Algebraic Procedures in Need of a Conceptual Makeover

Karen McPherson

NCTM Annual Conference 2018

Washington, DC

Instructional Shifts in Mathematics

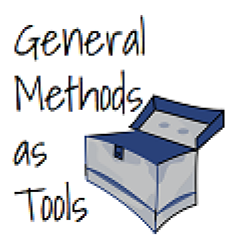
- Focus strongly where the standards focus
- Coherence think across the grades and link to major topics within the grade
- Rigor in major topics pursue with equal intensity:
  - conceptual understanding,
  - procedural skill and fluency, and
  - application

Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Procedural Fluency Defined

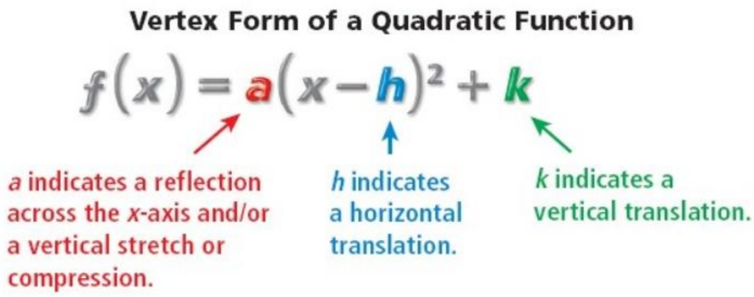
Conceptual Understanding → **Procedural Fluency**



WARNING:

Procedure: Identifying Vertex of a Quadratic from Vertex Form

Traditional Procedure



The vertex of the parabola is at (h, k).

A Conceptual Makeover

Identify the smallest quantity an expression can represent by substituting values for x

Prior Understandings and Skills

- Understand expressions represent quantities
- Evaluate an expression by substituting values for the unknown variable

Identify a real number that when placed into the blank will produce the smallest possible value. Also, identify the smallest value of the expression.

$\square^2$ $\quad$ $\square^2 + 3$ $\quad$ $\square^2 - 5$ $\quad$ $(\square + 7)^2$

$(\square + 6)^2 + 1$ $\quad$ $2(\square - 3)^2 + 1$ $\quad$ $5(\square - h)^2 - 2$

$5(\square - h)^2 + k$ $\quad$ $a(x - h)^2 + k$

Discussion

What strategy did you use to identify the real number to substitute into the expression?

Justify why the strategy works to provide the smallest possible value for the expression.

Identify a real number that when placed into the blank will produce the largest possible value. Also, identify the largest value of the expression.

$-\square^2$ $\quad$ $-\square^2 + 8$ $\quad$ $-(\square - 4)^2$

$-(\square + 3)^2 - 6$ $\quad$ $-2(\square - h)^2 + k$

Discussion

What strategy did you use to identify the real number to substitute into the expression?
Justify why the strategy works to provide the largest possible value for the expression.

Vertex Form of a Quadratic

Identify the vertex of the quadratic given the quadratic expression

$$a(x - h)^2 + k$$

Describe the conditions for when the vertex is a minimum and when it is a maximum.

Reasoning Strategies

- Focus on the value for x for which the evaluation of (x - h) is 0.
- Understand that when x – h is 0 then the evaluation of the entire expression is k.
- Recognize when the expression represents a minimum or a maximum quantity and consider how adding k to the expression $a(x - h)^2$ affects the quantity for various values of x.

Procedure: Writing the equation of a line given two points.

Traditional Procedure

Write the equation of the line that passes through the points (-1,2) and (7, 6).

$$m = \frac{6-2}{7-(-1)} = \frac{4}{8} = 0.5$$

- Use the slope formula to find m.

$$y = mx + b$$
$$6 = 0.5\,(7) + b$$

- Use the formula $y = mx + b$ and substitute slope for m and one of the points for (x, y).

$$6 = 3.5 + b$$
$$6 - 3.5 = b$$
$$2.5 = b$$

- Solve for b.

$$y = 0.5x + 2.5$$

- Substitute m and b back into the equation

A Conceptual Makeover: Use a Table to Build a Generalized Procedure

Prior Understandings and Skills

- Defining characteristics of a linear relationship are the constant rate of change (slope) and the initial value (y-intercept)
- Connections between how the slope and y-intercept are represented among mathematical representations (graph, table, description and symbols)

For each of the following tables, write the equation of a line that passes through the two given points.

x	y
4	3
8	1

Analyze the Strategy

---

x	y
3	1
9	5

Analyze the Strategy

Analyze the Strategy

x	y
3	7
8	27

Analyze the Strategy

x	y
14	23
23	77

Generalized Procedure

Write the equation of a line passing through the points (-1, 2) and (7,6).

Analyze the strategy

What do the points look like in a table?

What is the slope of the line?

What calculations produce the y value for when x = 0?

Reasoning Strategies

- Students transition from using Δx and Δy separately to the Δy when Δx=1 and relate this to slope.
- The y value changes as the x value changes. The amount of change in the y value from the y-intercept is *m*x where *m* is the slope
- The y-intercept is any given y value minus the amount of change in the y value.

Procedure: Solving Exponential Equations Using Logarithms

Traditional Procedure

Solve $5(2^x) = 240$

$2^x = 48$	Isolate the exponential part of the expression
$\log_2 48 = x$	Convert the equation to logarithmic form
$x = \dfrac{\log 48}{\log 2}$	Use change of base rule since 48 is not a rational power of 2
$x \approx 5.585$	Evaluate the expression using calculator

A Conceptual Makeover: Rewriting Quantities as a Power of 10

Prior Understandings and Skills

- Writing an expression into equivalent forms reveals properties of the quantity it represents.
- Power to a Power property for exponents; $(b^m)^n = b^{mn}$

Express each of the numbers as a *Power of 10*. You can find exact values for some of the required exponents. Others might require some calculator exploration.

10^x

Power of 10

a. 100	b. 1,000	c. 10,000
d. 2	e. 500	f. 4,782
g. 0.01	h. -0.001	i. 0.0023
j. 3.45	k. 34.5	l. 345

Discussion

Describe the numbers that can be written as a power of 10.
What numbers cannot be written as a power of 10 and why?

Use your results from rewriting numbers as a *Power of 10* to solve each of the following equations.

a. $10^x = 100$

b. $10^{x+2} = 1{,}000$

c. $10^{3x+1} = 10{,}000$

d. $3(10)^{x+3} = 300$

e. $10^{2x} = 500$

f. $2(10)^x = 6.90$

Common Logarithms **Look for and make use of structure**

$$\log_{10} a = b \text{ if and only if } 10^b = a$$

Without the use of a calculator, find each of the following logarithms.

a. $\log 10^{2.5}$

b. log 100

c. log 10,000

d. log 3.45

e. log 500

f. log -0.001

Use what you know about logarithms to help find the exact solution for these equations.
(Note: The exact solution will use "log 45" rather than the decimal approximation of 1.6532)

a. $10^x = 15$

b. $5(10^{2x}) = 60$

c. $3(10)^{x+4} = 21$

d. $3^x = 729$

e. $5(2^x) = 240$

f. $3(5^x + 12) = 60$

Reasoning Strategies

- Rewriting values as a Power of 10.
- Reasoning that when there are equivalent expressions with like bases then the exponents must be equivalent.

Here's the Challenge

When you are teaching a procedure

- approach it through the perspective of a student
- focus on why and when
- make connections explicit
- compare procedures and problems
- reach out to colleagues + do this work together

@nc_teach

www.mcphersonmath.com

karen.mcphersonmath@gmail.com

References

Bay-Williams, Jennifer M., and Amy Stokes-Levin. "Teaching to Build Procedural Fluency." *Enhancing Classroom Practice with Research Behind Principles to Actions.* Reston, VA: NCTM, 2017: 61-72

Boston, Melissa; Frederick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12.* Reston, VA: NCTM, 2017

"College and Career Ready Shifts in Mathematics." Student Achievement Partners. https://achievethecore.org/page /900/college-and-career-ready-shifts-in-mathematics

National Council of Teachers of Mathematics (NCTM). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM, 2014