Congruence, Similarity, and Proof: A Hands-On Transformation Approach Using Web Sketchpad

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The CCSSM expects students to understand congruence and similarity in terms of transformations and superposition, an approach that's both clearer and more generally applicable than the traditional approach.

Another benefit of the transformational approach is that SAS, SSS, and ASA are no longer postulates: now students can actually prove them!

To take advantage of this opportunity, we need new approaches to both transformations and proofs. Proof has always been a difficult area for geometry students to master, and almost all of us who teach secondary math today had minimal exposure to transformations during our own high school days.

Richard Feynman famously said *What I cannot create, I do not understand.* In this session we'll present lessons designed to overcome students' difficulties with transformations and proof, in part by providing opportunities for students to manipulate and experiment with mathematical objects that they themselves have created.

Concreteness Fading

Math-education researchers suggest that students' understanding of abstract concepts is enhanced by connecting those concepts to students' concrete experiences, and that explicitly linking and fading from concrete experiences to the abstract (usually symbolic) form can help students make important mathematical connections and strengthen their understanding of the target concept.

Accordingly, we've designed the lessons we show today to gradually link students' concrete experiences to their understanding of transformations, congruence, and similarity.

Dancing to Understand Transformations

Donovan will describe how his students explored reflection and rotation through both dance and Web Sketchpad (WSP) dynamic-geometry constructions. In these lessons students' work in the virtual environment is intentionally "faded" (less concrete) relative to their dances.

Geometric Proof as a Card Game

Wayne has used a progression of components to make proof accessible to geometry students. (Visit his session #463 and/or see his article in this May's MT.) Here he'll show one of his "Proof Without Words" lessons: students play a card game in which winning plays form a flowchart proof of an important theorem.

SSS Proof as a Construction Problem

Scott provides students with dynamic-geometry transformation tools that they use to superpose the image of one triangle on another, given that corresponding sides of the triangles are equal in length. The steps of a successful construction correspond to the steps required to prove the SSS Theorem.

Presentation web page: geometricfunctions.org/links/nctm2018/

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Concreteness Fading References

Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2014). Concreteness fading in mathematics and science instruction: A systematic review. *Educational Psychology Review*, *26*(1), 9-25.

The benefits of concreteness fading may be explained by the fact that it starts with a well-understood concrete format, and explicitly links and fades it to the abstract symbols. Concrete materials are advantageous initially because they allow the math concept to be grounded in easily understood, real-world scenarios.

Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction*, *35*, 104-120.