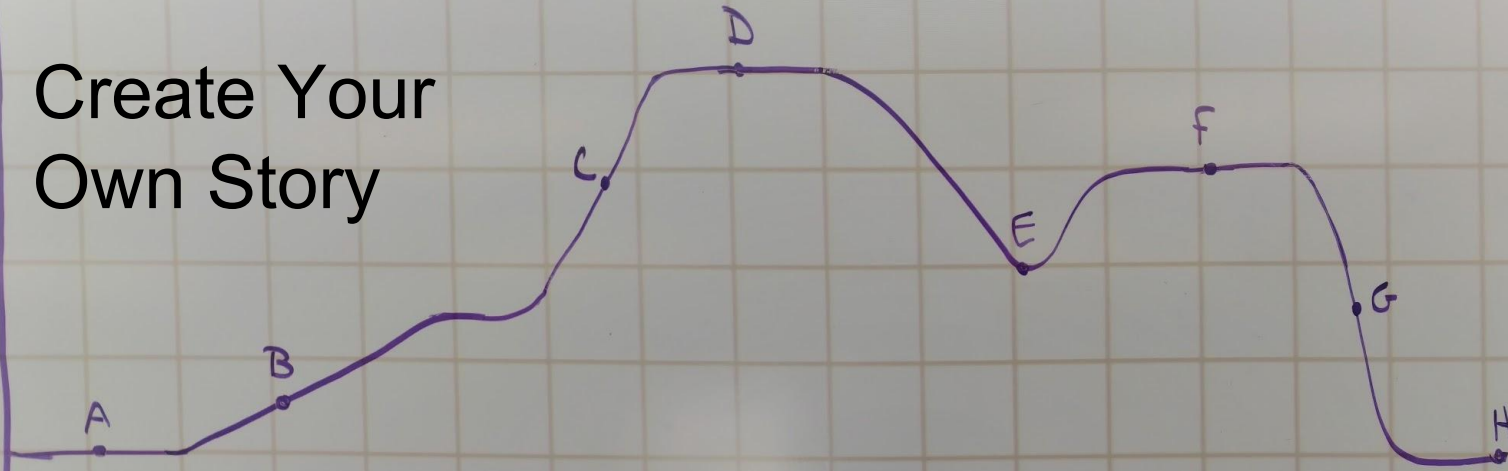
Analytical

Graphical

Numerical



Create Your Own Story



Verbal - explain what is happening in the situation at each labeled point

Numerical - estimate the slopes' values

Graphical - draw tangent lines, graph of the slopes of the tangent lines

Analytical - make a conjecture regarding the relationship between the position of the newly graphed points and the behavior of the original graph

Basic Continuity/IVT Precalculus Question

Where are the roots of $f(x) = \lceil x \rceil + 0.5$?

Verbal

Analytical

Graphical

Numerical

IVT Introduction Activity

Groups of 4 students are given envelopes filled with cards. They sort the cards into two piles depending on an answer of “yes” or “no” for each question. The teacher then facilitates a conversation about continuity, leading to a formal definition, and challenges the students to create their own scenario that includes both positive and negative y-values.

Ammon had \$20 when he went to the store. When he came home, he had \$13.25. Must there be a time when Ammon had \$17.32?

After the first day of spring break, Jamie’s husband noticed that the odometer on the car read 285 miles. After the 5th day of spring break, the odometer read 725 miles. Was there a day when the odometer showed 400 miles?

Basic Riemann Sum Precalculus Question

What is the area of the region bounded by the graph of $f(x) = -x^2 + 4x + 12$ and the x-axis using a left Riemann sum of eight equal subintervals?

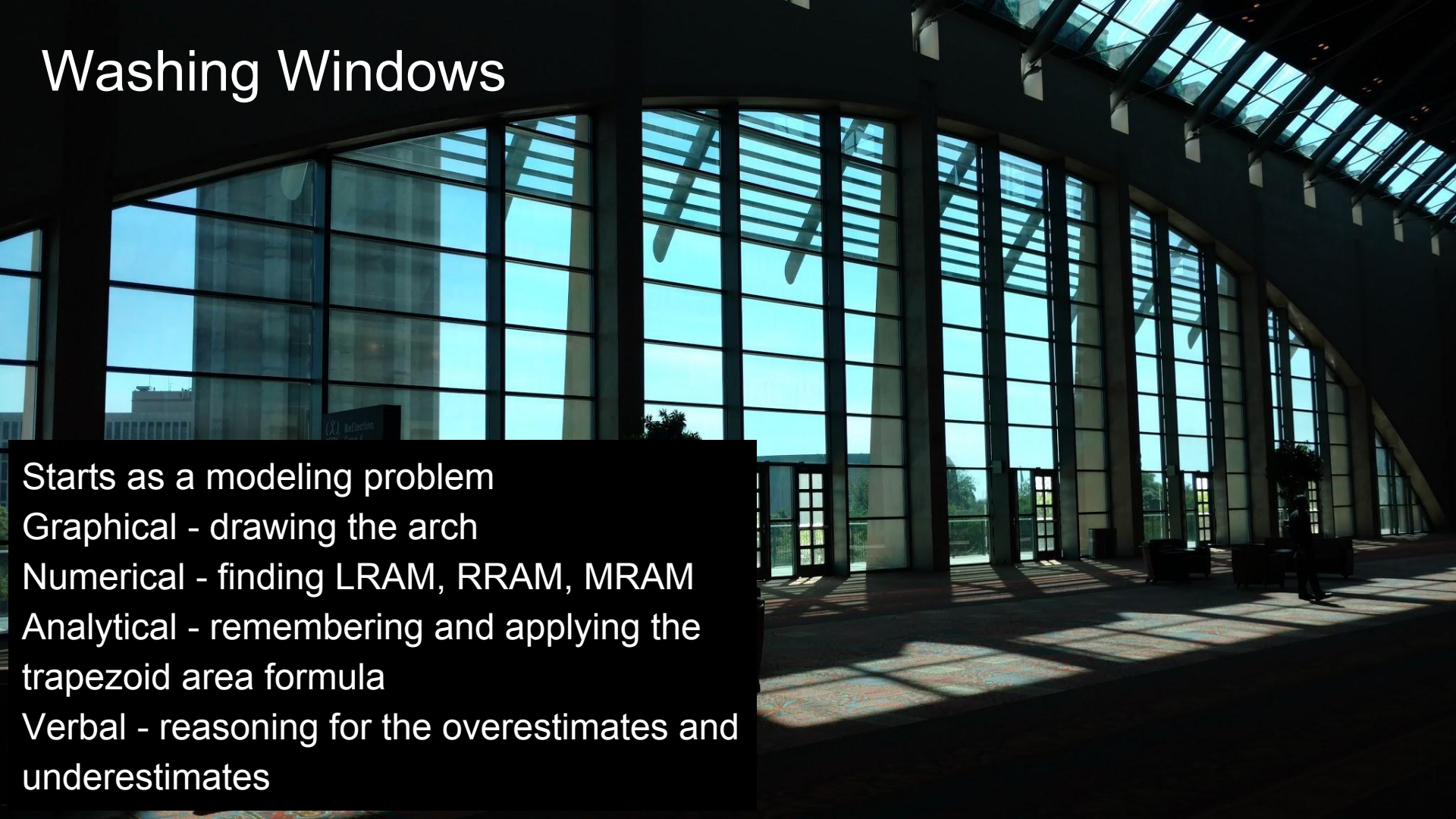
Verbal

Analytical

Graphical

Numerical

Washing Windows



Starts as a modeling problem

Graphical - drawing the arch

Numerical - finding LRAM, RRAM, MRAM

Analytical - remembering and applying the trapezoid area formula

Verbal - reasoning for the overestimates and underestimates

Now you try...

Try to enhance your given precalculus problem to include multiple representations.

Verbal

Analytical

Graphical

Numerical

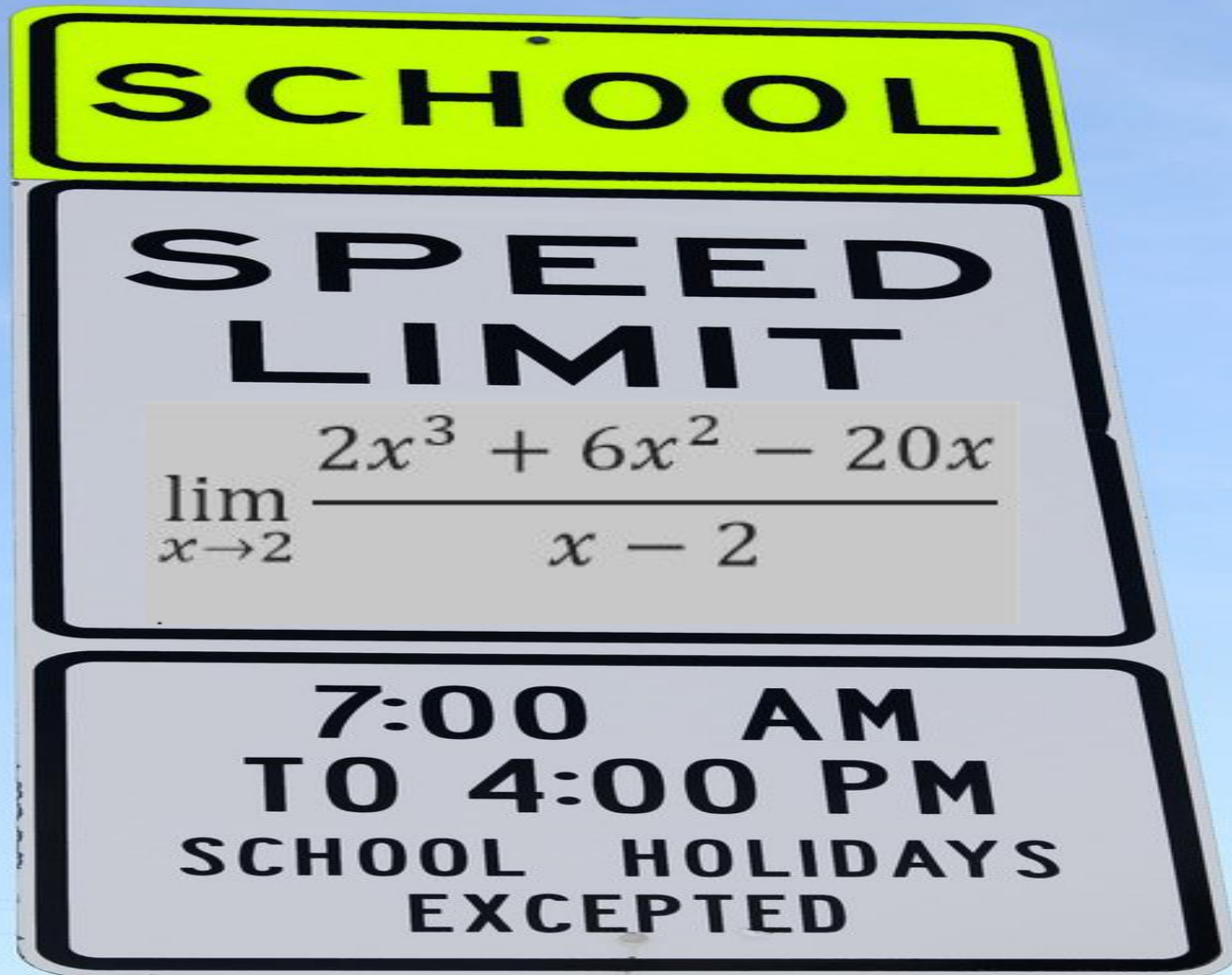
Exponentials

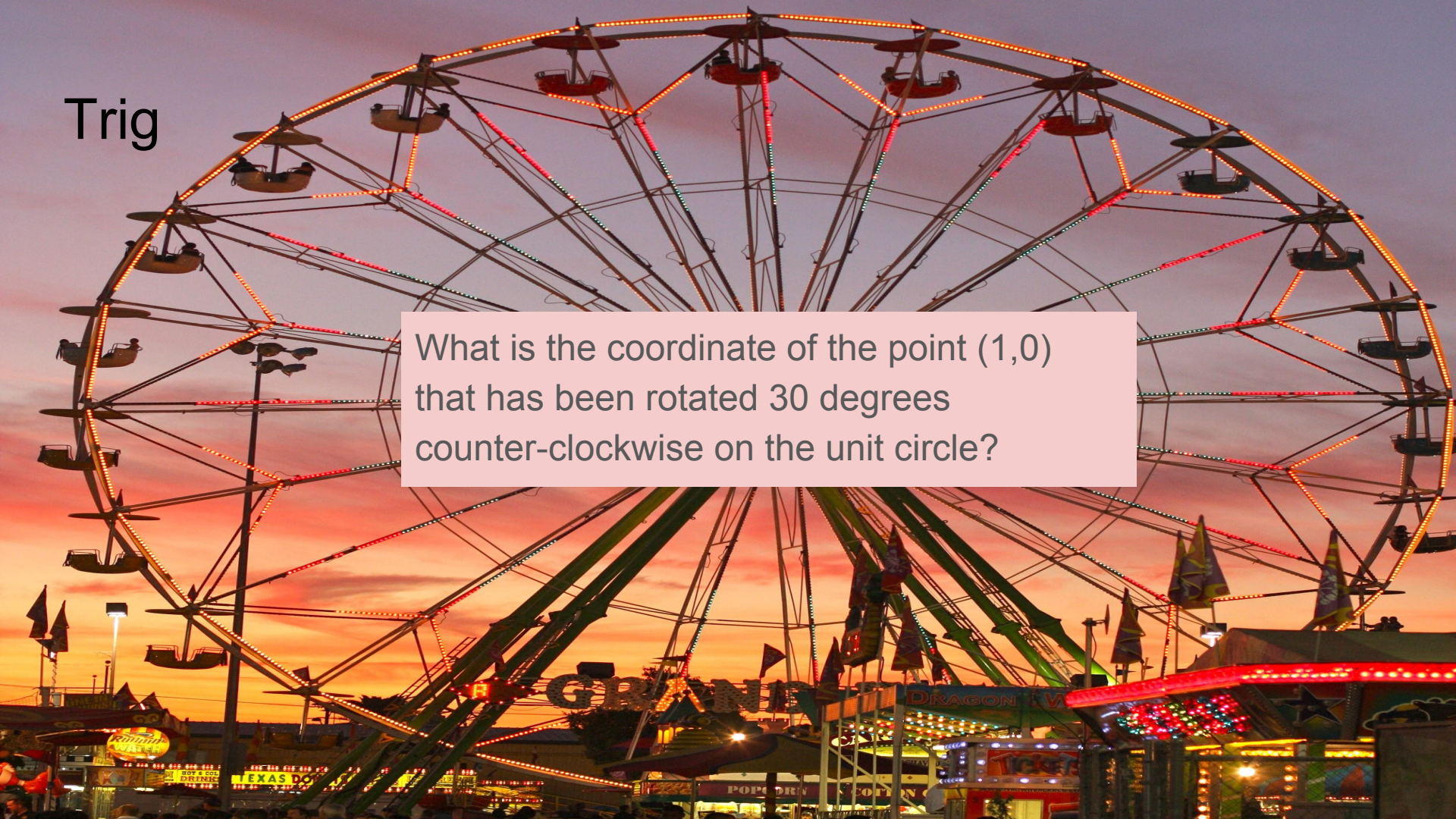
A 2012 Ford Mustang GT is valued at \$11,000 and depreciates 10% per year. What is the value of the car in 4 years?



Limits

Find the limit.



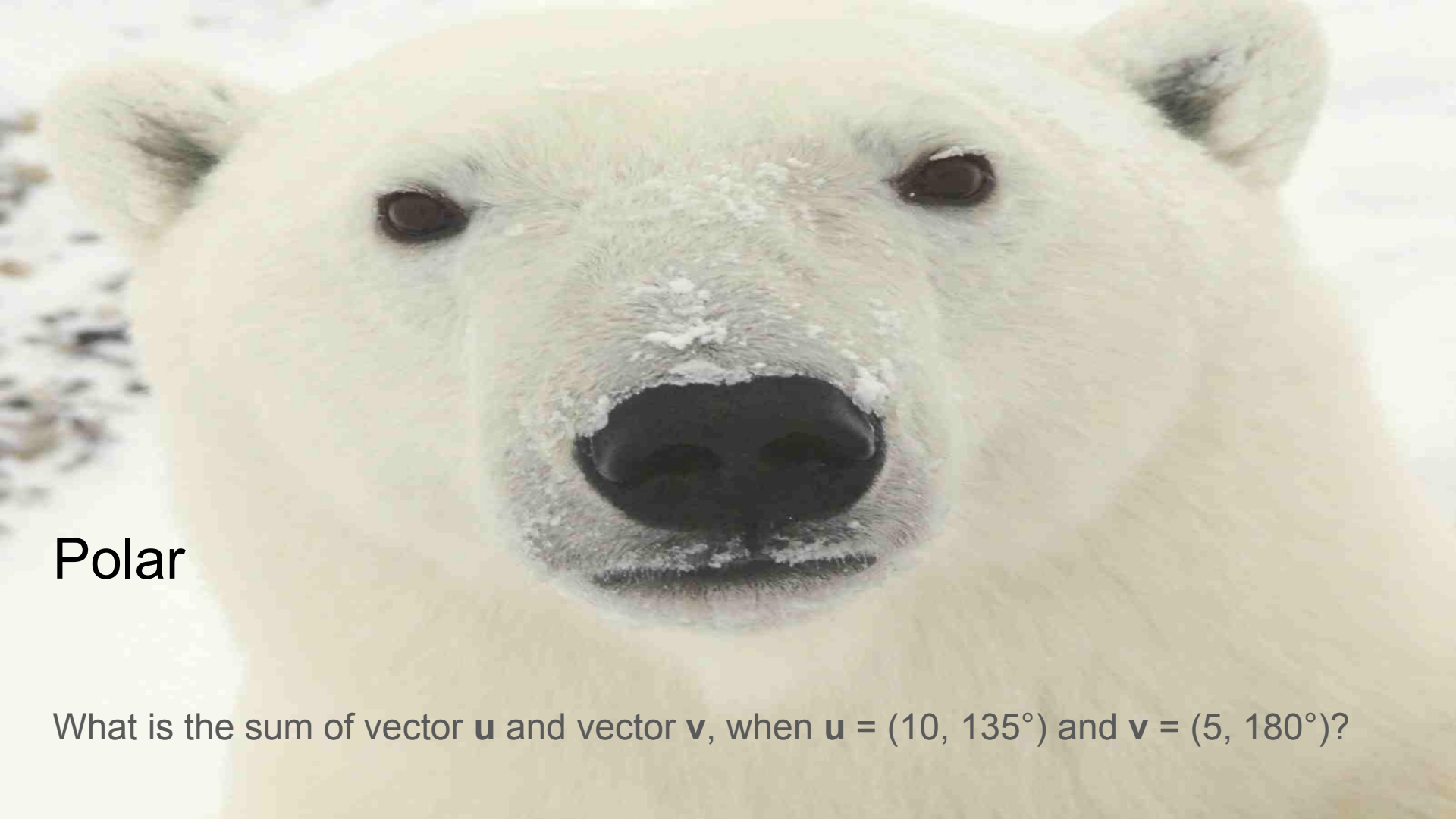
A large Ferris wheel is the central focus of the image, set against a twilight sky with orange and pink hues. The wheel's structure is illuminated with red and green lights. Below the wheel, various fairground structures and signs are visible, including one that says "GRAND" and another "DRAGON".

Trig

What is the coordinate of the point $(1,0)$ that has been rotated 30 degrees counter-clockwise on the unit circle?

Sequences and Series

Consider the sequence $30, 33, 36, \dots$ What is the sum of the first 35 terms?



Polar

What is the sum of vector $\mathbf{u}$ and vector $\mathbf{v}$, when $\mathbf{u} = (10, 135^\circ)$ and $\mathbf{v} = (5, 180^\circ)$?

Basic Algebra Question

What is the equation of the line containing the points (2,5) and (6,15)?

Embellished Algebra Question(s)

Consider the ordered pairs (2,5) and (6,15).

- a) Describe a situation in which these points could represent collected data.
- b) What is the rate of change? What does it represent in your situation? What units are being used?
- c) Plot these on graph paper and connect with a line. Does the point (3,8) lie on it? Check/Justify your answer by utilizing the equation of this line.
- d) Extrapolate a y-value for when $x = 112$. Would this make sense in your situation from part a)? Why or why not?

Basic Geometry Question

What is the volume and surface area of a sphere with a radius of 3?

Embellished Geometry Question(s)

Consider a spherical balloon with a current volume of 36π cubic inches.

- a) What is the current surface area of this balloon? Include the correct units.
- b) If the balloon's radius is doubled, how would this affect the surface area?
How would this affect the volume?
- c) What is the rate of change of the surface area and volume with respect to the radius at the points in parts a) and b)? Are they equal?
- d) Graph the surface area equation for a sphere. Explain why we cannot use the whole parabola when dealing with the balloon.



Questions?

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Thank you!