

Not Just Answering Someone Else's Questions: Making Math Class More Like Mathematics

Here's an un-answered question I
asked today. "Are shapes everything?"
Or, "is everything made up of shapes?"

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2018

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Should students spend valuable class time learning how to ask mathematical questions?

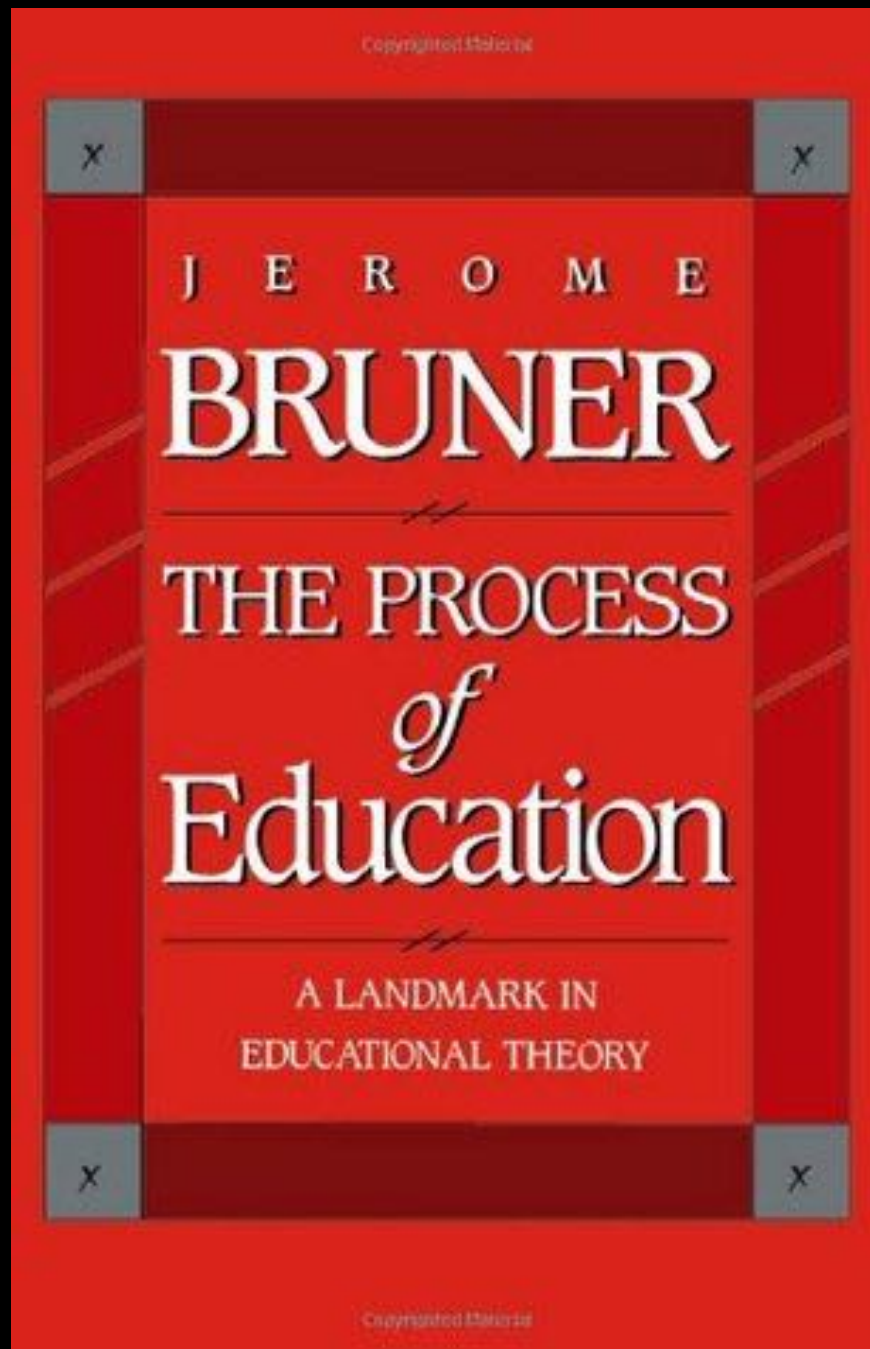


If so, why?
If not, why not?

Three reasons I'd offer for why we should teach mathematical inquiry:

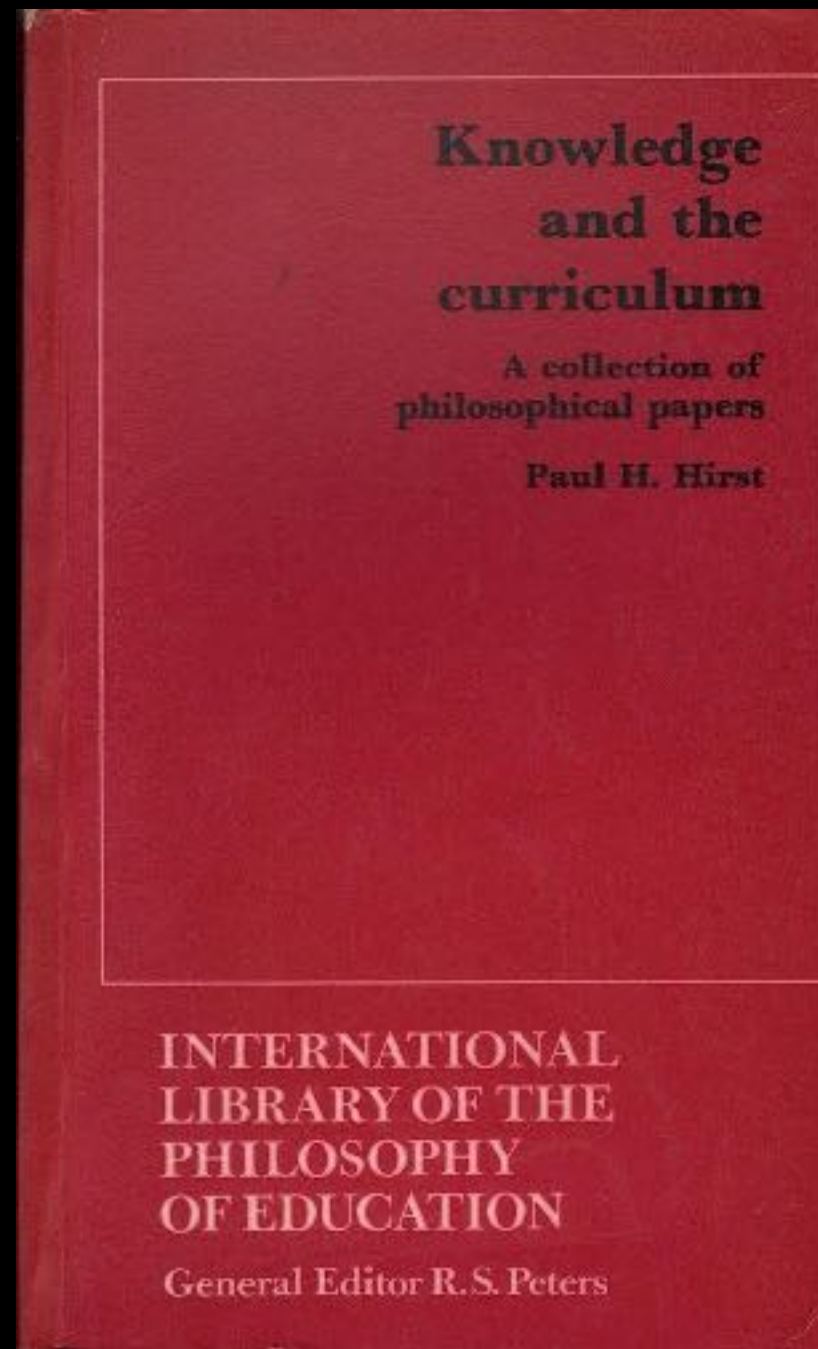
- It's the “intellectually honest” thing to do.
- It's a darn good way to teach mathematical content and practices.
- It's an equity and access issue.

Bruner



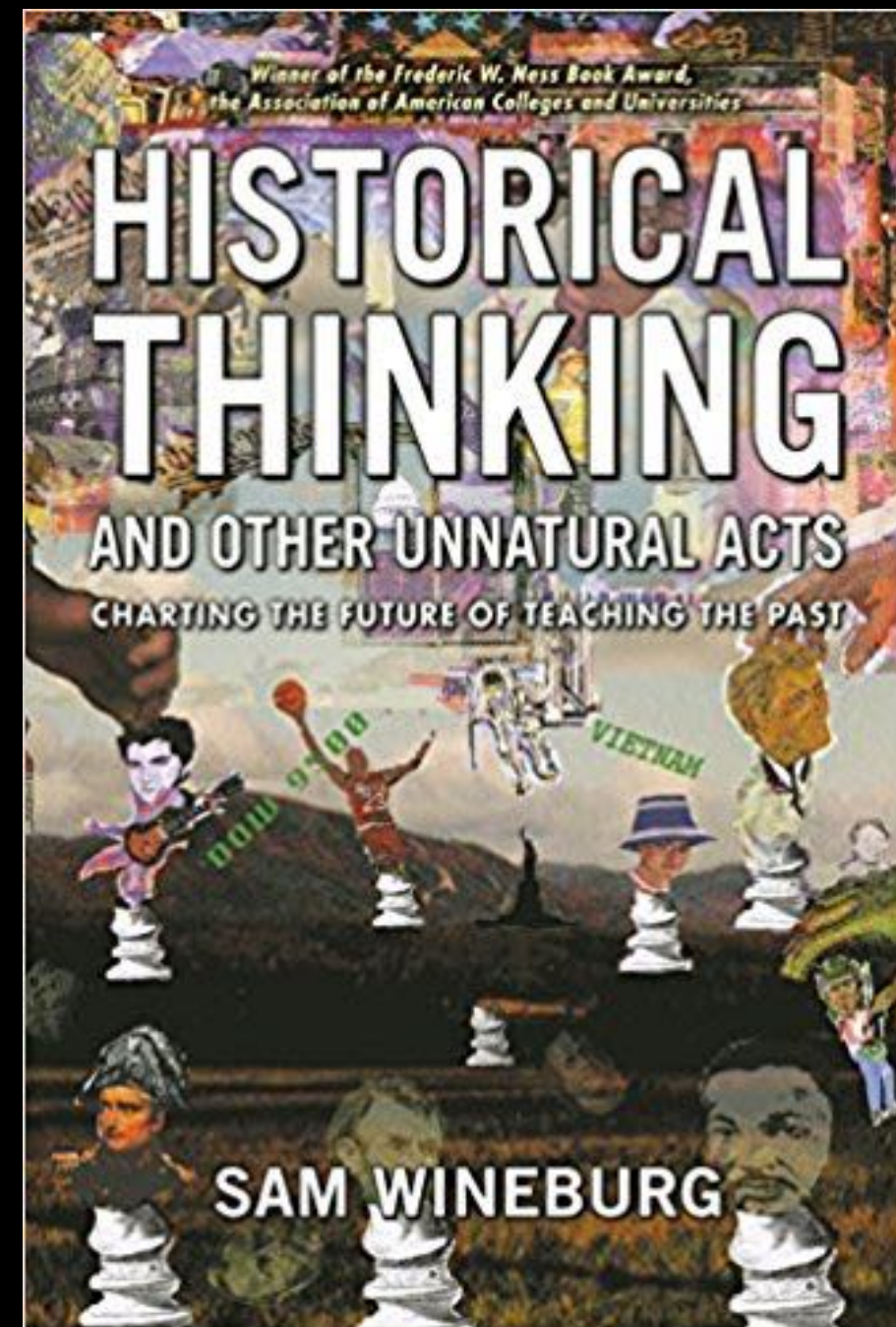
1960

Hirst



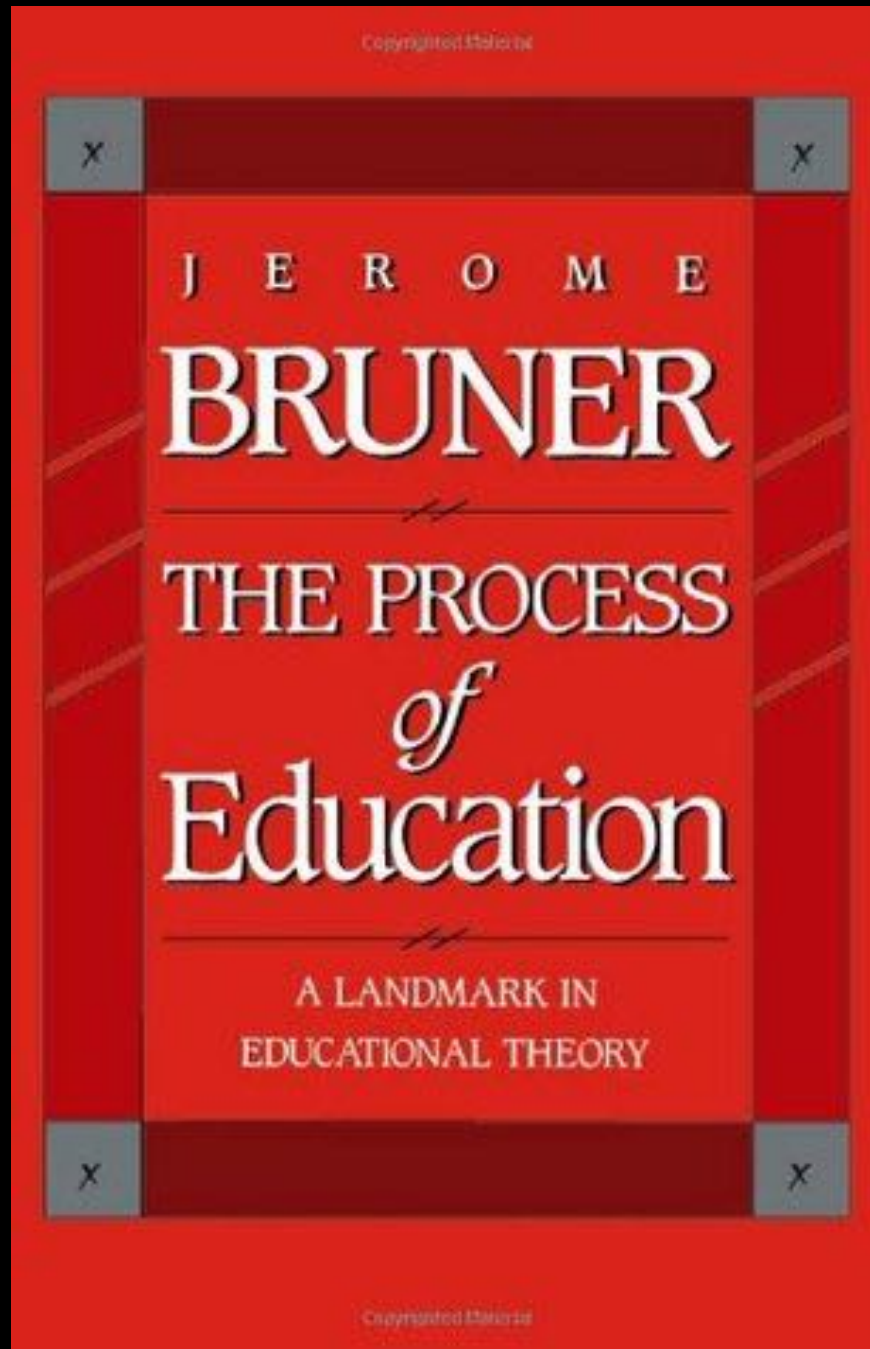
1965-1974

Wineburg



2001

Bruner

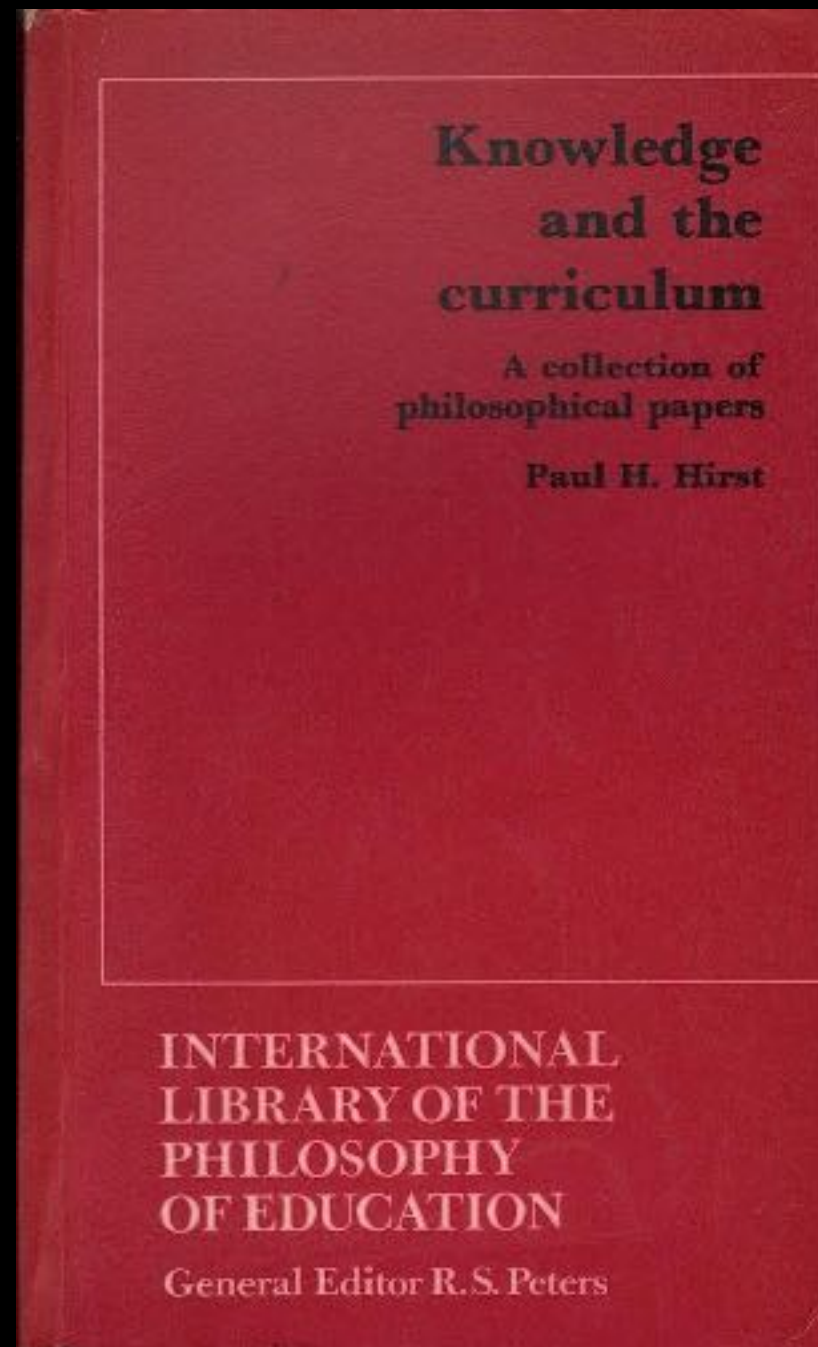


“We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development.”

1960

Hirst

- Concepts
- Logical structure
- Truth criteria



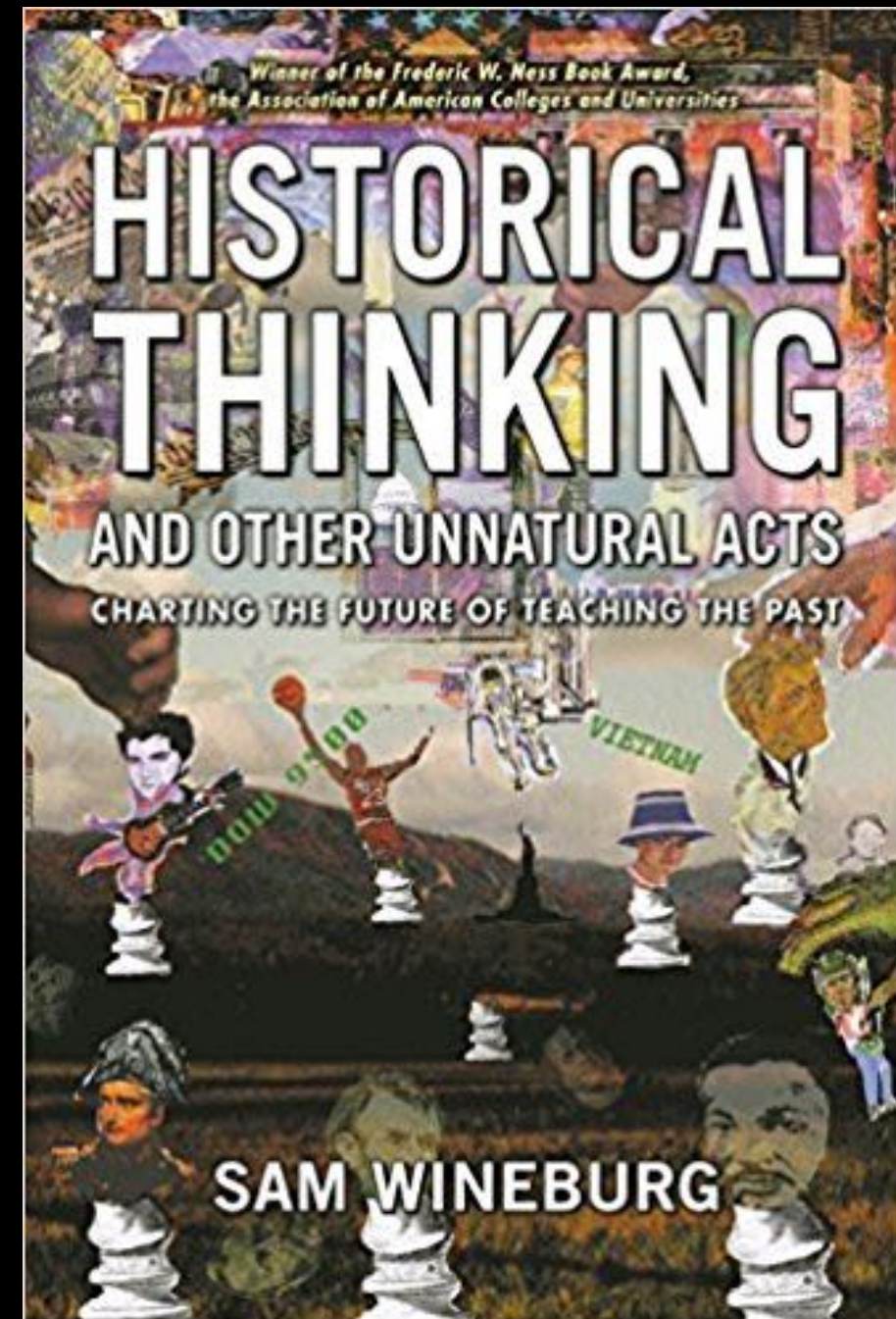
“Understanding a form of knowledge is far more like coming to know a country than climbing a ladder.”

1965-1974

Wineburg

“And where should our standard come from? To me, there is only one defensible answer.

“We must look to the discipline.”



2001

experts and novices?







Splash

@SplashSpeaks

I've got an idea! Let's put just the greens in here.
We can sort them.

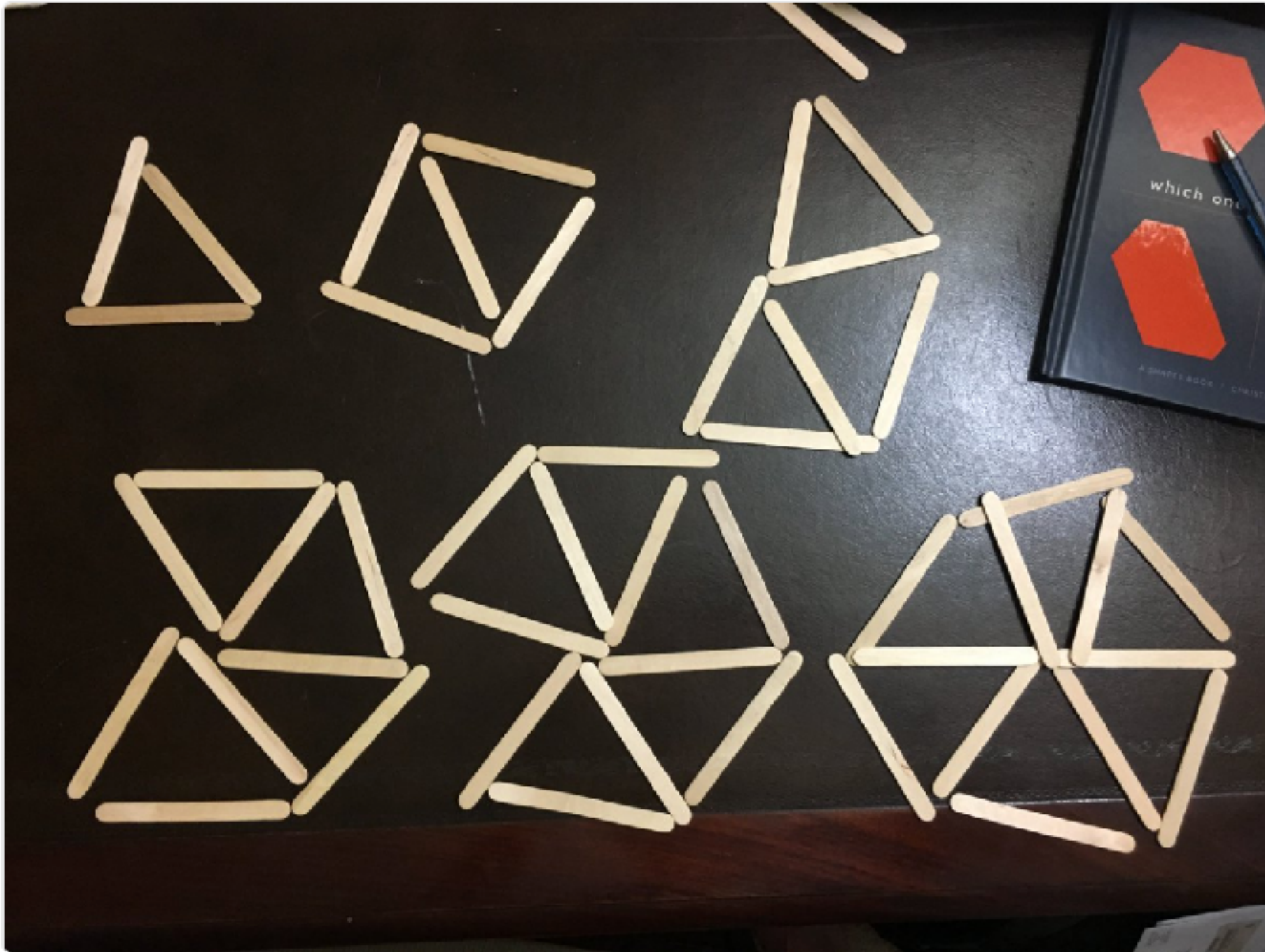




Michael Fenton

@mjfenton

I bought some tiny popsicle sticks. Here's what my son Tyler made. So fun. [#tmwyk](#)





John Stevens

@Jstevens009

L: "daddy, what's the first number?"

Please tell me I'm not the only one who's been stumped by a 5 year-old.

#tmwyk

Daphne, age 8, a few weeks ago:

“What would be halfway between zero and infinity?”

Daphne, age 8, a few weeks ago:

“What would be halfway between zero and infinity?”

...

“Oh! I think half of infinity must be infinity. Because infinity goes on forever, so half of infinity would also go on forever.”



Kassia Wedekind

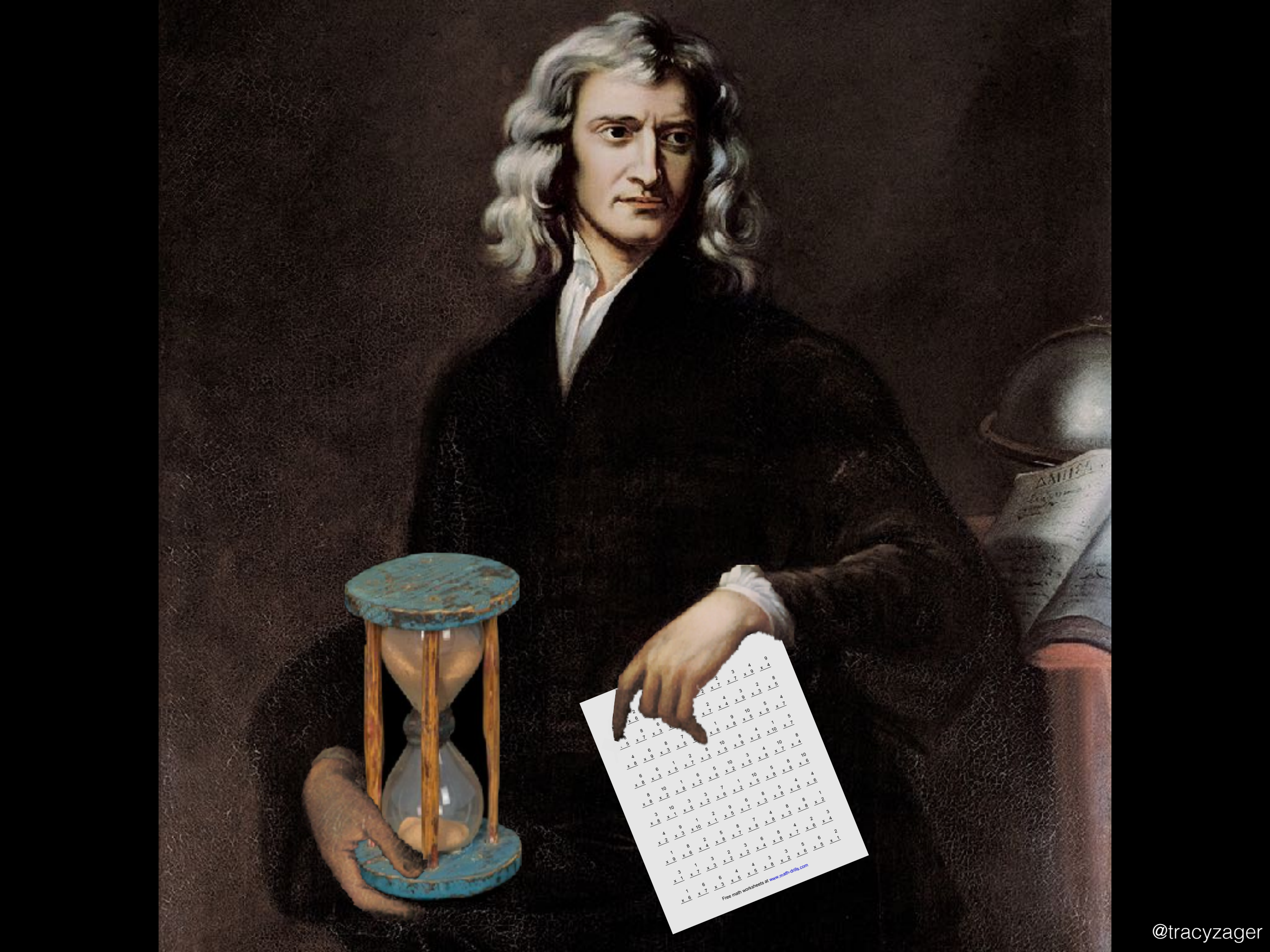
@kassiaowedekind

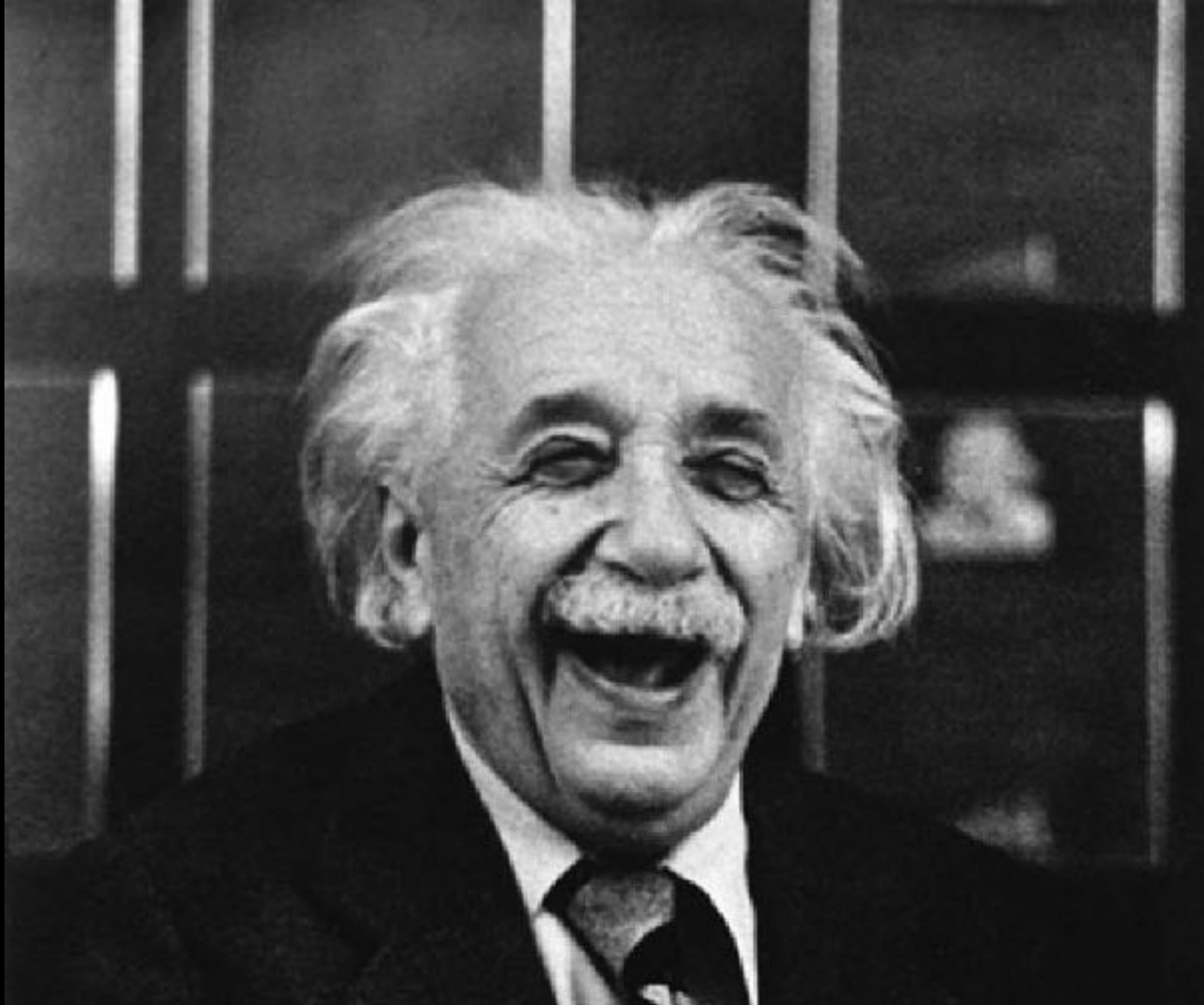
3 yr old to her baby brother:
"Frankie, do you know what a double is? That's
when mama gives me 2 mango pieces but you
only get 1." #tmwyk

“...any subject can be taught effectively
in some intellectually honest form
to any child at any stage of development.”

math class  mathematics

“The difference is in degree, not in kind.”





Words into Math

ADDITION

Add
Sum
Increased By
More
Plus
Combined

SUBTRACTION

Subtract
Difference
Decreased By
Minus
Take away

MULTIPLICATION

Product
Times
OF
Multiply
Three

DIVISION

Quotient
Shared
Ratio
Per
Divided By

EQUALS IS

Under The Sea

Count, color, and graph.

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

Fraction

What fraction does the color in the bubble add?

Copyright © free

Color By Code

Name: _____

Directions: Color By Code

Code

6 = Black

7 = Red

8 = Blue

9 = Light Blue

10 = Pink

11 = Yellow

12 = Gray

Bee Mine

2-DIGIT MULTIPLICATION

MULTIPLY. REGROUP IF NEEDED.

97 X 30	58 X 46	85 X 34	72 X 47	97 X 18
6 X 1	92 X 38	44 X 12	83 X 49	
47 X 29	65 X 23	58 X 2		

APPLE ATTRIBUTES

Name: _____

Write how many sides each apple shape has.

Sides

m&m's

Graphing Activity Booklet

Chris spent 2 hours working on his spelling and math homework. If he spent 1 hour on his spelling homework, how much time did he spend on his math homework?

Name: _____

Division

Directions: Solve each problem. Color the pictures.

42 ÷ 7 =

12 ÷ 2 =

18 ÷ 3 =

48 ÷ 8 =

40 ÷ 8 =

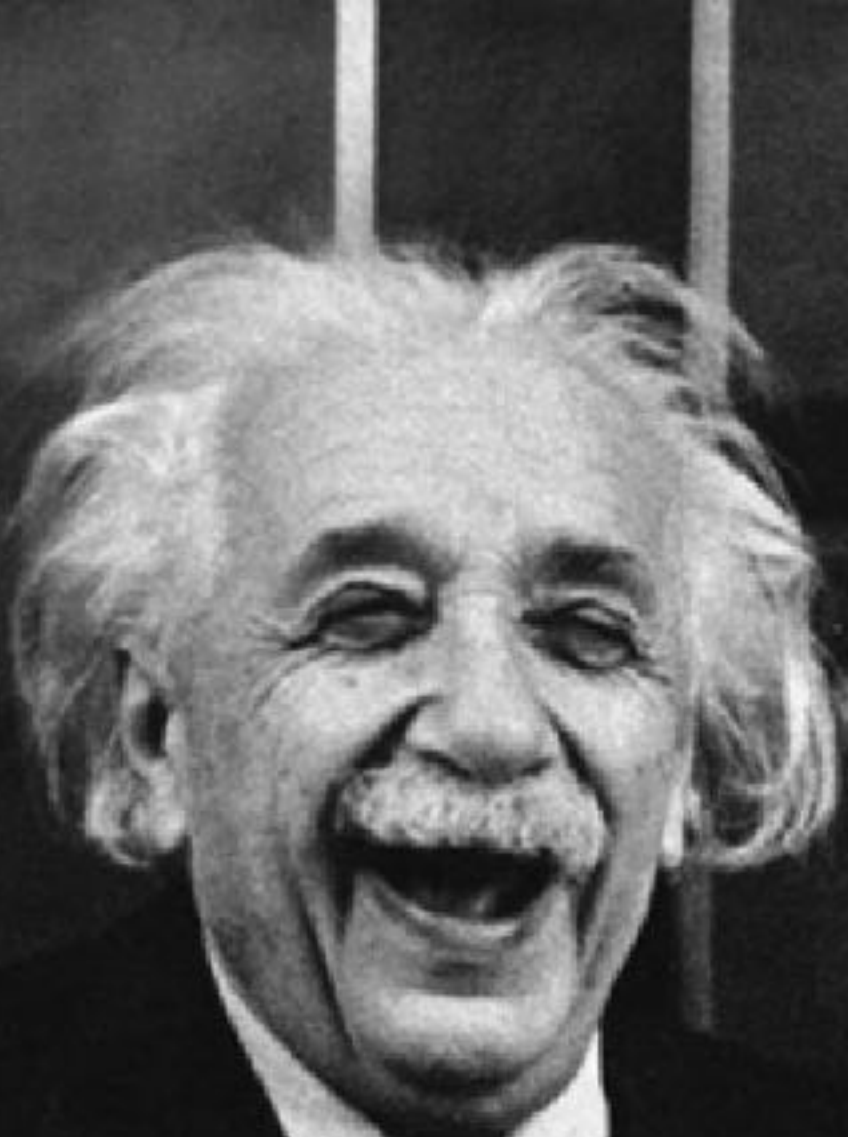
36 ÷ 6 =

50 ÷ 5 =

18 ÷ 2 =

72 ÷ 9 =

49 ÷ 7 =



math class



mathematics

Mathematicians.....

- Wonder
- Explore
- Think Inquisitively
- Build/Create
- They Code
- Have Persistence
- Work Collaboratively
- Problem Solve/Discuss
- Struggle
- Make connections

Solve
 play
 create
 discover
 fix
 read
 compose
 figure out

decide
 learn
 arrange
 think
 wonder
 develop
 realize
 see

practice
 show
 imagine
 build
 add
 cut in half
 express
 ask questions

try
 replace
 explore
 do
 pay attention
 spell
 finish
 fill

What Do Mathematicians Do?

Wonder
 imagine possibilities
 Examine different perspectives
 Question
 Look for patterns
 Explore
 Have fun discovering
 Create
 Collaborate
 Try new things
 Struggle

wonder
 interview
 challenge someone
 postulate
 question
 emotive
 imagine
 communicate
 Compare
 trace/write
 make connections

fragmentary
 tentative
 maybe
 a dark mansion
 artist
 adventurer



Asking questions
 Figured it out
 discovered
 how do you think about...
 what do you see?
 pictures of what math looks like
 tossed riddles
 head was swimming
 day dreaming
 notice
 wonder
 realized

amazed
 stumbling
 bump bumping
 being different
 maybe
 being creative
 challenged me



$\lambda = 5 \cdot 10^{-9}$ $\Delta t = 10^{-9}$ $x^2 + y^2 = 0$ $m_1/m_2 = G_1/G_2$ $(x - \frac{p}{2})^2 + y^2 = (x - \frac{p}{2})^2$ $\Delta t = 10^{-9}$ $\Delta E = \frac{h}{\Delta t}$

MATHEMATICAL PROBLEMS

$\sqrt{(x - \frac{p}{2})^2 + y^2} = \sqrt{(x - \frac{p}{2})^2 + y^2}$ $v_e = \sqrt{\frac{2T}{m}}$ $v_e = 1.7 \cdot 10^6 \text{ m/s}$ $\Delta T = 125$

$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$ $\int \frac{1}{x^3} dx = -\frac{1}{2x^2} + C$ $\int \frac{1}{x^4} dx = -\frac{1}{3x^3} + C$ $\int \frac{1}{x^5} dx = -\frac{1}{4x^4} + C$ $\int \frac{1}{x^6} dx = -\frac{1}{5x^5} + C$ $\int \frac{1}{x^7} dx = -\frac{1}{6x^6} + C$ $\int \frac{1}{x^8} dx = -\frac{1}{7x^7} + C$ $\int \frac{1}{x^9} dx = -\frac{1}{8x^8} + C$ $\int \frac{1}{x^{10}} dx = -\frac{1}{9x^9} + C$ $\int \frac{1}{x^{11}} dx = -\frac{1}{10x^{10}} + C$ $\int \frac{1}{x^{12}} dx = -\frac{1}{11x^{11}} + C$ $\int \frac{1}{x^{13}} dx = -\frac{1}{12x^{12}} + C$ $\int \frac{1}{x^{14}} dx = -\frac{1}{13x^{13}} + C$ $\int \frac{1}{x^{15}} dx = -\frac{1}{14x^{14}} + C$ $\int \frac{1}{x^{16}} dx = -\frac{1}{15x^{15}} + C$ $\int \frac{1}{x^{17}} dx = -\frac{1}{16x^{16}} + C$ $\int \frac{1}{x^{18}} dx = -\frac{1}{17x^{17}} + C$ $\int \frac{1}{x^{19}} dx = -\frac{1}{18x^{18}} + C$ $\int \frac{1}{x^{20}} dx = -\frac{1}{19x^{19}} + C$

BY DAVID HILBERT

$y = p(x - x_0)^2$ $a + c = b + d$ $\int f(x, y, z) dz$ $\text{Arth } t = \ln\left(\frac{1-t}{1+t}\right) (a-b)$ $\text{ch}^2(a-b)(c-d) = (ac - bd)$ $\text{sh}^2 x = 1, x = \frac{2t}{1-t^2} (-1 < t < 1)$ $\text{th} \frac{x}{2} = t, L = \frac{1}{24}(80k^2 - 7)$

Six Strategies to Include Mathematical Questioning in Mathematics Classes

- 101 questions
- Notice and Wonder
- Problem Posing
- Riffing off Solutions
- Question-Centered Unit
- DREAM: Drop Everything and Math

1. 101questions



101qs.com

101questions



Questions without pressure.
Takes just a few minutes.
Promotes curiosity.

2. Notice and Wonder

Annie Fetter and Max Ray-Riek
The Math Forum

Give students an image or scenario *without a question*.

Ask students what they notice and wonder.

Allow plenty of time before selecting questions to wonder about, mathematically.

Follow these steps for each problem.

- a.** Decide which two numbers need to be multiplied to give the exact answer.
Write the two numbers.
 - b.** Estimate whether the answer will be in the tens, hundreds, thousands, or more.
Write a number model for the estimate. Circle the box to show your estimate.
 - c.** On the grid below, find the exact answer by multiplying the two numbers.
Write the answer.
- 1.** The average person in the United States drinks about 61 cups of soda per month.
About how many cups of soda is that per year?

a. _____ * _____

numbers that give
the exact answer

b. _____

number model for your estimate

c. _____

exact answer

10s	100s	1,000s	10,000s	100,000s	1,000,000s
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1. The average person in the United States drinks about 61 cups of soda per month. About how many cups of soda is that per year?

a. _____ * _____

numbers that give
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number model for your estimate

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exact answer

10s	100s	1,000s	10,000s	100,000s	1,000,000s
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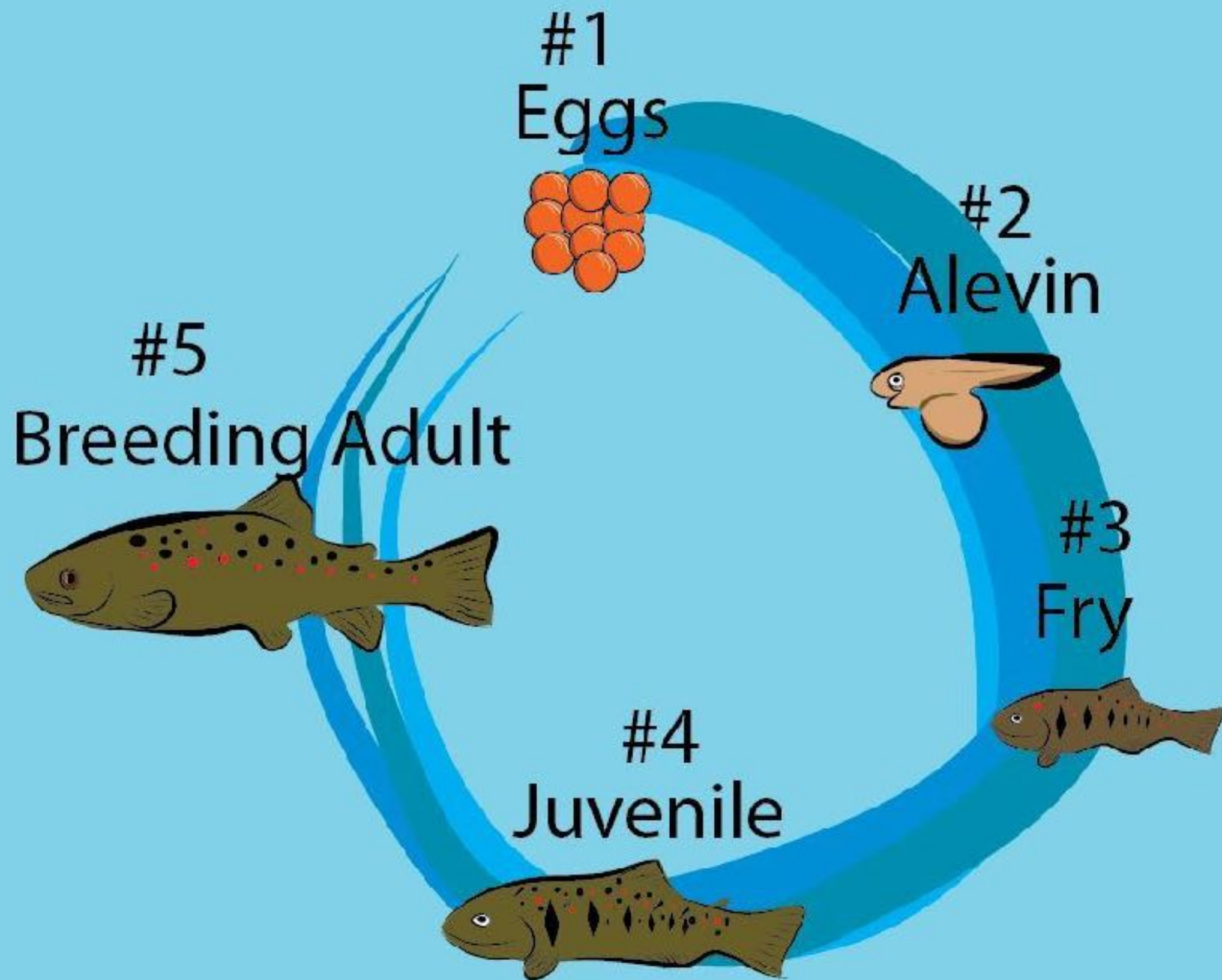
1. The average person in the United States drinks about 61 cups of soda per month.

3. Problem Posing

Expect students to pose problems,
not just solve someone else's.

One variant: give the students data
and ask *them* to generate questions.

Be prepared for some blowback.

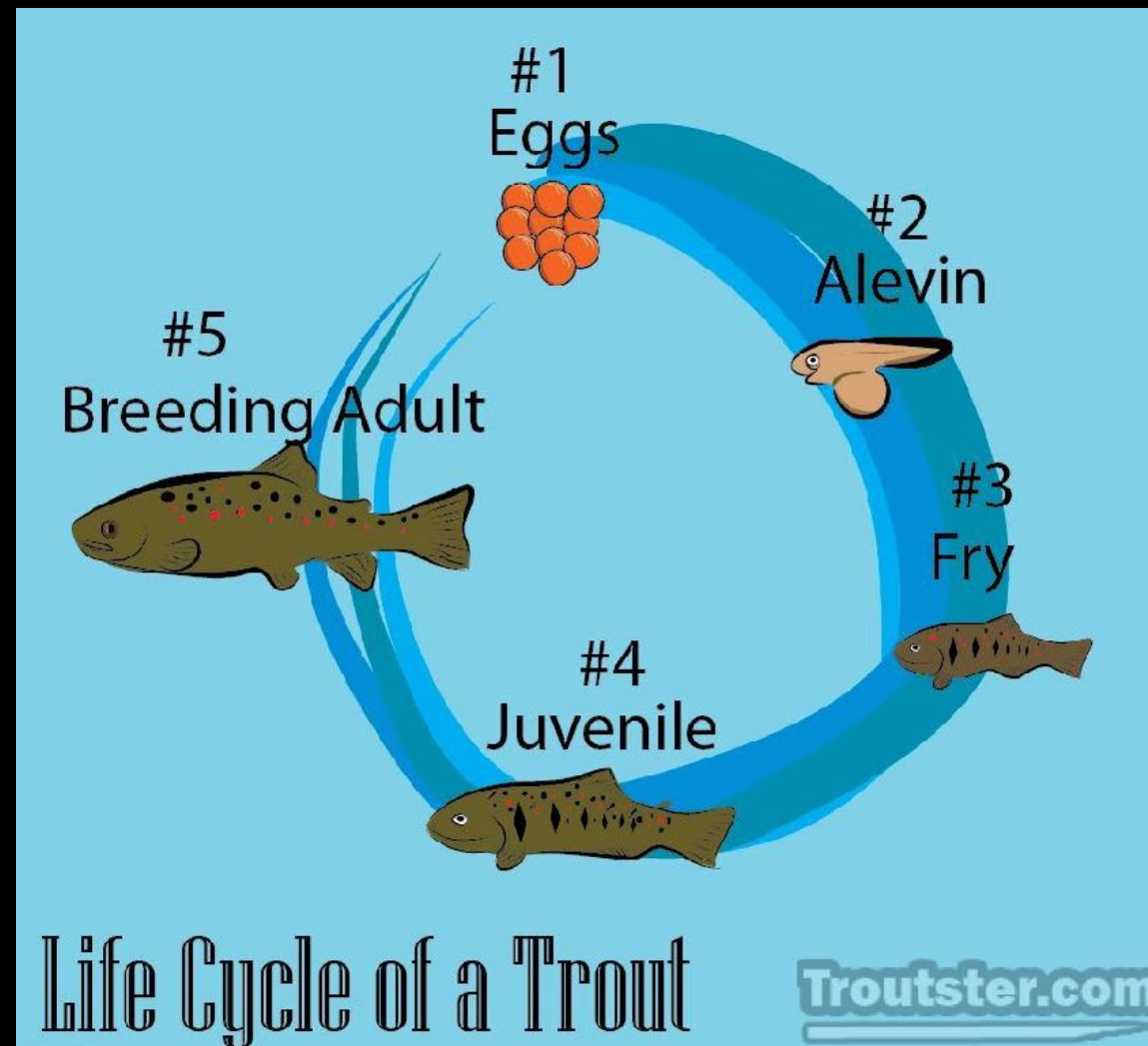


Life Cycle of a Trout

Troutster.com

The fish hatchery gave us 200 trout eggs. Right now we have 75 live alevin, 25 live eggs, and 20 dead eggs in the tank.

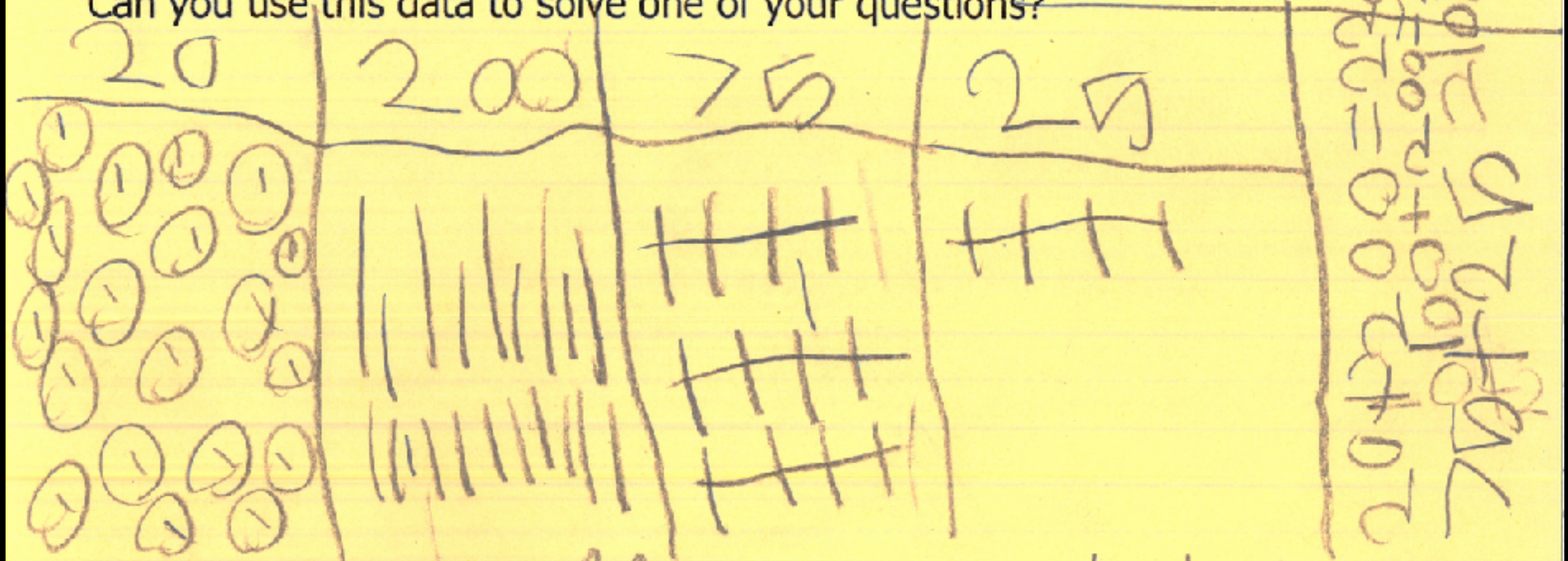
What questions could you ask using this data?



Sheryl Horton's teaching

The fish hatchery gave us 200 trout eggs. Right now we have 75 live Alevin, 25 live eggs, and 20 dead eggs in the tank. What questions could you ask using this data?

Can you use this data to solve one of your questions?



How many things
are in the tank?

300

“That’s what you do in math problems. See these numbers up here? What you do is you take those numbers and you add them together.”

The fish hatchery gave us 200 trout eggs. Right now we have 75 live Alevin, 25 live eggs, and 20 dead eggs in the tank. What questions could you ask using this data?

Can you use this data to solve one of your questions?

20	200	75	25	
				$\begin{array}{r} 200 \\ + 20 \\ + 75 \\ + 25 \\ \hline 320 \end{array}$

How many things
are in the tank?

300

4. Riffing Off Solutions

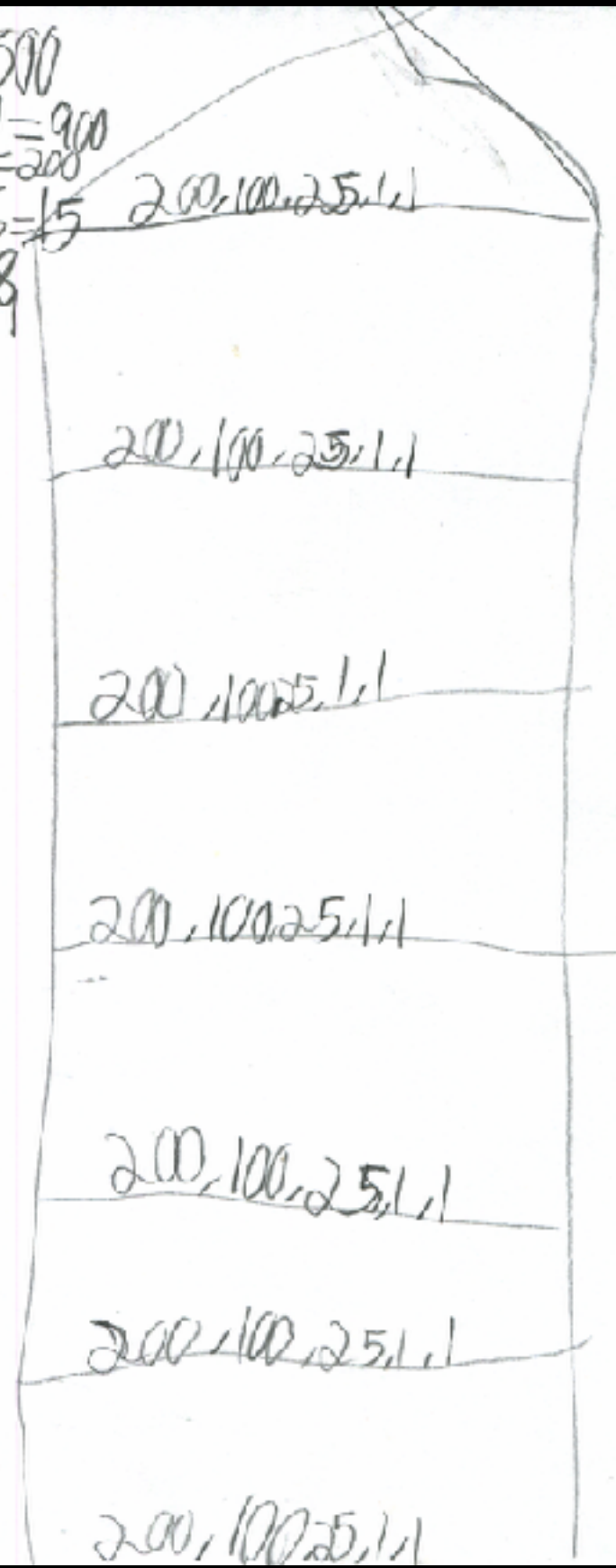
“Mathematical curiosity also includes a desire to explore mathematical ideas through posing mathematically interesting problems after one has ‘finished’ a problem. The solution or aspects of the problem can become springboards for further exploration.”

-Knuth 2002

~~2,300~~
 - 1000 = 1300
 - 500 = 800
 - 500 = 300
 - 250



2,300
 - 1,400 = 900
 - 700 = 200
 - 15 = 15
 - 7 = 8



7 floors
 2,300 people

327 people on each floor
 how many people in the yard?
 There is one person in the yard

2,300

- 1200 = 1200, 100, 50, 25, 5, 2, 1

- 600 = 500

- 300 = 200

- 150 = 50

- 30 = 20

- 12 = 8

- 6 = 2

200, 100, 50, 25, 5, 2, 1

200, 100, 50, 25, 5, 2, 1

200, 100, 50, 25, 5, 2, 1

200, 100, 50, 25, 5, 2, 1

200, 100, 50, 25, 5, 2, 1

2,300 P2

- 900 = 1,000

- 900 = 500

- 400, 50 = 10

- 40 = 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

100, 100, 50, 5

Maya, what are you wondering about now?

“Is there a pattern? I think I’m starting to notice a pattern. If it’s an odd number of floors, then people are outside...I want to know about even and odd. I want to do them all up to ten and then figure out if there’s a pattern.”

Later on...

What questions are you asking now?

“Six floors didn’t work. I wonder if that’s because three floors didn’t work?”

Fencing a Garden

Ms. Lights' fifth-grade class will plant a garden in the school yard. The garden must be a rectangle, and the principal has given them 30 feet of fencing. Each side of the rectangle has to be a whole number.

Use grid paper, color tiles, or drawings to design at least 4 garden plots that would be enclosed by 30 feet of fence. Find the area for each garden plot.

Attach drawings of your rectangular gardens to this sheet. After you have designed at least four, fill out the table and answer the questions.

Dimensions	Perimeter	Area
1. 1×14	30 feet m	$14 m^2$
2. 5×10	30 feet m	$50 m^2$
3. 8×7	30 feet m	$56 m^2$
4. 13×2	30 feet m	$26 m^2$
5. 6×9	30 feet m	$54 m^2$
6. 12×3	30 feet m	$36 m^2$
7. 4×11	30 feet m	$44 m^2$

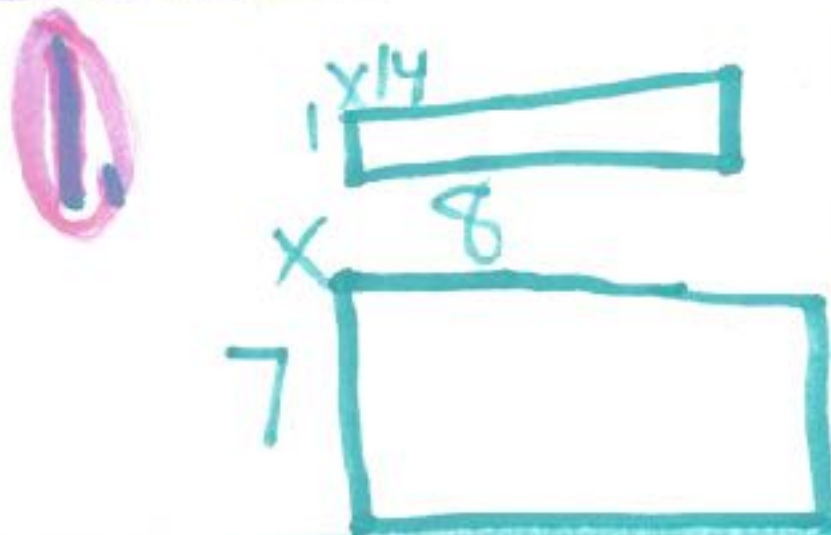
- What are the dimensions of the rectangle with the largest area?
- What are the dimensions of the rectangle with the smallest area?
- What do you notice about the shape of these rectangles?

Problem from Investigations 2nd Edition
Kristin Gray's teaching (@mathminds)

Table 6 Garden Rectangles!

① ^(rectangles) They get wider as the dimensions keep going. (Keep going up.)

② The rectangles are larger when they are skinnier. But when the are ^(rectangles) ~~FAT~~ they are Short



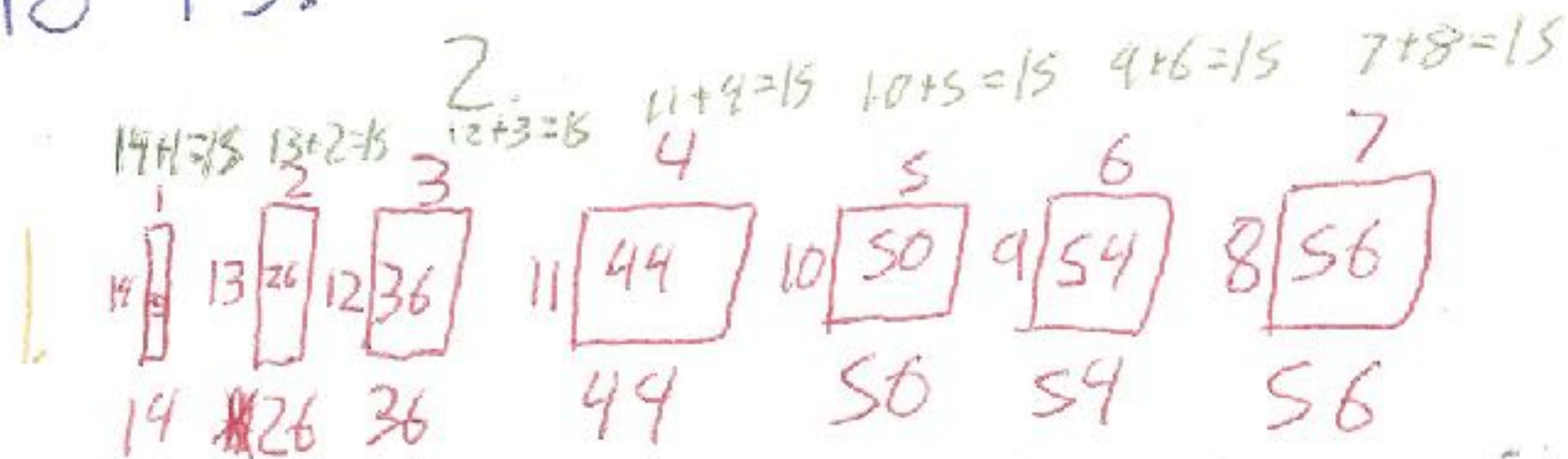
③ If you keep making bigger rectangles it turns into a big square.



Signed, Table

1. The closer the shape gets to being a square, the greater the ^{area} ~~perimeter~~ is.

2. The dimensions of every rectangle that has a perimeter of 30 add up to 15.



To teach students to generalize, look for patterns, and make claims, we need to view solutions as the **beginning** of new, important mathematical thinking, not only the end.

Once we have solutions:

What do you notice?

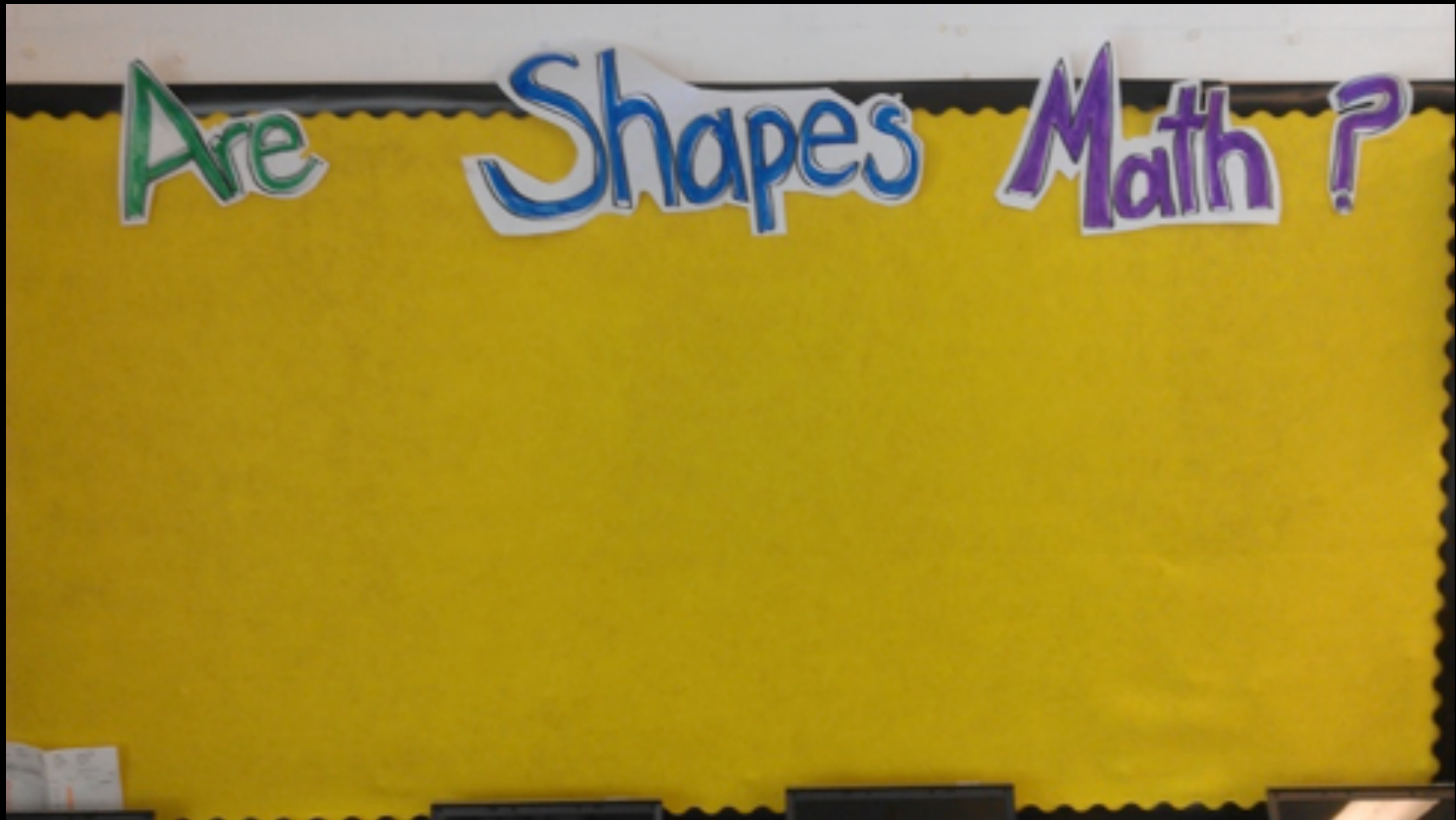
What do you wonder?

What are you asking now?

What new questions can you ask?

What if we...?

5. Question-Centered Unit



Debbie Nichols's teaching

How big can a circle go?

Are shapes life? Nature? Water?

How many shapes are there?

How do shapes get their form?

What was the first shape made?

Are shapes even math?

What do shapes do?

Are shapes fractions?

What are the kinds of shapes?

How many shapes are in a house?

How round can a circle be?

Are shapes fragile?

How long can rectangles be?

How big can a square get?

Are numbers and letters shapes?

Where can you find shapes?

Are there shapes in other shapes?

What can you make with shapes?

How long does it take to learn to make them?

Can shapes be in the sky?

Are shapes time?

Are shapes permanent?

Are shapes the same all around the world?

Can circles change their form?

Can shapes be found in insects? Food? Planets?

Is a heart a shape?

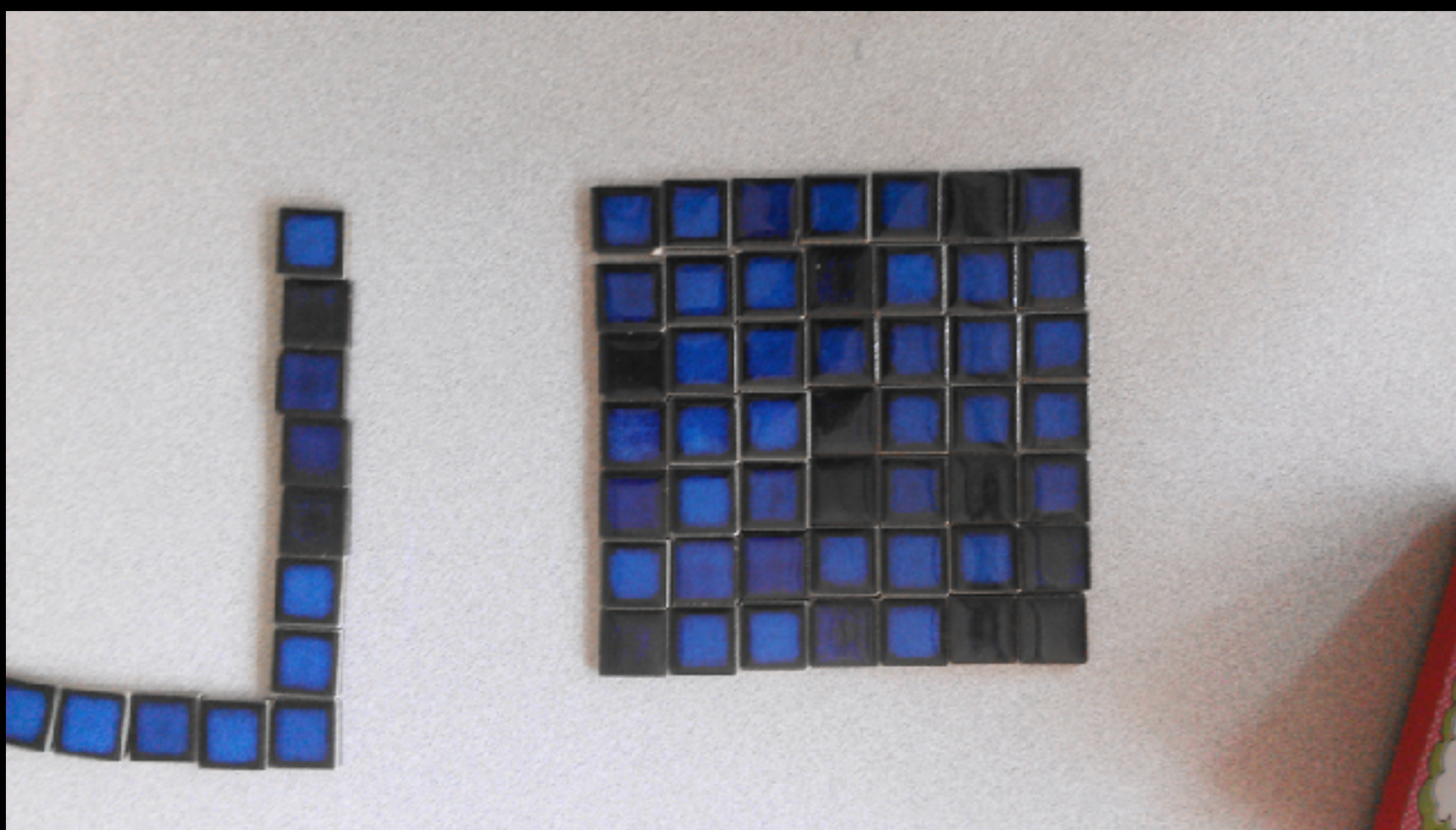
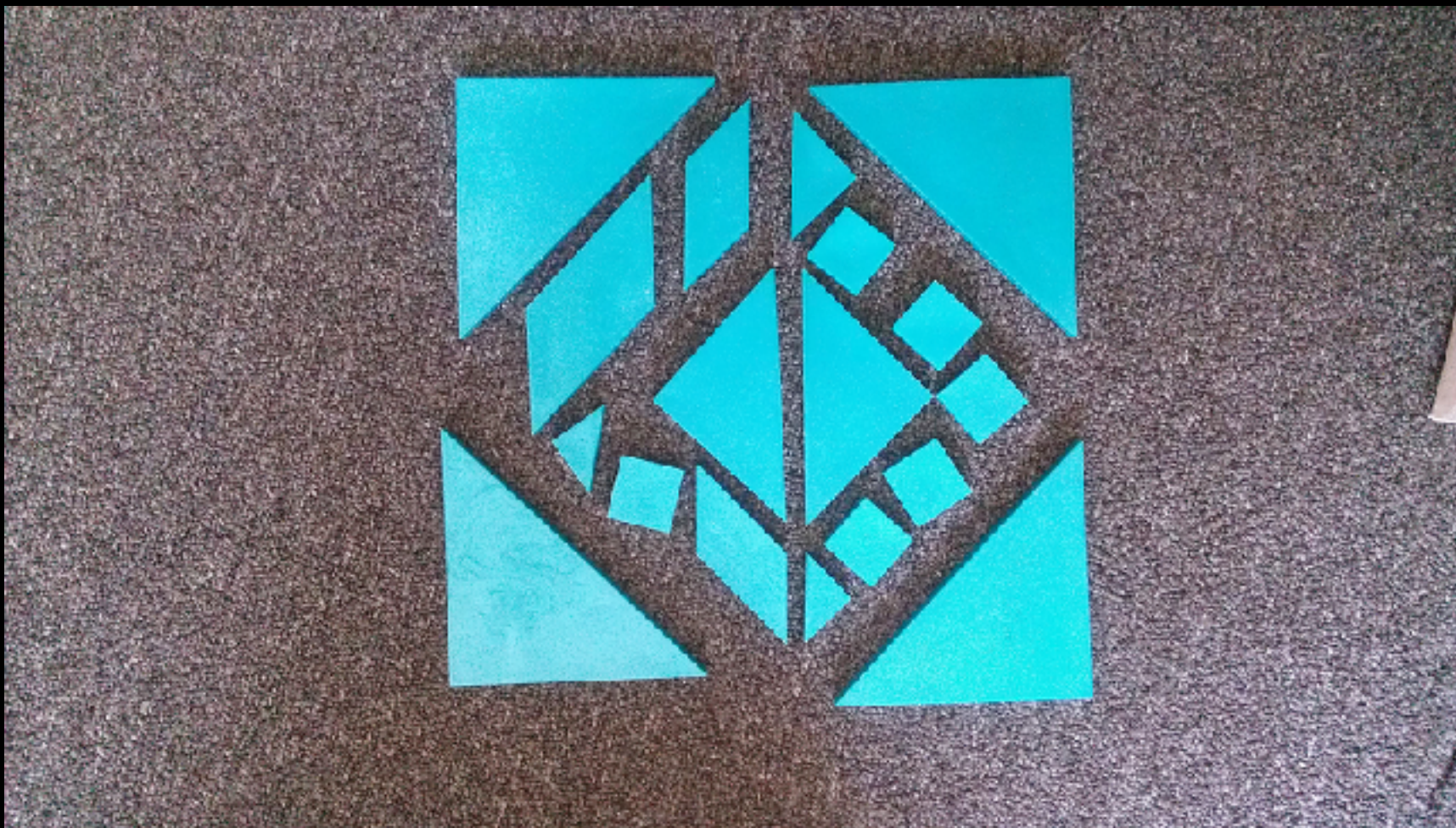
Are there shapes in other shapes?

Are shapes fractions?

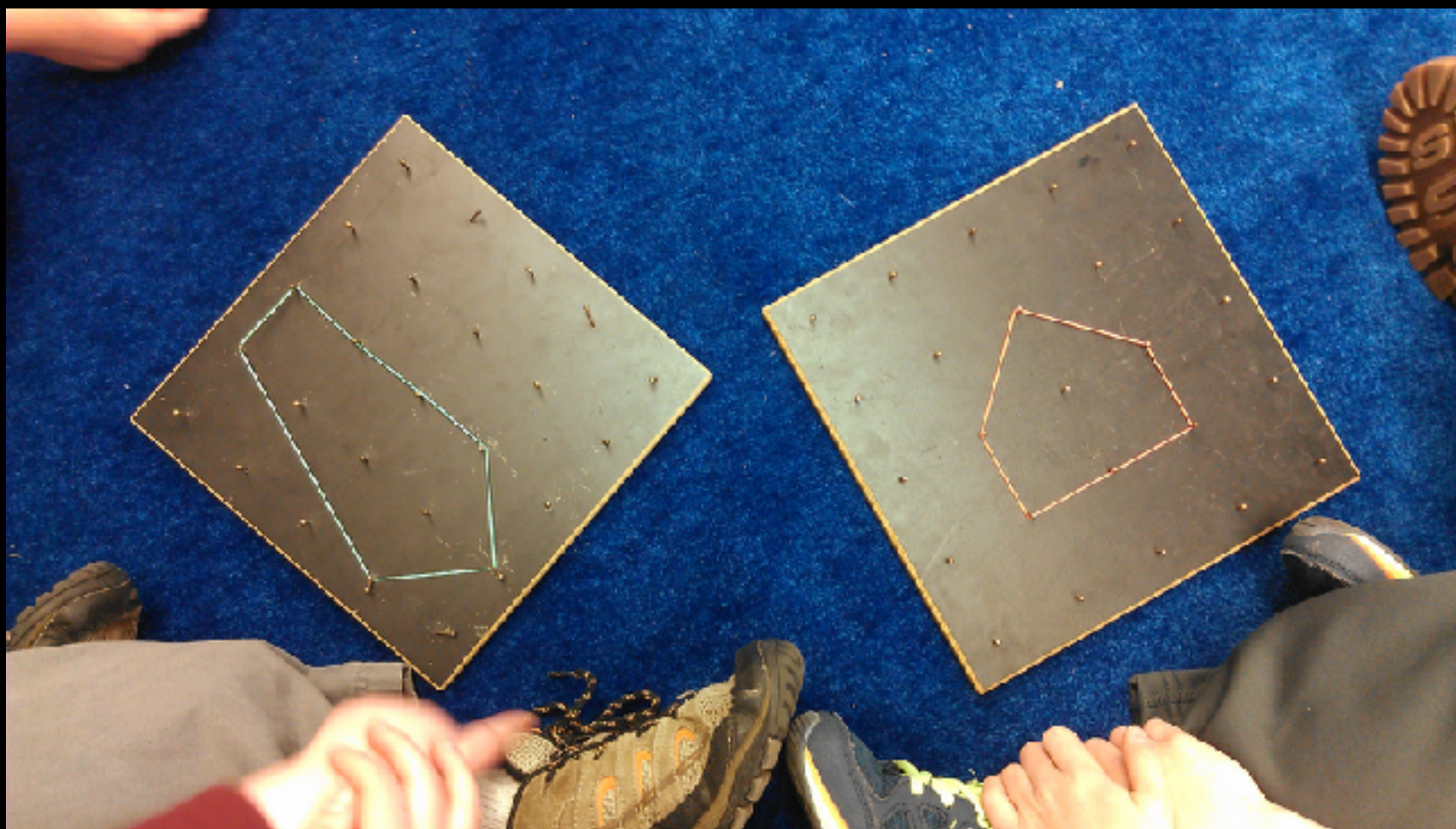
How do shapes get their form?

Are shapes permanent?

are shapes
fractals?
What are
fractals?
What are
shapes?







Andrew:

We need to know the names of these shapes so we know what we're talking about.

Debbie:

How can I help you?

Shapes Are Math

straight

angles

hexagon

horizontal

line

diagonal

trapezoid

slanted

Triangle

size

intersect

cross

vertex

segment

square

sides

different

rectangle

parallel

2D

flat

length

corners

Reason with shapes and their attributes.

CCSS.MATH.CONTENT.1.G.A.1

Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

CCSS.MATH.CONTENT.1.G.A.2

Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

CCSS.MATH.CONTENT.2.G.A.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

What are you wondering now?

Are shapes life?

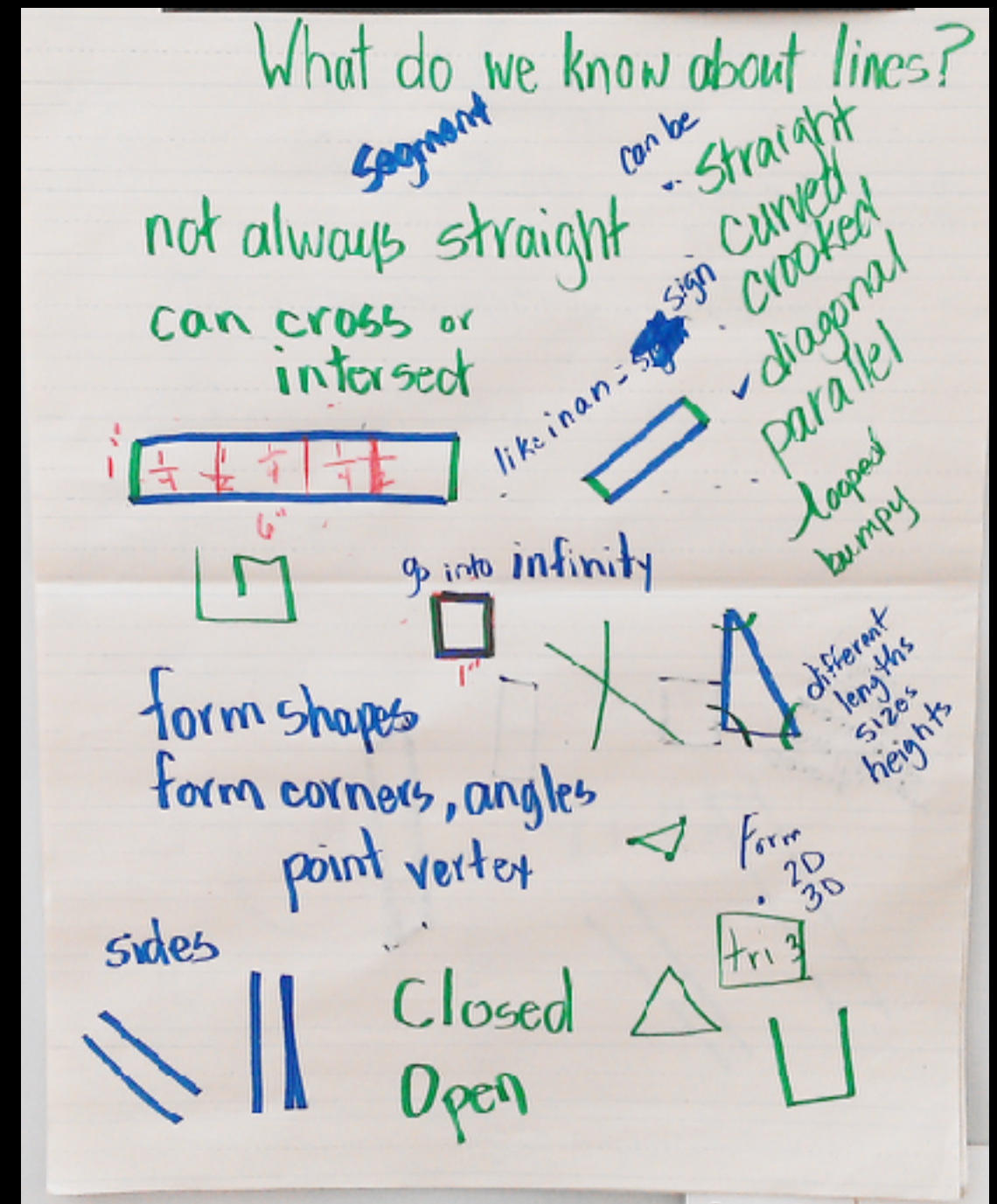
How big can the number of sides go?

Is a heart a shape?

Do curved sides count as sides?

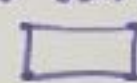
Can there ever be a perfect circle?

What about a line? Is a line a shape?



Shapes

What do you
NOTICE .

- Shapes are Everywhere
- Shapes are cool because there are many different shapes
- Shapes are math
- shapes are fun
- some shapes have corners and some shapes don't  
- Shapes are on your body
- shapes are in the sky like stars and clouds
- Shapes are at school, in the ocean, on houses, inside your body
- shapes are action figures

What do you 
Wonder? .

- What kind of shapes are there?
- How many shapes are there in the world?
- Are shapes everywhere?
- Why were shapes created?
- What shapes are in the future?
- Are shapes letters?
- Are shapes in space  or in the midnight zone?
- How many names are there for shapes?
- Are shapes in the sky?
- Can a cloud make a shape?
- Are shapes animals?
- What makes a shape a shape?
- Do shapes have little "specks" in them -> like atoms?

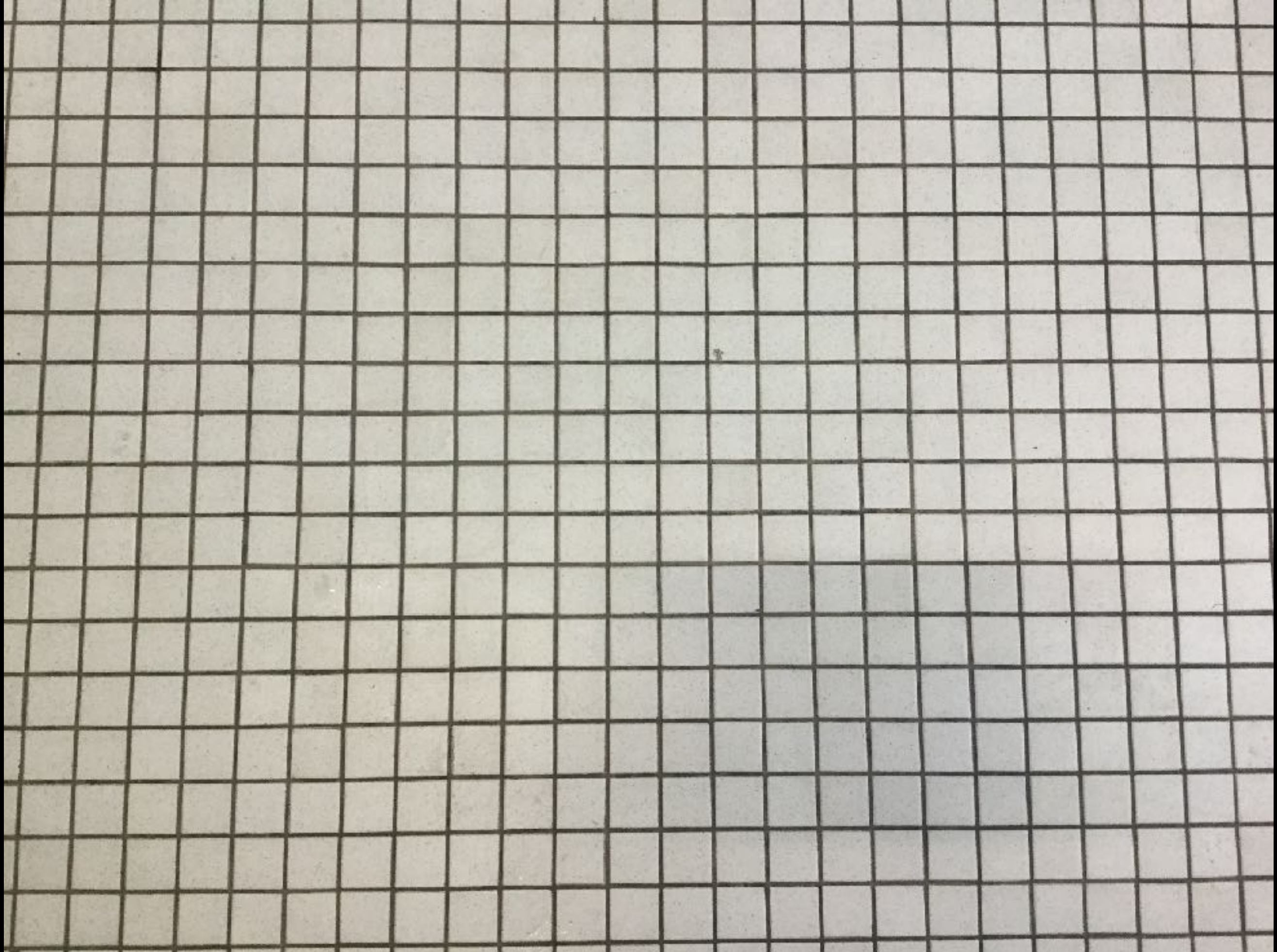
6. Daphne's DREAM: Drop Everything and Math

I wonder, you know how they have DEAR - Drop Everything and Read? ... Why don't they have it for math? I wish we could Drop Everything and Math. Instead of worksheets or tests or problems, the kids could just look around and see what interests them. We could find a question of our own and ask it and work on figuring it out until we're satisfied.

That's the kind of math I like.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



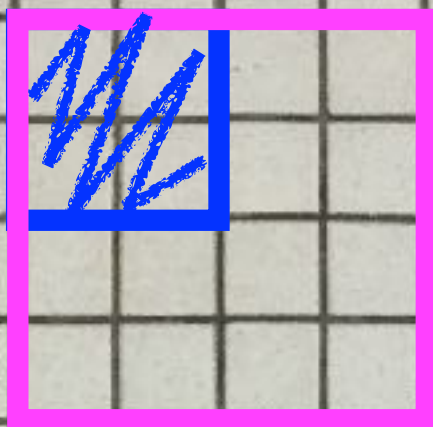


1



1

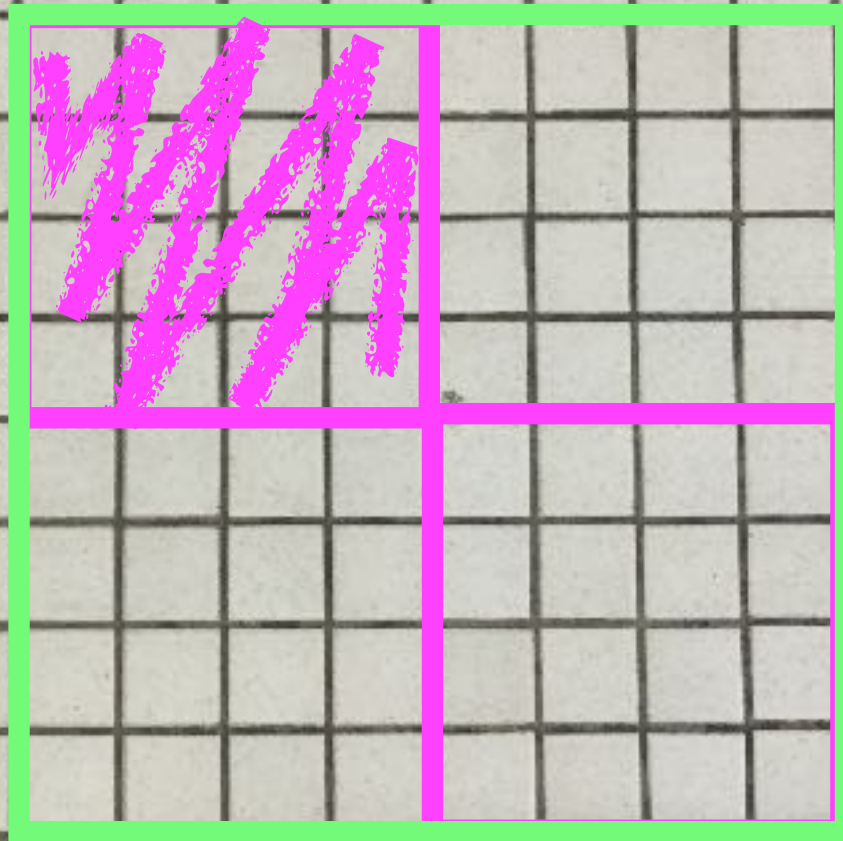
2



1

2

3

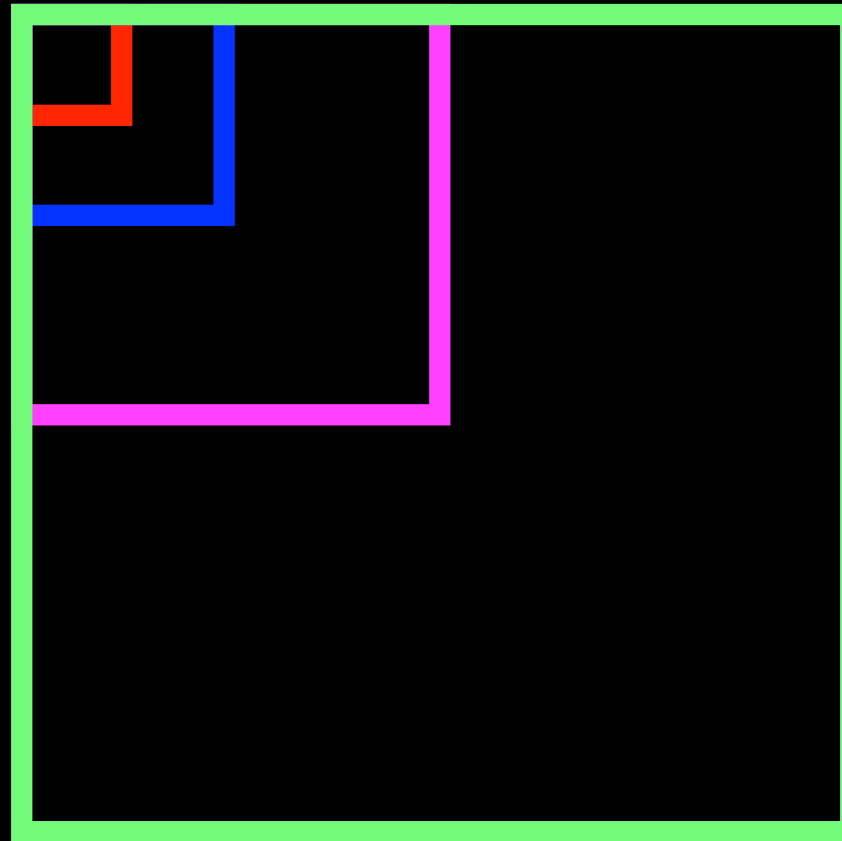


1

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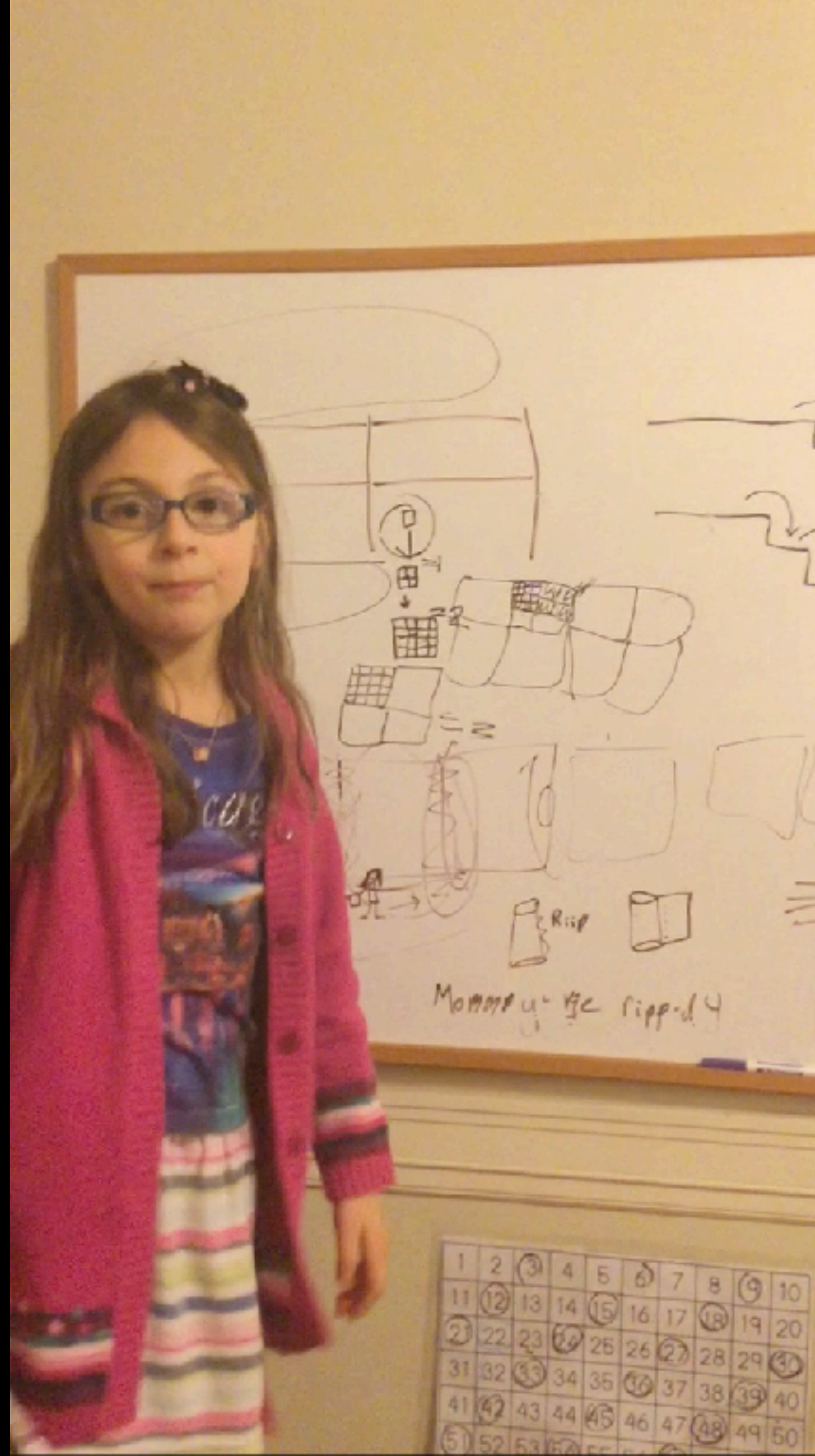
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4



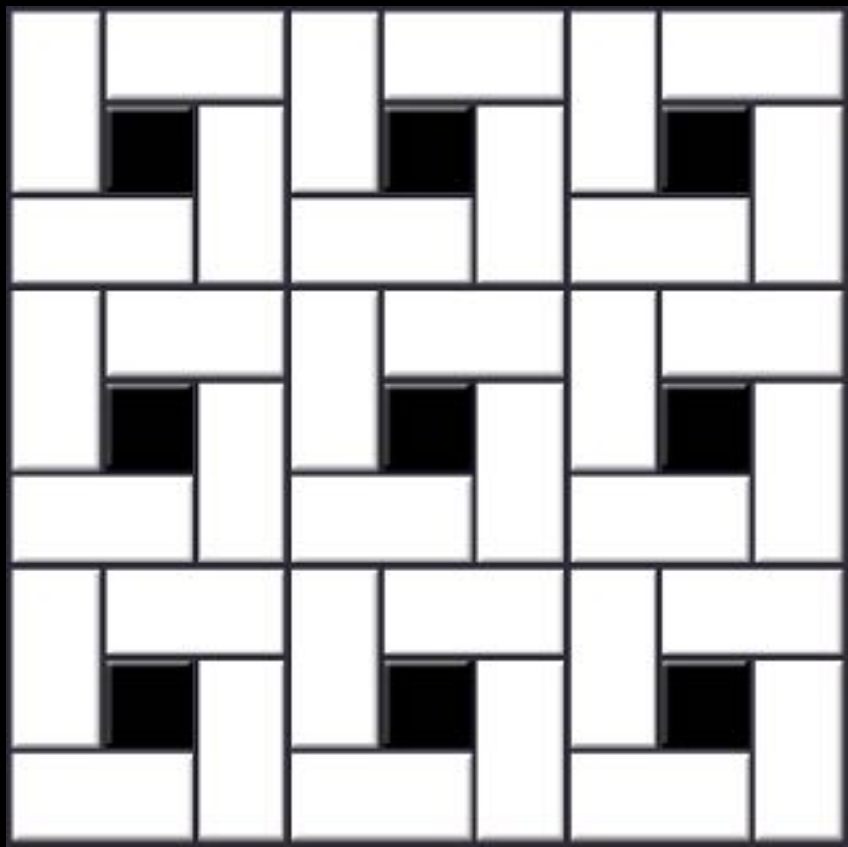
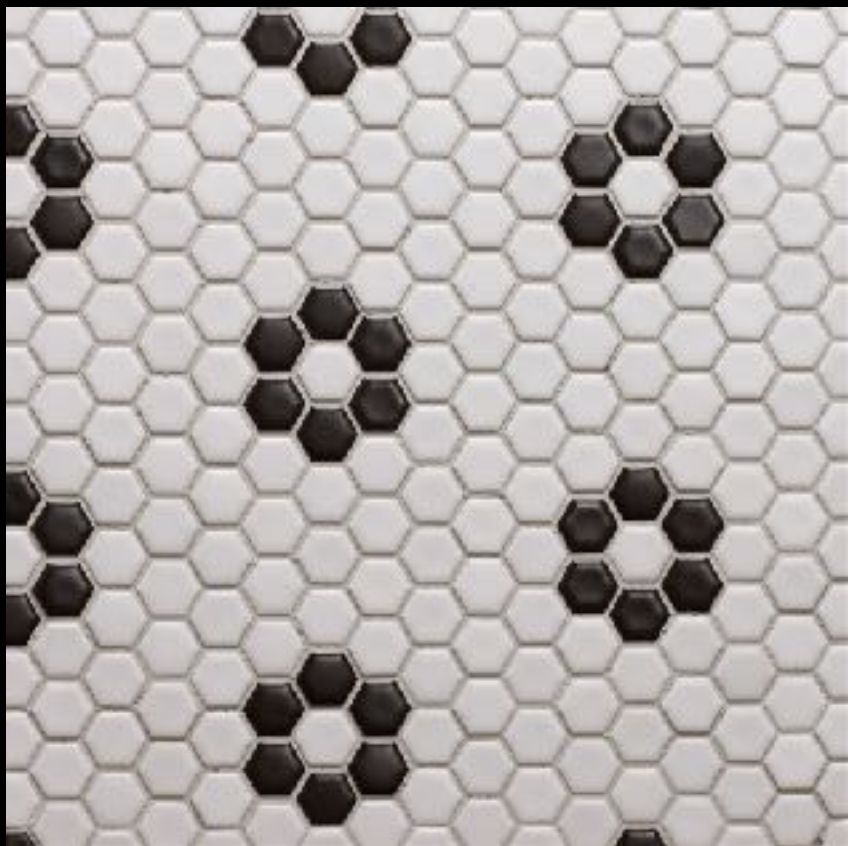
1
2
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4





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60





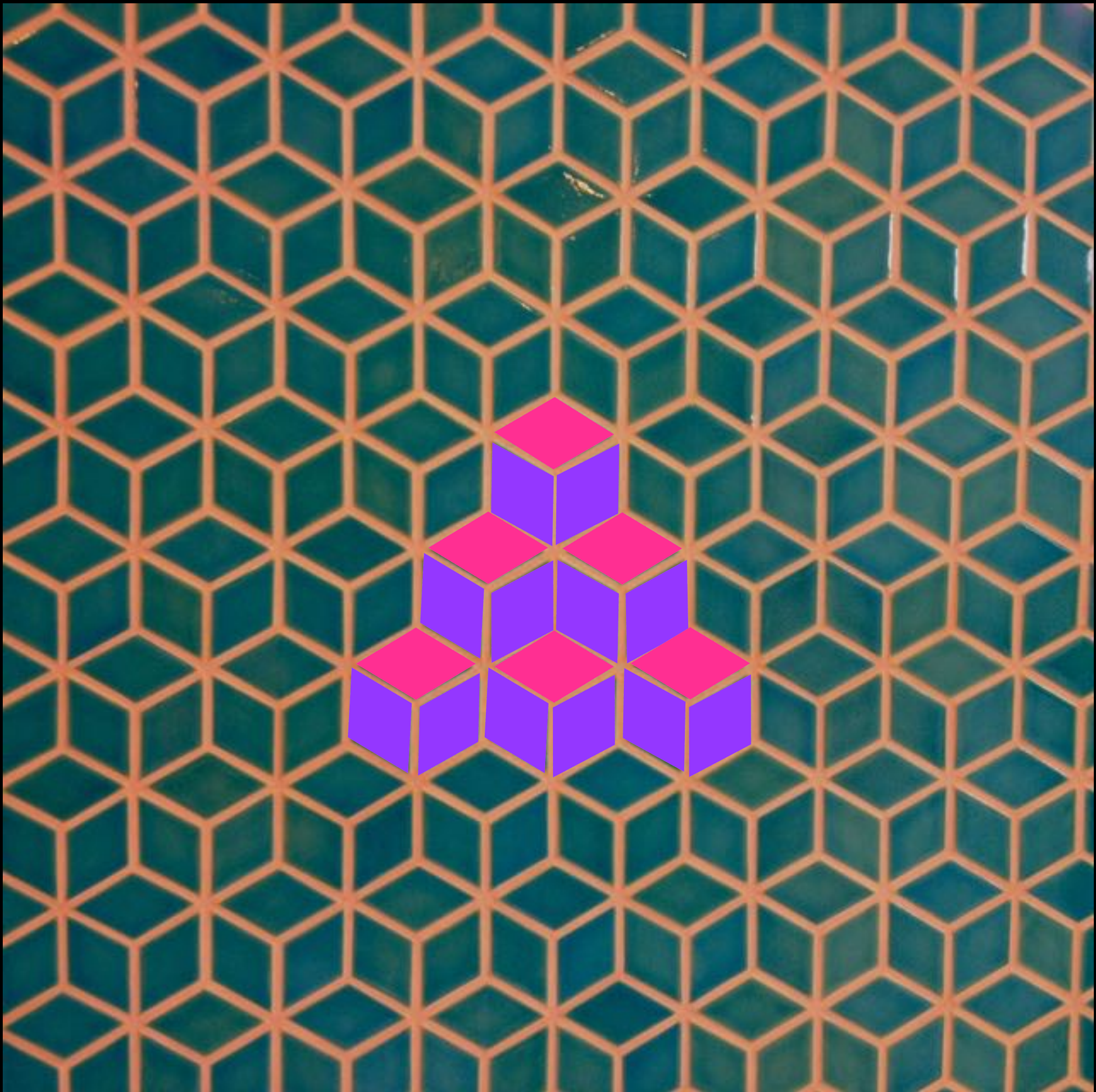


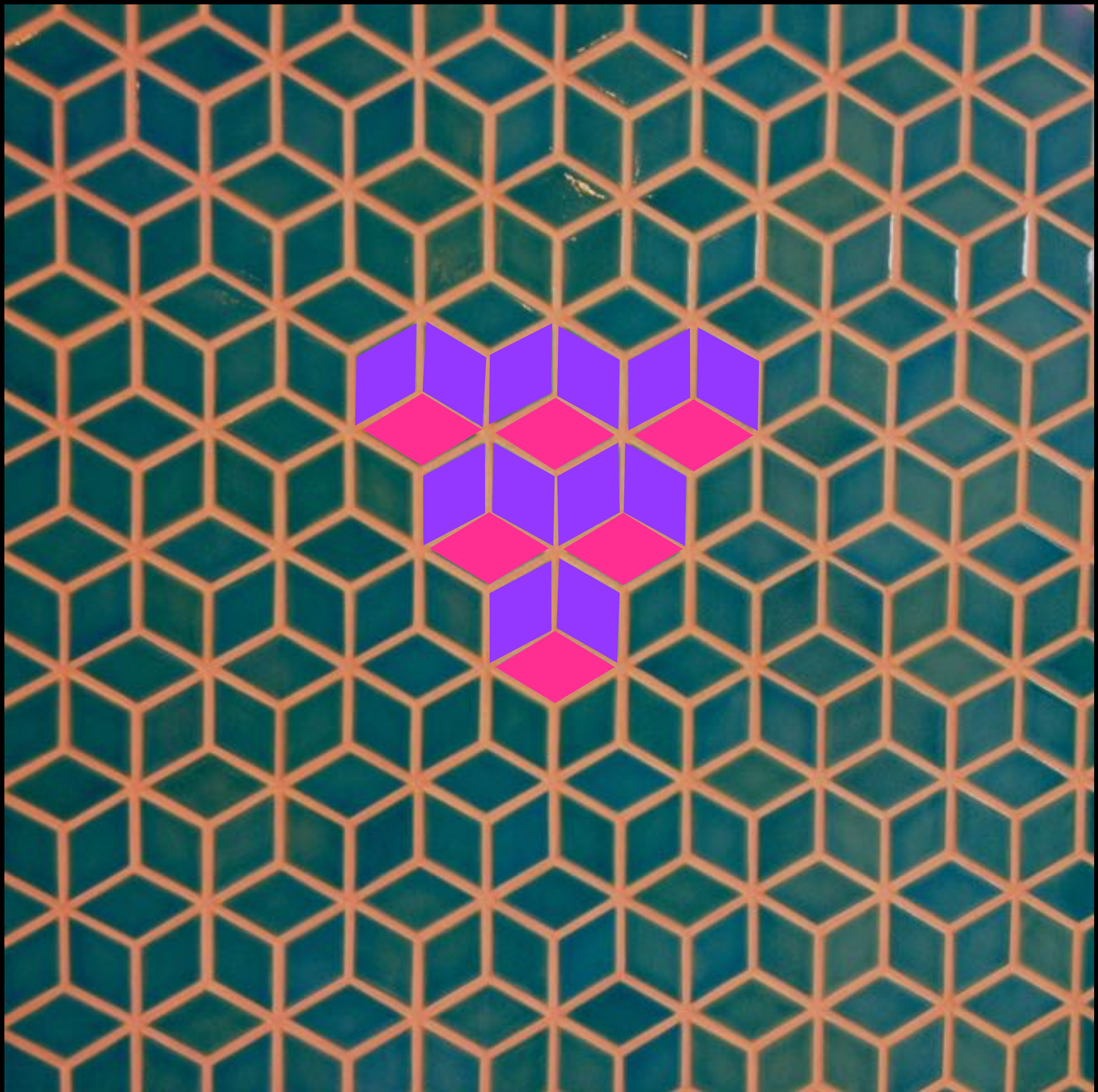


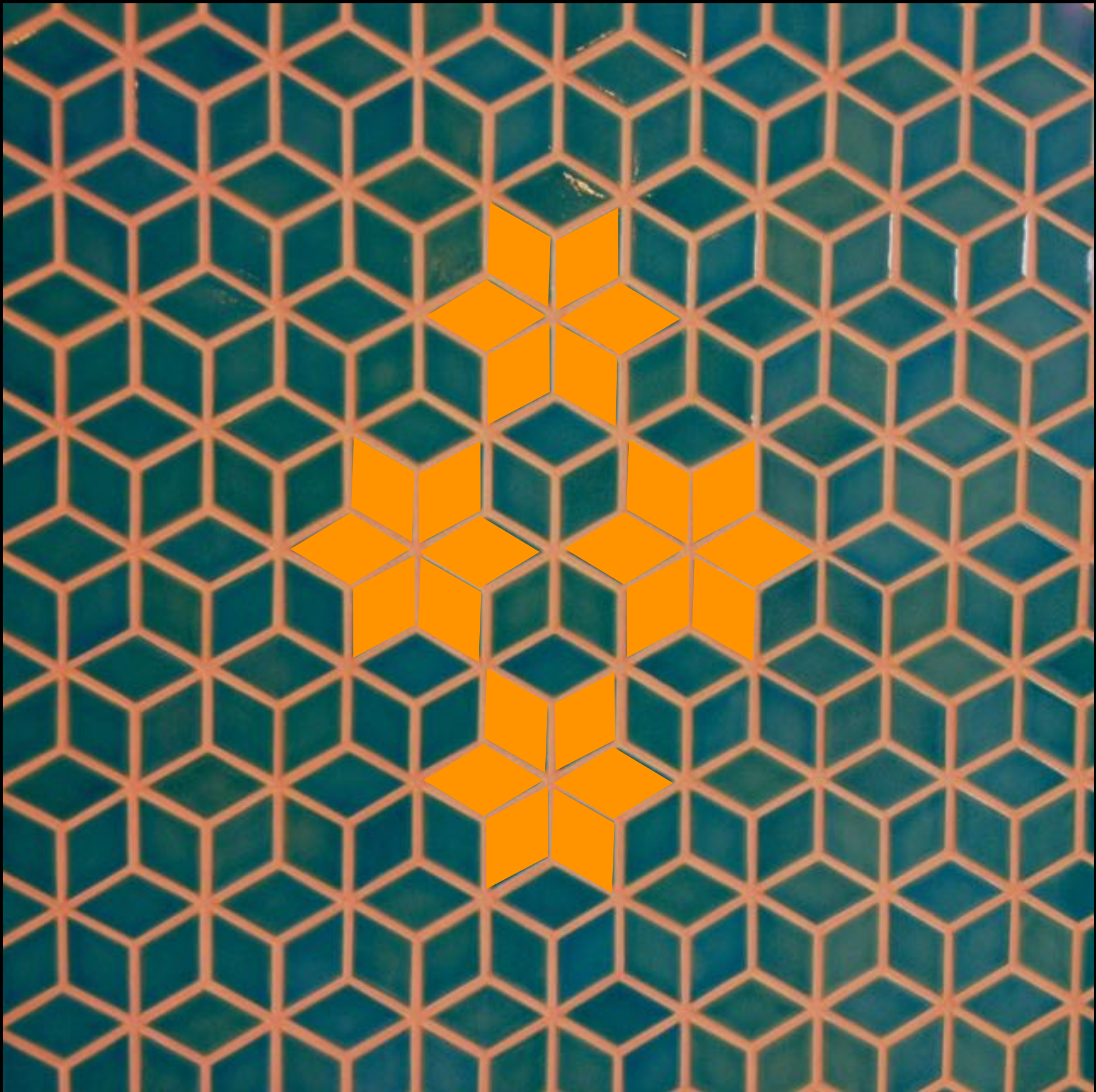


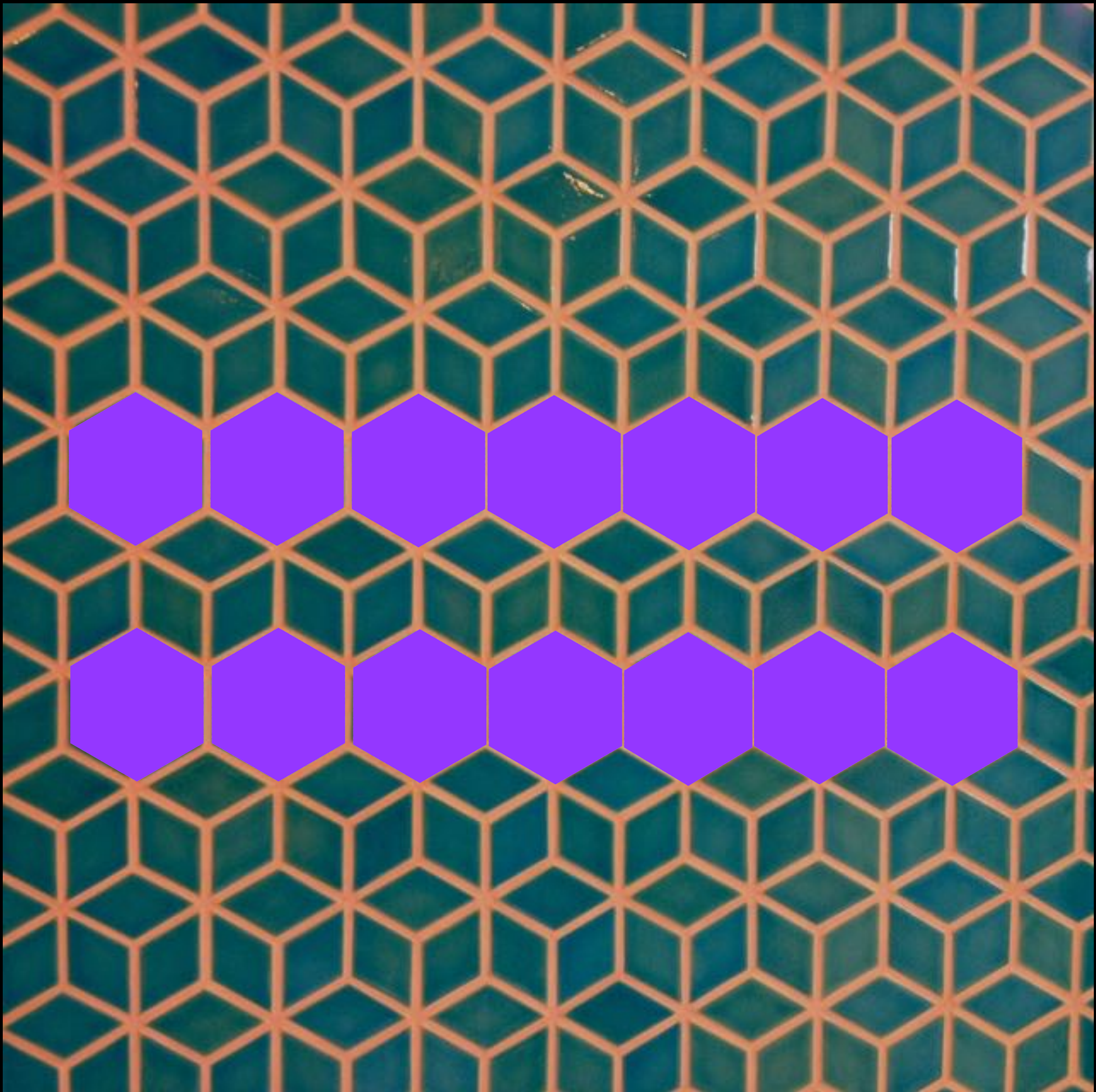


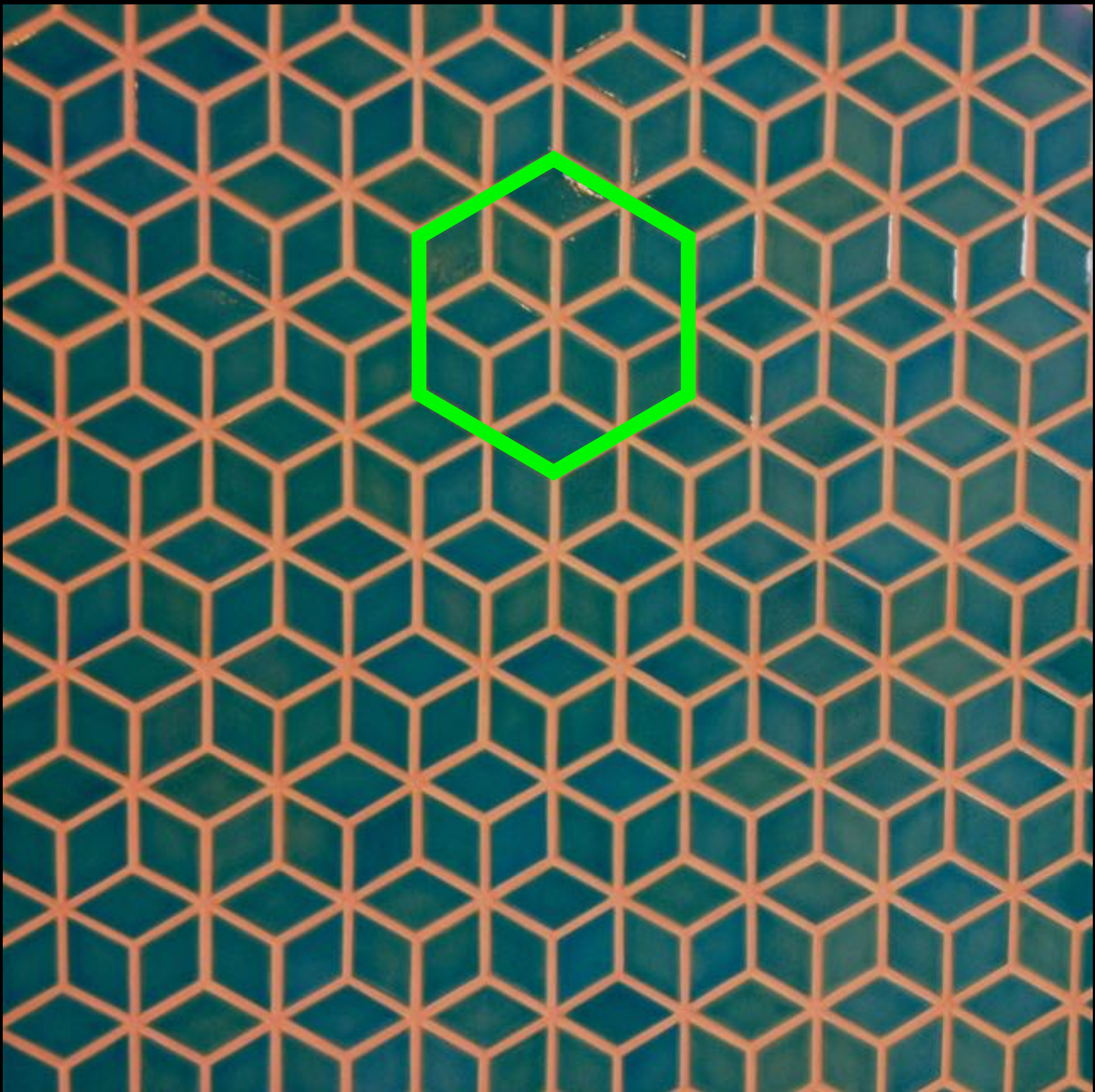


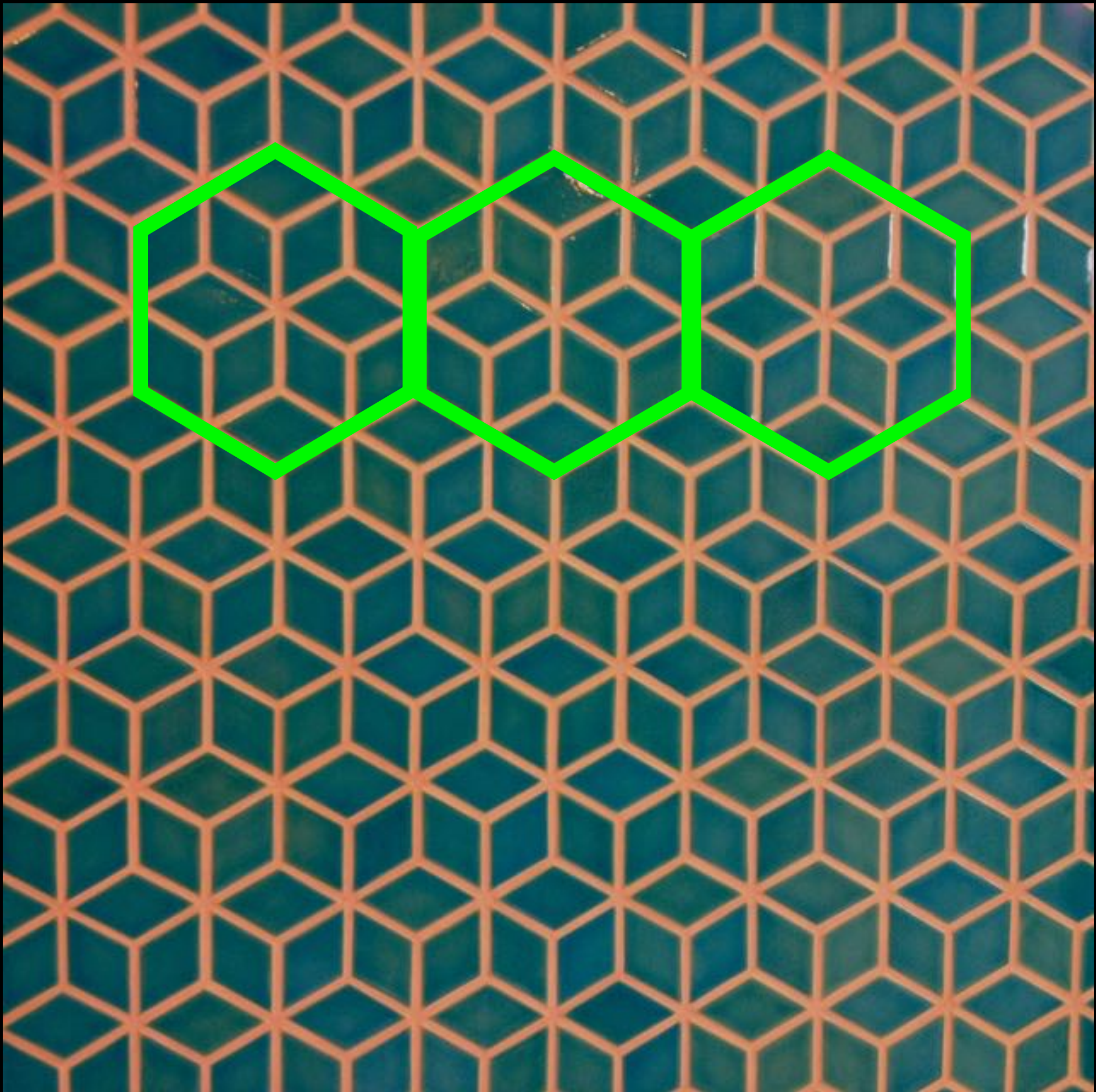


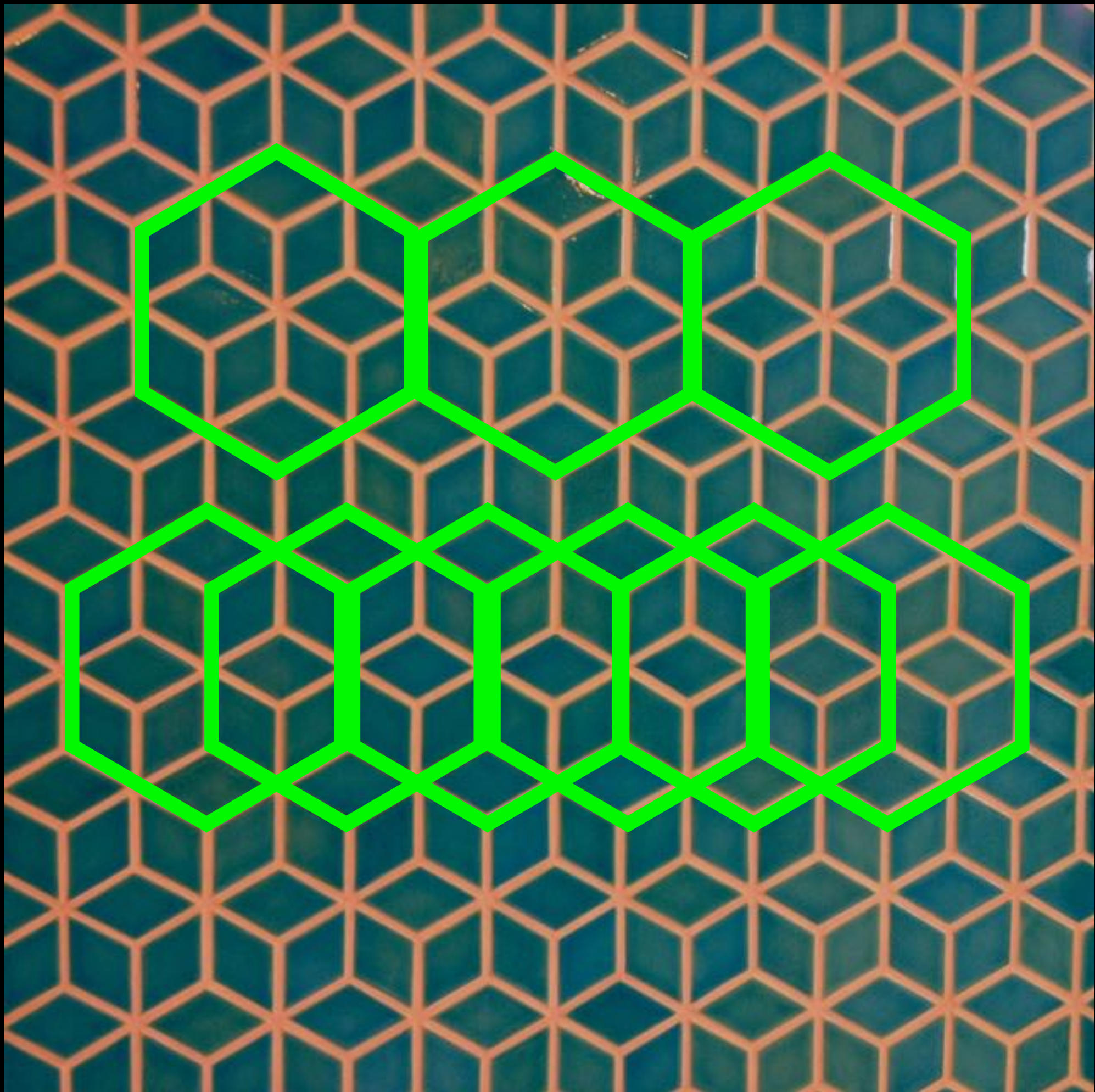


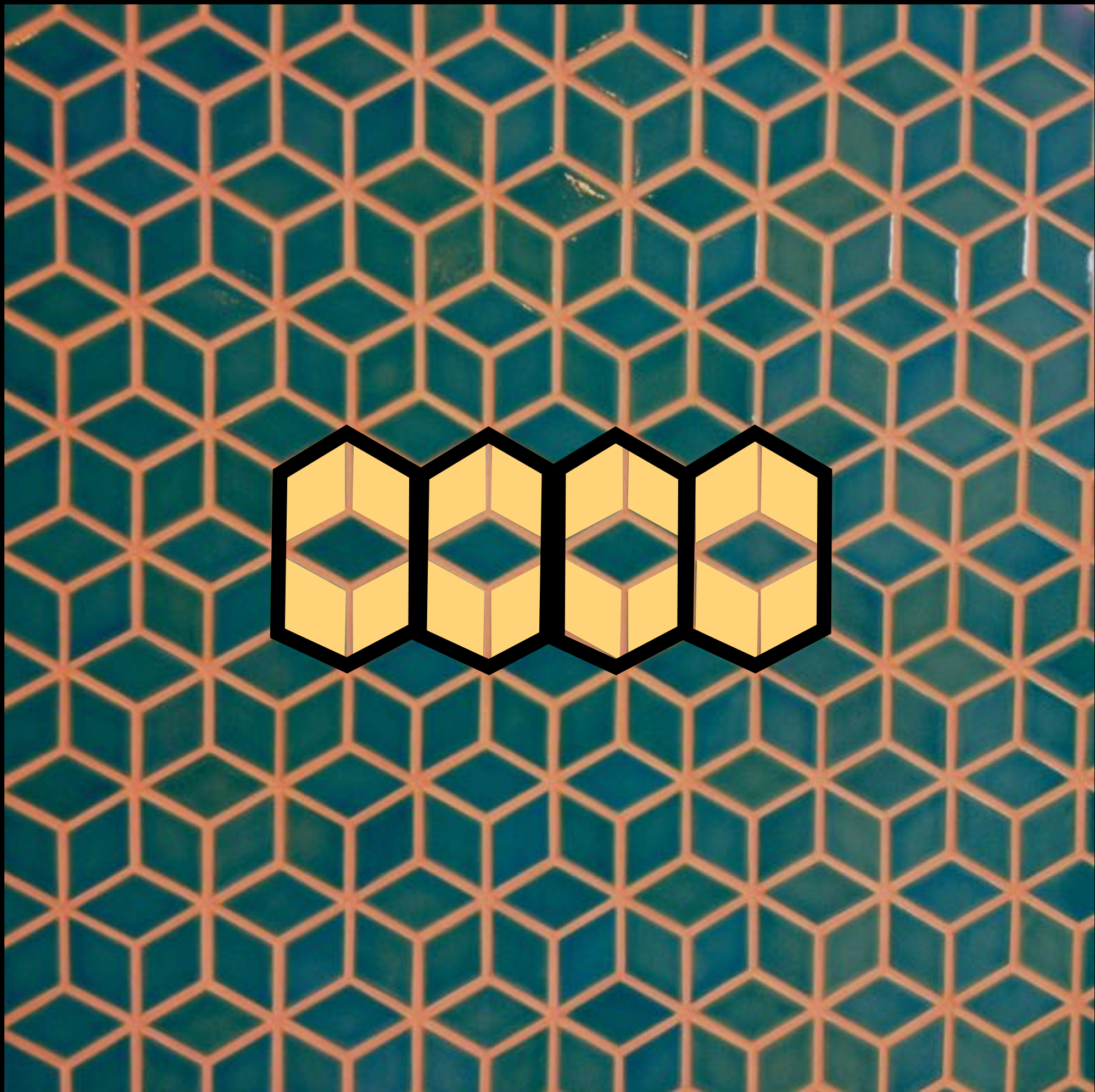




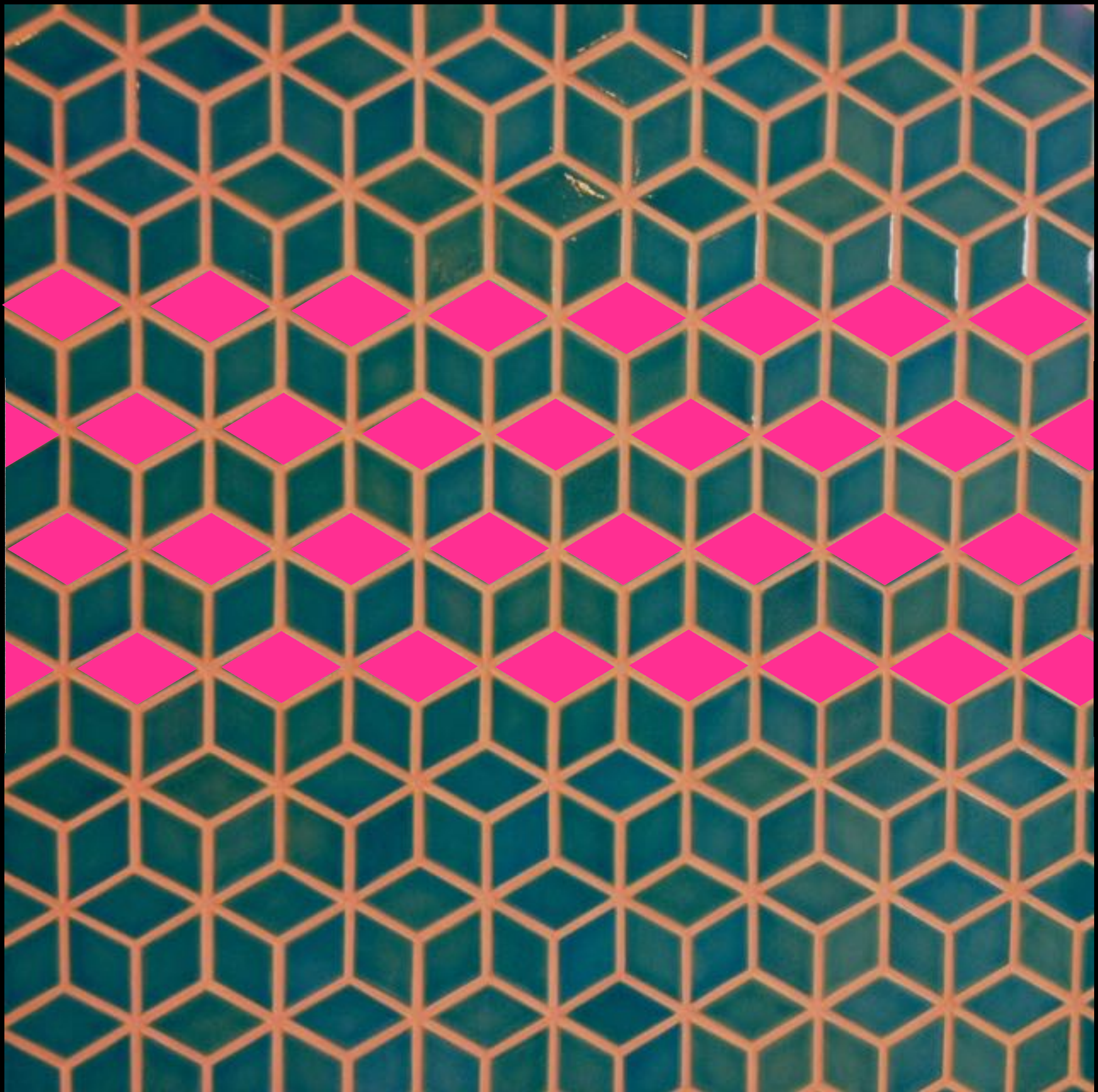


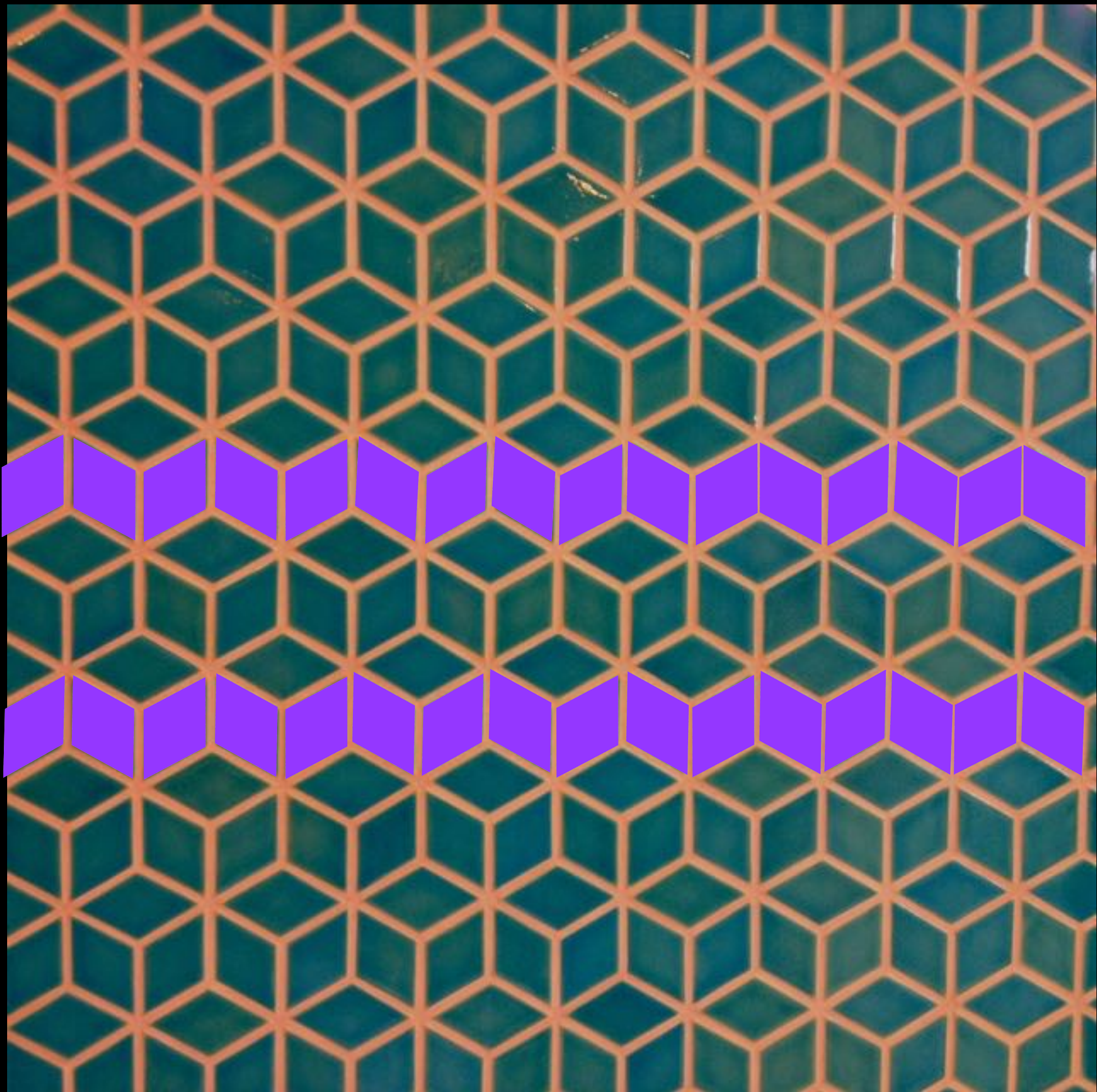


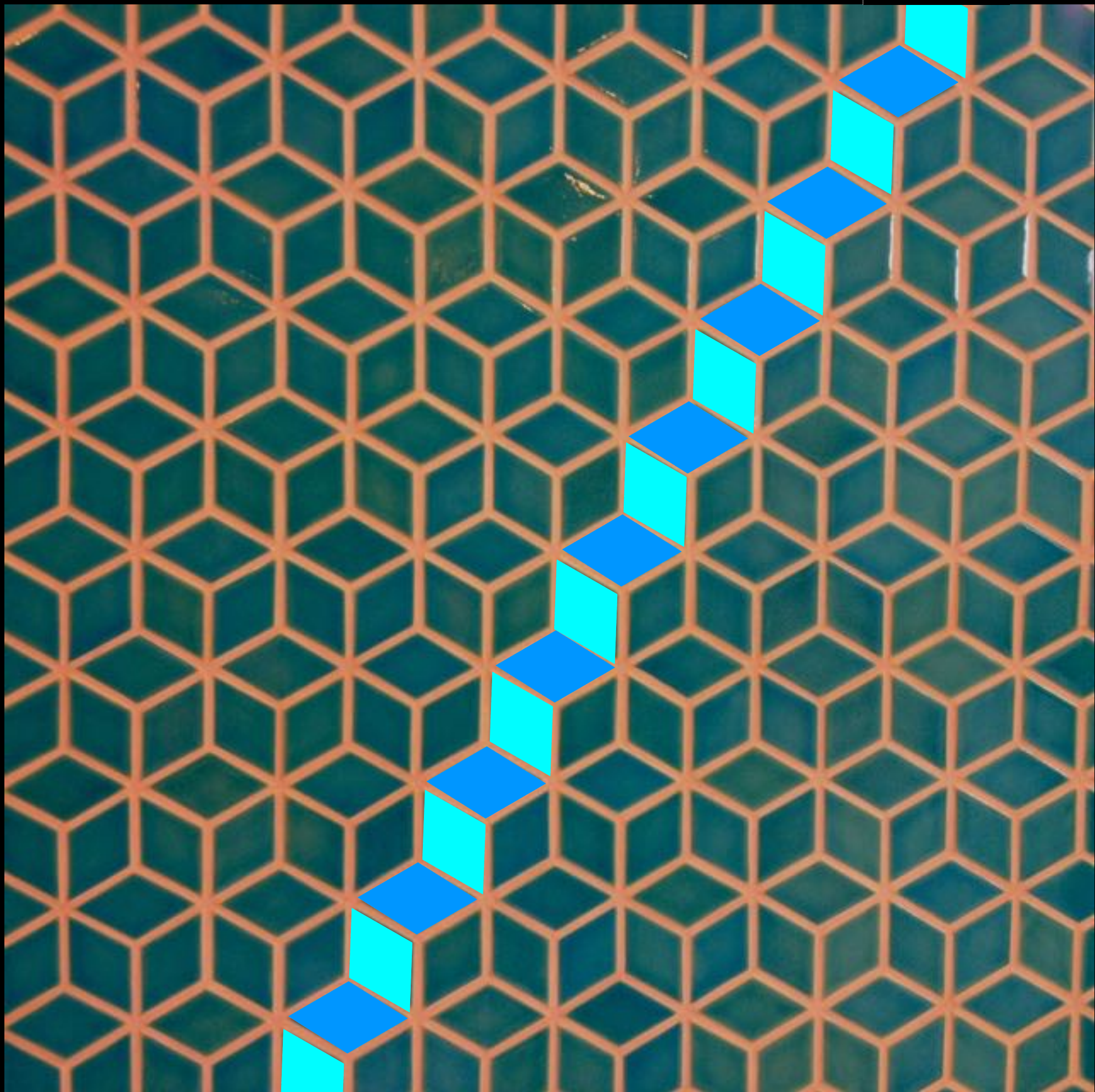














The Floor Game

August 6, 2012

When I was at the grocery store yesterday morning, I watched two little girls play a game that my sister and I used to call "the floor game." The goal of "the floor game" was to stay on the colored tiles as long as possible as we accompanied our mom in the grocery store while she shopped. If you stepped on a white tile, you'd lose and get eaten by [sharks](#) or something. We never really figured that part out, but the challenge was to stay on the black tiles and avoid the white ones.





1	2	3	4	5	6	7	8	9
11	12	13	14	15	16	17	18	19
21	22	23	24	25	26	27	28	29
31	32	33	34	35	36	37	38	39
41	42	43	44	45	46	47	48	49
51	52	53	54	55	56	57	58	59

Name _____

Reteaching
11-6

Mental Math: Ways to Find Missing Parts

Count on by hundreds and tens to find the parts of the whole.

$$260 + \underline{\hspace{2cm}} = 700$$

First, count on by hundreds. 4 hundreds

260, 360, 460, 560, 660

100 200 300 400

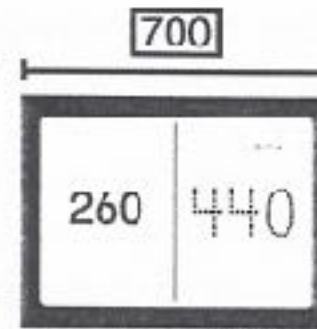
Next, count on by tens. 4 tens

660, 670, 680, 690, 700

10 20 30 40

4 hundreds and 4 tens is 440.

So, $260 + \underline{440} = 700$.



1. $350 + \underline{\hspace{1cm}} = 600$

Count on by hundreds. 2 hundreds

350, 450, 550

Count on by tens. 5 tens

550, 560, 570, 580, 590, 600

2 hundreds and 5 tens is 250.

So, $350 + \underline{250} = 600$.





The 5/6 Tacos

@mrsverveyclass

Follow



@TracyZager Drop Everything And Math occurring for the first time in our room. #DSBN #DREAM #mathrocks



7:32 AM - 29 Sep 2017



Class of Rockstars

@OfRockstars

Follow



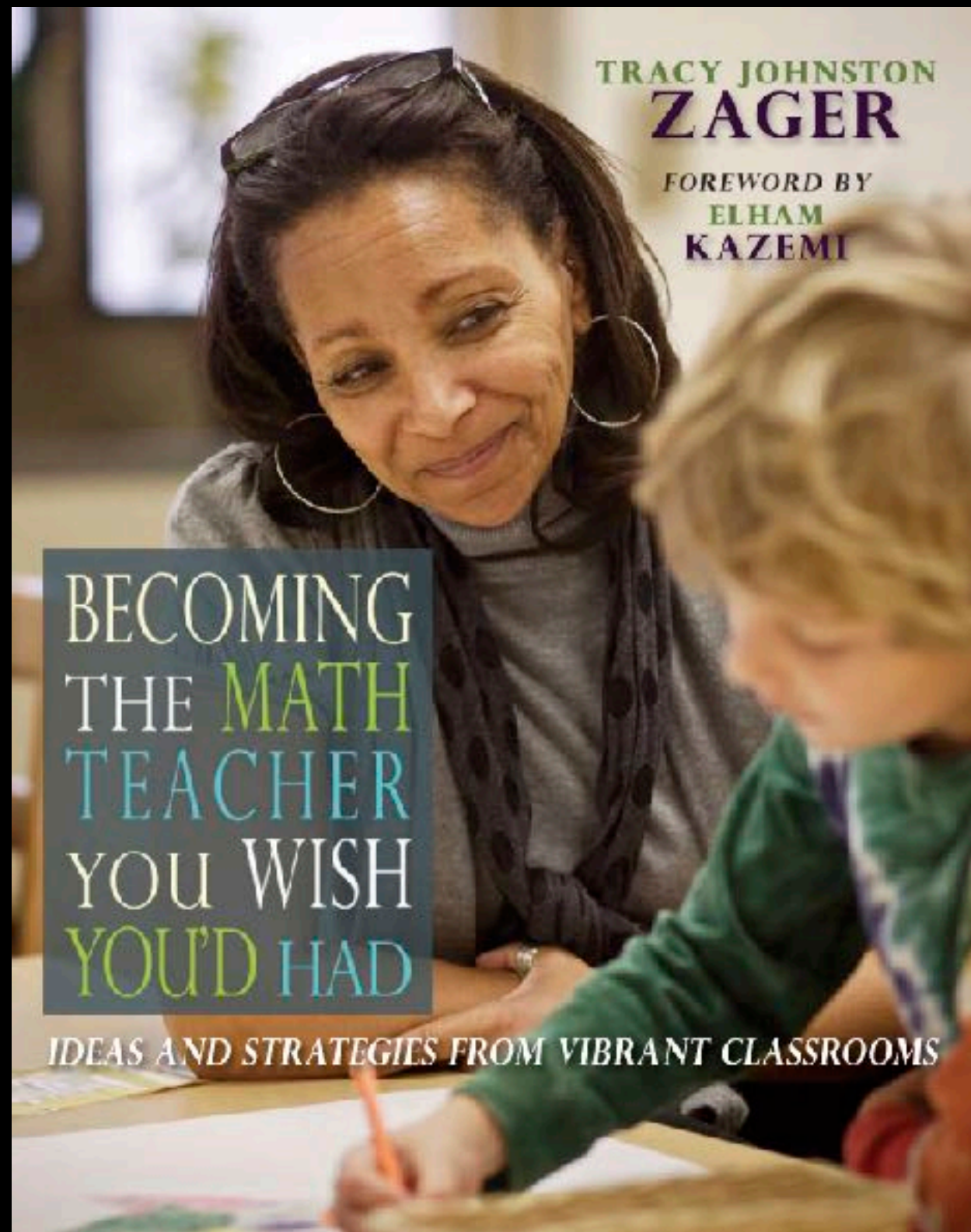
DRop Everything And Math-ing....aka D.R.E.A.M.-ing...building that ❤️ of Math with choice of Math activity! #BecomingMath



7:54 PM - 16 Oct 2017

Six Strategies to Include Mathematical Questioning in Mathematics Classes

- 101 questions
- Notice and Wonder
- Problem Posing
- Riffing off Solutions
- Question-Centered Unit
- DREAM: Drop Everything and Math



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