Mathematical Modeling: Using Authentic Problem-Solving Tasks to Interpret the World Around You

What is Authentic Problem Solving?

- Learning is an active process
- Students construct knowledge through inquiry learning, investigation, sense-making, and critical thinking
- Students solve problems with real-world connections

Benefits of Authentic Problem Solving

- Making connections
- Critical Thinking/Decision-Making
- Developing mathematical autonomy
- Encouraging collaboration
- Depth of understanding

Stages of Mathematical Modeling

- Understand and identify the problem
- Devise a plan & collect data
- Solve the problem
- Check reasonableness of solution and revise the model if necessary
- Communicate the solution

Questions to Ask During Authentic Problem Solving

- What are you looking to find?
- What are the unknowns?
- What information are you given? It is necessary or unnecessary?
- Have you seen this before (or have you seen something like it?)
- Can you use a strategy you've used before?
- Does your solution answer the question(s)?
- Does your solution make sense?

Polya, *How to Solve It*, 1957

Teachers' Notes: The Shoe Drive

This task provides students with a real-life application of measures of central tendency. In the real-life context, students will decide the best way to determine the "average" size (or range of sizes) of shoes (or feet) of middle school students. The task asks students to apply their knowledge of mean, median, and mode, and involves demonstrating flexibility with numbers, making decisions, and justifying their reasoning.

Ask students how they might determine the "average" size of middle-schoolers' shoes (I have put the word "average" in quotation marks because the most common measure of average is mean, but we may also use median and/or mode). Let students work in groups to find this information. Provide students with tape measures or yard/meter sticks. Do not direct students to use a particular method: let them "play" with the numbers and decide which is the best representation of average school side. Ask them to explain their reasoning.

When groups have determined their estimate of the average shoe size, ask them to compare their answers with other groups for reasonableness (they may even search online for similar information). Give students the opportunity to revise their work if they decide their answers might be incorrect.

To implement and communicate their solutions (assuming you are not actually holding a shoe drive), ask students to "bid" for the job, communicating with the principal (or the teachers, the school board...) their work and justifying their reasoning. They may write a letter, make a poster, make a commercial...

The Shoe Drive: Different Ways to Average

Our school is participating in a shoe drive. We will be collecting gently used shoes for those in need. When the shoes are collected, our class will be asked to sort out the shoes that might fit middle school students.

Working with your group, decide how you would identify the shoes that might fit middle school students. Think about our work on measures of central tendency (mean, median, mode) to report the "average" size of the shoes (or the feet) of middle school students.

What size or length shoes would you sort out to put in the middle school group? Show your work and explain your reasoning:

If you used measures of central tendency (mean, median, and mode), which one did you choose? Why? If you did not use measures of central tendency, why not? What did you do? Why?

Shaquille O'Neal was a very popular basketball player. He had size 22 feet! His feet measure 14.6 inches long (or 35.5 cm). If Shaq were to join our class, would your method still work? Why or why not?

If you have to adjust your model since Shaq joined the class, which method would make more sense? Explain your reasoning. Run your numbers for mean, median, and mode (including Shaq's numbers). Explain what happens to the value of each when you include those big feet!

Teachers' Notes: Family Fun Night

This activity provides an engaging, multi-layered modeling problem where students will make decisions, work collaboratively to consensus, check for reasonableness of their answers, and implement their solutions.

In this activity, students are told that they are helping to make important decisions about Family Fun Night (FFN) – an evening where school families come to learn, dance, play, and interact with each other and with school personnel. The class has been asked to make a decision regarding which treats should be sold at FFN. Students are asked to find the treats that would be the best-sellers at the event, given a list of possibilities.

Part I

This task requires students to collect data, and to make decisions regarding how to assess that data. Some questions you might ask them to think about as they make their decisions and set up their models:

Does it matter whether a treat is chosen first, second, or third? If it matters, how will you measure that?
What else might you want to take into consideration other than the rankings of your classmates? (cost, similar items…)

When students have recorded their preferences and collected whole-class data, ask them to work in their groups to make a decision regarding which two treats should be sold at FFN, and explain why they made the decision. Talk to students about "consensus" and the need for each group to reach a consensus.

Some groups will merely decide by the number of first-choice votes, others might find a total for each item of 1st, 2nd, and 3rd choices in order to choose, while others might create some sort of weighted average model where 1st choice is worth more than 2nd, etc. Encourage students to make decisions that they can justify (without directing them in any particular direction).

When they are finished, ask each group to share out their choices. After all groups have explained and defended their choices, give groups the opportunity to reconsider and revise their models if they desire.

Part 2

When students have made their choices of the two treats they are recommending, ask them to find **all possible combinations** of each of the treats that they could buy

for $100. Encourage them to find a system in order to be able to find all combinations without missing anyone.

Finally, students will look at all of the possible combinations, and decide which combination makes the most sense, based on the data (looking back at the survey). Students will justify their reasoning.

Students may implement and communicate their solutions (assuming you are not have a Family Fun Day!) by communicating their proposals to the class, the principal, the PTA…. They may communicate through an infographic, a poster, a letter, a commercial…

Family Fun Night

Our class is going to sell 2 different treats at Family Fun Night. We have $100 to spend on the treats and have to decide which treats to sell, and how many of each we should buy.

Part I:

The choices of treats are listed below. Please number your first 3 choices of treats (1 is your first choice, 2 is your second choice, 3 is your third choice).

_____Gourmet Popcorn
_____Caramel Corn
_____White Cheddar Popcorn
_____Kettle Corn
_____Karm'l Dapples (candy apples)
_____Ice Tickles
_____Cotton Candy

Record the class totals below:

Treat Item	# of students First Choice	# of students Second Choice	# of students Third Choice
Gourmet Popcorn			
Caramel Corn			
White Cheddar Popcorn			
Kettle Corn			
Karm'l Dapples (candy apples)			
Ice Tickles			
Cotton Candy			

Working with your group, decide on a method to figure out which two items our class should sell at Family Fun Night.

Which two items should we sell?

Explain how you made your decisions:

Part 2:

The prices for each item are listed below (all prices are for 24 items – 24 bags of popcorn, 24 caramel apples, or 24 cotton candies).

Treat Item	Price for 24 (tax is included)
Gourmet Popcorn	$8
Caramel Corn	$10
White Cheddar Popcorn	$9
Kettle Corn	$12
Karm'l Dapples (candy apples)	$22
Ice Tickles	$6
Cotton Candy	$15

List your two choices below:

_____   _____

Find all possible combinations of the two items that we could buy with our $100. Show your work and explain your reasoning.

Which combination of the two treats do you think we should buy? Explain your reasoning.