Teaching Probability with Purpose

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Activity 1 – The Last Banana

Activity 2 – Smelling Parkinson's Disease

Activity 3 – Is LeBron Streaky?

copies of this handout can be found at MrTysonStats.com

THE LAST BANANA (ADAPTED FROM *TED ED*)

(SOURCE: HTTP://ED.TED.COM/LESSONS/THE-LAST-BANANA-A-THOUGHT-EXPERIMENT-IN-PROBABILITY-LEONARDO-BARICHELLO)

Suppose that you're on a desert island playing dice with another castaway. The winner's prize will be the last banana. Here are the rules of the game:

- Each player rolls a die
- If the largest value shown is a 1, 2, 3, or 4, then player A wins
- If the largest value shown is a 5 or 6 then player B wins

1. Who has the advantage in this game: Player A, Player B, or neither? Make your best guess and explain your choice.

2. Get a partner and play this game. Play the game 20 times and record the winner of each game by tallying in the table below.

Player	**A**	**B**	**Total**
Tally/Count of Wins			20
Proportion of Wins			1.00

3. The process of imitating chance behavior is called a **simulation**. The long-run proportion of times that an event happens is called the **probability** of that event. Simulations are powerful tools that can be used to estimate complex probabilities. Based on the simulation by you and your partner, what is your estimate of the probability that Player A wins? that player B wins?

   P(Player A wins):

   P(Player B wins):

4. How do you suppose we could use simulation to obtain a more precise estimate of the probabilities in question 3?

5. Fill in the table below, combining the class data.

Player	A	B	Total
Count of Wins			
Proportion of Wins			1.00

Laws and Myths About Randomness

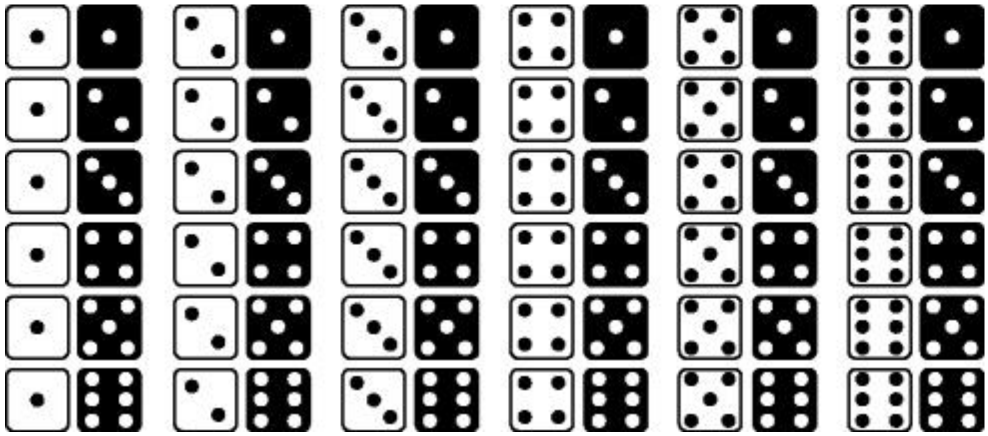
6. It turns out that random (or chance) behavior, rather than being haphazard, displays long-run patterns. Mr. Tyson will now simulate this process 5000 times using Fathom.

Player	A	B	Total
Count of Wins			
Proportion of Wins			1.00

7. Are proportions from the Fathom simulation in question 6 likely to be closer to or farther from the true probabilities than the ones you calculated in question 3? Why?

The **law of large numbers** states that as the number of repetitions of a random process (chance process) increases, the proportion of times that an event occurs will approach a particular value. That value is called the **probability** of the event.

8. It turns out that we can calculate the exact (theoretical) probabilities in this situation by listing all possible outcomes for rolling two dice. This list of all possible outcomes is called the **sample space**. Use the sample space below to calculate the theoretical probability that each player wins.



P(Player A wins): P(Player B wins):

9. Probabilities and the law of large numbers describe long-term behavior. Short term behavior often displays behavior that is very different than what the law of large numbers predicts. Look at the file called *CoinFlippingUnfair.xlsx*. Imagine that the game you played was replaced by flipping a coin that has a 20/36 = 0.555 chance to come up heads. What does the graph reveal about short-term and long-term behavior as they relate to probability?

SMELLING PARKINSON'S DISEASE

INTRODUCTION

As reported by the Washington Post (http://tinyurl.com/SmellPark), Joy Milne of Perth, UK, smelled a "subtle musky odor" on her husband Les that she had never smelled before. At first, Joy thought maybe it was just from the sweat after long hours of work. But when Les was diagnosed with Parkinson's 6 years later, Joy suspected the odor might be a result of the disease.

Scientists were intrigued by Joy's claim and designed an experiment to test her ability to "smell Parkinson's." Joy was presented with 12 different shirts, each worn by a different person, some of whom had Parkinson's and some of whom did not. The shirts were given to Joy in a random order and she had to decide whether each shirt was worn by a Parkinson's patient or not.

1. Why would it be important to know that someone can smell Parkinson's disease?

2. How many correct decisions (out of 12) would you expect Joy make if she couldn't really smell Parkinson's and was just guessing?

3. How many correct decisions (out of 12) would it take to *convince* you that Joy really could smell Parkinson's?

SIMULATING THE EXPERIMENT

Although the researchers wanted to believe Joy, there was a chance that she may not really be able to tell Parkinson's by smell. It's logical to be skeptical of claims that are very different than our experiences. If Joy couldn't really distinguish Parkinson's by smell, then she would just have been guessing which shirt was which. The researchers were not willing to commit time and resources to a larger investigation unless they could be convinced to that Joy's wasn't just guessing. When researchers have a claim that they suspect (or hope) to find evidence against, it's called the **null hypothesis**.

4. What claim were the researchers hoping to find evidence *against*? That is, what was their prior belief (**null hypothesis**) about the ability to smell Parkinson's?

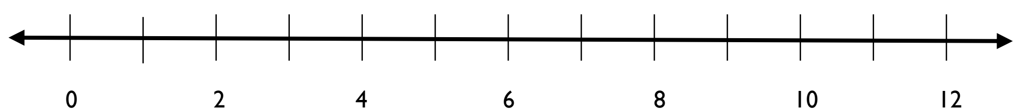
5. What claim were the researchers hoping to find evidence *for*? This is called the **alternative hypothesis** or the **research hypothesis**.

To investigate the idea that Joy was just guessing which shirt was worn by which type of person, we will assume that the null hypothesis is true.

6. Your instructor will hand you 12 cards (shirts) that have been shuffled into a random order. Don't turn them over yet! On the back of some of them is "Parkinson's" and on the back of others is "No Parkinson's." For each card, guess Parkinson's or No Parkinson's. Once you have made your guess, turn the card over and see if you were correct. Repeat this for each card and record the number of correct identifications (out of 12) below.

Tally of correct identifications	**Number of correct identifications**	**Proportion of correct identifications**

7. Create a dotplot of the number of correct identifications with the rest of the class. Record the results below.



8. In the actual experiment, Joy identified 11 of the 12 shirts correctly. Based on the very small-scale simulation by you and your classmates, what proportion of the simulations resulted in 11 or more shirts correctly identified, assuming that the person was guessing?

9. The proportion you just calculated is a crude estimate of a true probability called a ***P*-value**. How might we improve our estimate of the true probability?

Statistical Inference from the Simulation

10. Use the One Categorical Variable, Single Group applet at stapplet.com to run this simulation 10000 times. Then use that simulation to get a (likely) better estimate of the *p*-value for 11 or more shirts correctly identified, assuming that this person was just guessing. Is it *possible* that Joy correctly identified 11 shirts just by random chance (guessing)? Is it *likely*?

11. An interesting side note is that Joy's one "mistake" really wasn't a mistake. The shirt was worn by a person who supposedly didn't have Parkinson's even though Joy claimed that she could smell the telltale smell on that shirt. That person called the experimenters a little while after the experiment and reported that he had just been diagnosed with Parkinson's disease. That meant that Joy correctly identified 12 out of 12 shirts. What is the approximate *P*-value for 12 shirts correctly identified, assuming that this person was just guessing?

Note: A small *P*-value is considered strong evidence against the null hypothesis and in favor of the alternative hypothesis. But how small is small? As a rule of thumb, statisticians generally agree that *P*-values below 0.05 provide pretty strong evidence against the null hypothesis. Observed results with small *P*-values are said to be **statistically significant**.

Deeper Mathematical Connections

12. The true theoretical probability to get *k* successes in *n* trials when there is a true probability *p* of a success on each trial is given by the **binomial probability formula**: $\binom{n}{k} p^k (1-p)^{n-k}$. Compute the exact theoretical probability to get 11 or more successes in 12 trials when the true probability of success is 0.5. (*Hint*: calculate the probability for 11 successes and then do another calculation for 12 successes and then add these probabilities together.)

IS LEBRON STREAKY?

INTRODUCTION – IS LEBRON STREAKY?

LeBron James is certainly a great basketball player and arguably one of the greatest players of all time. There is no doubt he is a great shooter, but is he a streaky shooter? A streaky shooter is someone who has a greater chance to make the next shot, given that they made the previous shot. This is sometimes called having a "hot hand." We'll examine data from the 2016 NBA Championships to see if there is evidence that LeBron is a streaky shooter.

1. If a player is streaky, are shots independent or dependent?

2. We're going to examine LeBron's game 6 performance in the NBA championship to see if there is evidence that he was a streaky shooter in this game. State an appropriate null and alternative hypothesis. (*Challenge*: these hypotheses can be stated in two different ways.)

3. LeBron made 16 of his 27 shots in game 6 (excluding free throws). What is LeBron's shooting percentage in this game?

UNDERSTANDING RUNS AND STREAKINESS

4. For the sake of this investigation, let's suppose LeBron's true ability to make a shot in game 6 was really $16/27 \approx 0.593$. Here are the results of the 27 shots LeBron attempted, in order. ("S" means he succeeded in making the shot and "F" means he failed.)

   S S S S F S F F S F F S F S S S F F F S F F S S S S S

   A run (or streak) is defined as a consecutive subsequence of Ss or Fs. How many runs/streaks did LeBron have during game 6?

5. The *minimum* number of runs that LeBron could have had with these 27 shots is 2. Write a sequence of 16 Ss and 11 Fs that would result in 2 runs.

6. What is the maximum number of runs that LeBron could have had with these 27 shots?

7. If a shooter is ***streaky***, would you expect them to have a small number or runs or a large number of runs? Explain.

8. Would a large number of runs or a small number of runs provide evidence against the null hypothesis H_0: LeBron's shots were independent of one another?

Simulating Streakiness

9. Use a spinner split into two sections, 59.3% success and 40.7% failure to simulate 27 shots by LeBron during a simulated game 6, assuming that LeBron's shots are independent (that they are a result of random chance). Record your results and count the number of streaks in your simulation.

10. Make a dotplot by combining your simulated number of streaks with the rest of the class. Based on the dotplot, what is the estimated p-value that a 59.3% shooter gets 13 or fewer runs (which was LeBron's observed performance in game 6)?

11. Use the Streakiness applet (stapplet.com/) to conduct a larger simulation to get a (likely) better estimate of the p-value.

Statistical Inference from the Simulation

12. Is random chance a plausible explanation for LeBron's performance in game 6? Does your simulation provide convincing evidence that LeBron was a streaky shooter in game 6?

Note:

- LeBron made 9/24 shots in game 7 of the 2016 NBA Championship. His performance is shown below.
  S F F S F S F F S S F F F F F S S F S S F F F F