Mathematical Modeling as a Preassessment Tool

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Goals for this Session:

- Introduction
- Theoretical Background
- Effects of Research on My Teaching Practice
- Curriculum Changes and Sample Lesson
- Conclusions and Implications for Teaching

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Introduction:

- Who are you? (Quick Poll)
- Classroom Teacher from Columbus Ohio, and recently finished my Masters at Ohio State
  - Reflective
  - “Experienced” (according to my nametag)
  - Geek about mathematical modeling

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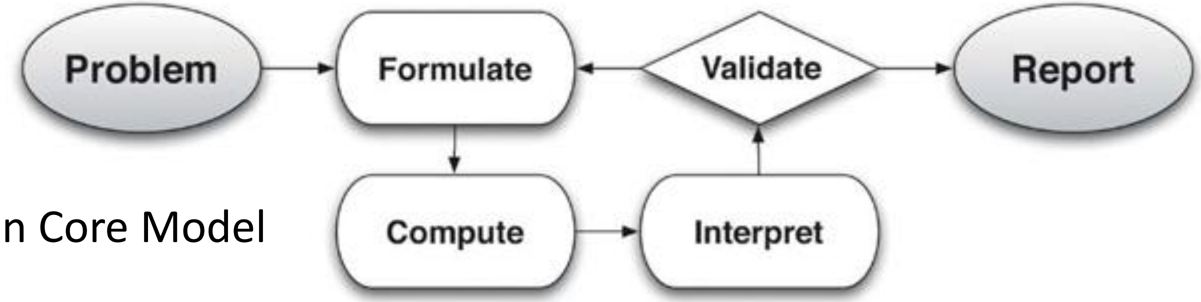
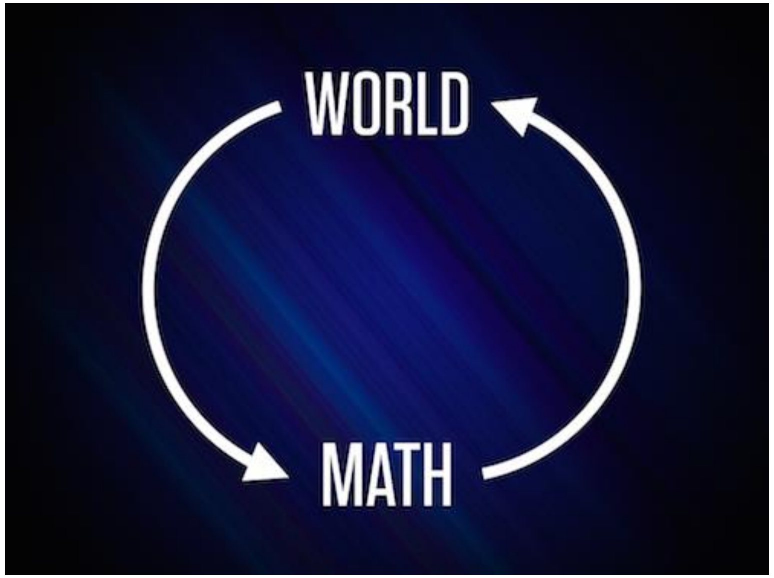
Theoretical Background:

- Teachers in the USA are surrounded by different models of mathematical modeling



Common Core Model



Dan Meyer's Model

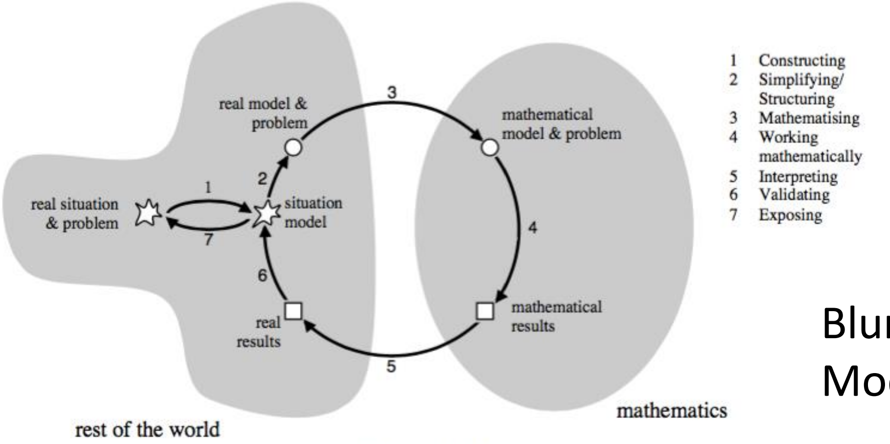


Figure 1 – Modelling cycle

Blum & Borromeo Ferri's Model (2009)

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Theoretical Background:

- Through mathematical modeling, teachers should:
  - Encourage students to exercise their mathematical curiosity and pose problems for real situations (Larson, et. al, 2013)
  - Use mathematical modeling as introductory tasks for new content to make connections with previously learned content (Galbraith, Stillman, & Brown, 2013)
  - Understand that “content knowledge in mathematics does have a significant influence on students’ performances in posing new mathematical problems” (Van Harpen & Presmeg, 2013)

Theoretical Background:

- Precautions that teachers should take when implementing modeling tasks are:
  - To be aware of the natural tendency to want to be too helpful during the modeling process. With some vagueness in the tasks, students will feel the “cognitive need” to explore the details the scenario and research important facts on their own (Chen, 2012)
  - To seek to find artful ways to help students find cognitive independence throughout the process (Stillman, 2012)
  - To be patient, since the process of connecting mathematical knowledge and the “activation” of real world knowledge at the same time is a complex mental process (Gravemeijer, 1997)

Theoretical Background:

- Teachers should consider beginning the study of new mathematical topics with modeling tasks because:
  - These tasks can assess students' prior knowledge, while engaging students with the mathematics (Mousoulides, Pittalis, Christou, & Sriraman, 2013)
  - It forces students to develop questions, then go back and refine their questions, both realistically and mathematically – an important step that is often not required in tradition word problems (Stillman, 2012).

Effects of Research on My Teaching Practice:

- Mathematical modeling tasks were used as preassessments to gain understanding of students' content knowledge through the questions they posed and how they answered them.
- My realization was that the tasks were helping me learn more about my students than just their content knowledge.
- Changes in curriculum.

Effects of Research on My Teaching Practice:

Chapter 1: Right Triangle Trigonometry

Prerequisite Quiz:

- 1.1 Angles
- 1.2 Trigonometric Functions of an Acute Angle (sine, cosine, tangent)

	Day 0	Modeling Pre-Activity	1.3	1.2	1.4	~~1.5~~	REVIEW	TEST
Lesson	The answer is $____. What is the question?	Applying problem solving and possibly triangle relationships to find height.	Give Prerequisite Quiz 1 1.3 Applications & Solving Right Triangles	1.2 Trigonometric Functions of an Acute Angle	1.4 Trigonometric Functions of any Angle	~~1.5 Rotations and Reflections of Angles~~	Review	Sections 1.2 - 1.4 Prereq Retake over missed sections
Activity	This activity is to get students to refine their questions and collaborate.	Find the heights of 2 objects outside that are taller than you. Use 3 different methods to find the heights.	Use students' modeling problems from previous day to intro the lesson.	FOCUS: Work on reciprocal function	INTRO: Use special right triangles 45-45-90 and 30-60-90 on coordinate plane	~~Guided Notes~~		

Effects of Research on My Teaching Practice:

Chapter 2: General Triangles

Prerequisite Quiz:

- none

	Modeling Pre-Activity	2.1	2.2	2.4	REVIEW	TEST
Lesson	Thinking about trig in triangles that aren't right triangles	2.1 The Law of Sines	2.2 The Law of Cosines	2.4 The Area of a Triangle	Review Sheet	Sections 2.1, 2.2, 2.4
Activity	CA Triangles - Intro to Using Trig Laws	NCTM Illuminations - Discover Law of Sines Wrap-up Notes for Discovery Activity	NCTM Illuminations - Discover Law of Cosines		Review Matching Puzzle Puzzle Answers	

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Effects of Research on My Teaching Practice:

Chapter 4: Radian Measure

Prerequisite Quiz:

- none

	4.1	4.2	4.3	Modeling Activity	4.4	REVIEW	TEST	Pumpkin Math
Lesson	4.1 Radians and Degrees	4.2 Arc Length	4.3 Area of a Sector Day 1: Area of Sector Day 2: Area by a Chord	Modeling Angular Motion	4.4 Circular Motion: Linear and Angular Speed	Extra practice with radians and trig measures	Sections 4.1-4.4	Data collection with pumpkins - relate to arc length
Activity	Discovering Radians Activity				Hidden Figures Clip: Trajectory			Notice/Wonder Google doc data collection

Effects of Research on My Teaching Practice:

Chapter 6: Additional Topics

Prerequisite Quiz:
- none

	6.4	Polar Graphs	Polar Graphs Group Quiz
Lesson	6.4 Polar Coordinates	TI-Nspire Polar Graph Exploration	
Activity	Real Life Example from Wright Air Force Museum Polar/Cartesian Grids	Polar Graph Exploration	

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An Activity:

- A case from the second month of school….
  - We were reviewing quadratics in Advanced Algebra 2 and Precalculus.
  - It was a Pep Rally day, so classes were shortened.
  - So we went to the football field…

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An Activity:

- Students were instructed to get into groups of 3, choosing a kicker, a photographer, and a timer.
- Each student was told to write 3 mathematical questions that you have about this experiment ("Push yourself to ask interesting questions!")
- Write 2 things you would like to try to determine using the TI-Nspire and the soccer ball path picture.
- Lastly, what did you observe?

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Some Results:

- Common Mathematical Questions from Precalculus:
  - What is the equation of the path of the ball?
  - What is the axis of symmetry?
  - What was the velocity of the ball?
  - How many yards per second did the ball travel? (The teacher pushed for clarification on this question. Did the student mean the number of yards per second through the air? Or did the student mean the linear distance traveled in that time period? The student was also asked to clarify if this was an average rate or a calculated rate for each interval in the photoburst.)

- Common Mathematical Questions from Advanced Algebra 2:
  - What is the maximum height of the ball?
  - How far did the ball travel? (Again, the teacher pushed for refinement and clarification of this question as to whether the student meant the distance through the air or the linear distance traveled. Several students chose to add the word “lateral” to their questions.)
  - What is the equation of the path of the ball?
  - What was the speed of the ball? (When pushed to refine this question, the group decided that they meant the average speed of the ball as it traveled through the air, which they calculated by using the TI-Nspire to calculate the distance between each point where the ball was in the picture. They added all of these distances together to approximate the distance of the curve. They then divided that total distance of the approximated curve by the time.)
  - Does the path of the ball have a line of symmetry?
  - How fast does the iPad take pictures?

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Some Results:

- Common things that students wanted to find on the TI-Nspires from Precalculus:
  - What is the equation of the path of the ball?
  - What is the maximum height of the ball?
  - How far did the ball travel? (The teacher pushed for refinement of this question. Did the student mean how far did the ball traveled along the ground or through the air?)
- Common things that students wanted to find on the TI-Nspires from Advanced Algebra 2:
  - What is the equation of the path of the ball?

Some Results:

- Unexpected (by the teacher) questions from Precalculus:
- Can we find the distance of a curve?
- Does it take longer for the ball to go up or down?
- Is the slope going up equal to the slope going down? (The teacher then asked the group of students who posed this question to refine their original question as to what they meant by “slope.” The students then began a conversation about the symmetry of the arc of the ball’s path, and explained that they meant that if they found the slope between points, would it be the same for the corresponding points on the path down.)
- Can you calculate the height of the following bounces from the original kick?
- What was the angle of the kick? (The student was asked to refine this question. At what point did they want to know the angle of the kick? Was the angle in relation to the ground? What was the vertex of the angle?
- What was the hangtime/yard ratio? (This is the question that a group developed when they overheard the teacher asking others to refine what they meant by the phrase yards/second.)
- Are the different forms of kicks related to the different forms of equations?
- How did the amount of air in the ball affect how high and how far the ball traveled?

- Are the different forms of kicks related to the different forms of equations?
- How did the amount of air in the ball affect how high and how far the ball traveled?

- [In the equation of the parabola, calculated by the TI-Nspire] What variable does time represent in the equation of the line or the kick? (The student, later, upon discussion and reflection with her group concluded that time was not an element of our graphical display, which was contrary to what she often observed in graphs for science class.)
- Examine the motion diagram aspect of the kick and how velocity and acceleration affect this. (This student was making attempts to try to bring his Physics knowledge together with his math knowledge. He was asked to discuss what he meant by this question with his group to clarify what he meant. After some time, he decided that he needed a little more Physics background knowledge to answer all of his questions.)
- At what point did the ball start travelling in a negative distance? (The student was asked to refine what she meant by “negative distance” and stated that she meant the ball’s journey back down to the ground. She used the term “negative” because she had related the idea to negative slope.)
- (Some of these questions were discussed, but not answered fully on paper. The teacher and students decided to revisit the questions later during the course as they gained more content knowledge in math and science class.)

Some Results:

- Unexpected (by the teacher) questions from Advanced Algebra 2:
- How fast does the ball reach its maximum height? How fast does the ball drop from its maximum height? (This group quickly realized that this problem was easily calculated based on the symmetry of the path of the ball.)
- At what time was the ball at its highest point?
- How hard are you kicking the ball? (This question was tabled for later in the year since we had not talked about force yet.)

Conclusions:

- When should mathematical modeling tasks be implemented in the curriculum to foster problem posing in students?
  - As soon as possible! I was much more successful at creating a modeling environment because this was established as part of our culture from the first week.
- What steps do I need to as a teacher to create an environment that supports mathematical modeling and problem posing?
  - Encourage students to truly be creative and problem pose by having them do modeling tasks as preassessments instead of activities at the middle or end of the chapter. Students will give insight into their creativity and content knowledge through modeling.

Implications for Teaching:

Traditional Structure of Teaching Math:

Teach Content → Practice Content → Teach Applications

Modeling Structure of Teaching Math:

Mathematical Modeling → Teach Needed Content → Revisit Modeling Task

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Implications for Teaching:

- Mathematical modeling tasks are valuable preassessments because they give us insight into the students':
  - Previous content knowledge
  - Creativity
  - Comfort with posing problems
  - Own personal experiences which affect their interpretations of problems.
  - Technology skills

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Implications for Teaching:

- Mathematical modeling tasks provide scaffolding for the entire course.
  - Keep the modeling projects on the walls and around the room. Refer back to them often and help students to see connections between what they see as disjoint mathematical topics.

Implications for Teaching:

- **Challenges:**
  - Class time needed had to be carefully planned
  - Teacher preparation time needed was more than teaching regular content
    - Must be proficient in possible math that arises
    - Must be proficient in many other content areas in order to guide students appropriately
    - Must be willing to admit the teacher doesn't know everything and research must be completed together as a learning community

Thank you for coming!

Questions?

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