Technology, Textbooks and Tasks: Understanding How Textbooks Integrate Technological Activities

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Thursday, April 26, 2018
NCTM Annual Conference

BOSTON UNIVERSITY

Cathy Seeley

President's Message

Technology Is a Tool

Cathy L. Seeley

Calculators, computers, and a growing array of technological innovations are important tools for doing mathematics, joining the ranks of the compass, straightedge, pencil, slide rule, and trig tables. Technology is neither a panacea for improving the teaching and learning of mathematics nor the source of all the challenges facing teachers and students. It is a tool, and how teachers use that tool greatly affects how well students learn mathematics.

Computers and calculators were invented to save humans time and to allow us to solve more challenging problems more easily than we could otherwise do. Yet, some observers have noted that some school mathematics curricula seem designed to make technology obsolete. Rather than discussing whether technology is good or bad, we might discuss instead how we can capitalize on this powerful tool to help every student learn more mathematics, not less.

The Influence of Technology on Curriculum

Technology is here to stay, and it generates at least three important effects on the school mathematics curriculum. First, some mathematics is now *more important* than in the past because it plays a role in the design and use of technology. For instance, discrete mathematics topics such as matrices take on new importance in light of their use in optimization problems and the organization of data in rows and columns. The ability to translate data from one form of representation to another, such as from numbers and symbols to graphs and tables, or from graphs to tables, is more important now than ever, as well.

At the same time, technology makes some mathematics *less important*. This idea can be controversial because some fear that we will omit from the school curriculum all the computations that calculators can perform. It remains important for students to know how to add, subtract, multiply, and divide mentally and with a pencil and paper. But we must determine how much valuable instructional time we should devote to helping students become proficient with lengthy or tedious calculations. How long a divisor is long enough in the study of long division, for example, is now an important educational decision. In the typical American mathematics curriculum, often described as a mile wide and an inch deep, deciding what to leave out to make room for other topics is crucial.

The third and most important way in which technology affects the mathematics curriculum is that it makes some mathematics *possible* for the first time. Using technology, students at all levels can tackle real problems that might arise from planning a field trip or from a news story, even if the numbers involved might make the problem unwieldy with only a pencil and paper. Middle school students can analyze data using a wide range of statistical tools, developing quantitative reasoning abilities far beyond what was once possible. Graphing calculators allow students to see the connections between visual representations and symbolic ones, and we are only beginning to scratch the surface of the potential of computer algebra systems. These are just a few examples of how technology can help us raise the level of mathematical knowledge and thinking that we can expect of all students.

Teaching with Technology

The teacher is a key decision maker in using technology in the classroom. The most important decision is when to use it and when not to. Students need to learn when to use technology, when to reach for a pencil, and when to do something in their heads. Using technology well, like making other instructional decisions about mathematics, calls for teachers to have a solid knowledge of mathematics and a working knowledge of the technologies available. There is nothing wrong with students knowing more than their teacher about how to use some features of a calculator or a piece of software. What is not acceptable is for a teacher to limit the use of technology because of the teacher's own discomfort or for a teacher to allow unlimited use of technology without first helping students master essential mental tools.

Providing appropriate access to technology should be the responsibility of the school and should not be left to students and their families. If we expect students to buy calculators that cost more than their shoes, society will surely face a digital divide greater and more dangerous than any economic gap we have yet seen in schools. The same can be said if some students have access to computers outside of school while others do not. This kind of economic imposition, and the disastrous inequity that would result, are unconscionable. Unfortunately, this means that math can no longer be the cheapest subject in the school budget. What was once done with little more than chalk or overhead pens now calls for a significant and ongoing investment in technology. In times of tight budgets, this is a real challenge, but one that must be faced head-on.

The question, then, is not whether to use technology, but how to use it in ways that support the mathematics learning of every student. If students do not learn appropriate ways to use technology in school, they will surely find inappropriate ways to use it outside of school.

What are the challenges that you face in using technology to support student learning? How have you addressed the financial challenge of providing students with technology or handled related management issues, like keeping equipment in working order and accounted for? What successes have you experienced in using technology to raise the level of mathematics your students learn? Join me for my next President's chat about these and related issues on March 21 at 4:00 p.m. EST or submit your comments beforehand by visiting www.nctm.org/news/chat.htm. Ω

Originally published in the *NCTM News Bulletin* (March 2006).

- "Some mathematics is now **<u>more important</u>** than in the past because it plays a role in the design and use of technology… At the same time, technology makes some mathematics **<u>less important</u>**…The third and most important way in which technology affects the mathematics curriculum is that **<u>it makes some mathematics possible</u>** for the first time."

(March 2006)

NCTM Position (Oct 2011)

- “It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. **<u>Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics.</u>** When teachers use technology strategically, they can provide greater access to mathematics for all students.”

Teachers and Curriculum

- Research has shown that teachers largely mediate the benefits and constraints of written curriculum (Eisenmann & Even, 2009; Herbel-Eisenmann et al., 2009).

Activity Sort

- Examine the different versions of the "same" technology activities in the handouts.
- What similarities and differences do you notice among the ways that these textbooks structure the mathematics and the technology of these activities?
- How would using each of these texts be different for students? For teachers?
- Try find different ways of sorting the activities along different dimensions.

Similarities/Differences

- Starting with fixed parallel & move transversal vs. starting with non-parallel lines.
- Variation in the detail of the instructions.
- By the end of all the tasks, students will see how changing one angle affects all the angles.
- Technology based activity vs. analogue based activity (or accuracy or ease of construction)
- Vocabulary was sometimes integrated and for others it was prerequsite.
- Some are telling (“this will add up to 180°”) vs investigating.

Texts

Title	Publisher	Text
Focus on Geometry: An Integrated Approach	Addison Wesley Longman	A
CME Project: Geometry	Pearson	B
Connected Geometry	Everyday Learning Corp.	C
Holt Geometry	Holt, Rinehart and Wilson	E
Discovering Geometry: An Investigative Approach	Key Curriculum Press	D
Geometry	McDougall-Littell	F
Prentice Hall Mathematics: Geometry	Pearson	G
Prentice Hall Geometry: Tools for a Changing World	Prentice Hall	H
UCSMP Geometry	Scott, Foresman & Co.	I

EXPLORE: PARALLEL CROSSING

MATERIALS

Protractor, Straightedge
Geometry software (optional)

Use geometry software or the opposite edges of a straightedge to draw two parallel lines. Draw a transversal through the two lines.

Measure the various angles, and make as many conjectures as you can about the angles that are congruent and supplementary in your figure. (You may want to check to be sure that your conjectures hold for the calligraphy guidesheet at the right.)

Text A

For You to Do

1. Use geometry software to construct two parallel lines and a moveable transversal. Measure all the angles and note any two angles that have the same measure and any two angles with measures that add to 180°.

   Move the transversal. Check which of the equalities found above is invariant. Is there a state of the sketch in which all the angles have the same measure?

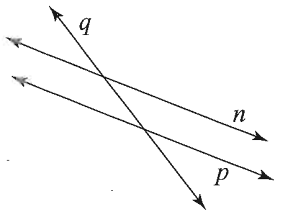
Text B

ACTIVITY

Explorations

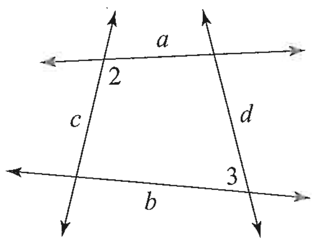
Your construction will look similar to the drawing of lines j, k, and m.

1. Use geometry software to construct a pair of intersecting lines along with a transversal. Measure the angles in your figure.
   a. Move the transversal while the other lines remain fixed. What invariants can you find? Look especially for pairs of angles that remain equal and for pairs that have a constant sum.
   b. Try moving one of the intersecting lines while the transversal stays fixed. Record and explain what you find.
2. Construct a pair of parallel lines with a transversal.

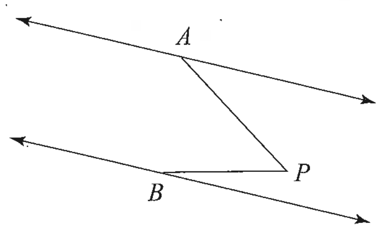


   a. Which angles stay equal when the transversal moves, or one of the lines moves while remaining parallel?
   b. What angle sums are invariant? Compare these results to your findings in Problem **1.**
3. Construct a figure with two parallel lines (like a and b) and two transversals (like c and d).

In the illustration here, lines a and b were drawn carelessly, and clearly are not parallel.

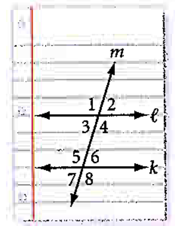


Angles 2 and 3 are not necessarily equal in measure. In the picture above, they are certainly not equal. Move the lines around to make $m\angle 2 = m\angle 3$.
   a. When $m\angle 2 = m\angle 3$, do lines c and d have any special relationship? Is that relationship invariant over changes in the equal measure of $\angle 2$ and $\angle 3$?
   b. If lines c and d are parallel, do the measures of $\angle 2$ and $\angle 3$ have an invariant relationship?
   c. How would your answers to the previous two parts of this problem be different if lines a and b were not required to be parallel?
4. Construct a pair of parallel lines with a movable point P between them. Draw $\overline{PA}$ and $\overline{PB}$ to connect the point to the parallel lines.
   a. What invariants can you find in this situation?
   b. In the figure below, what would you describe as transversals, alternate interior angles, and corresponding angles?



Text C

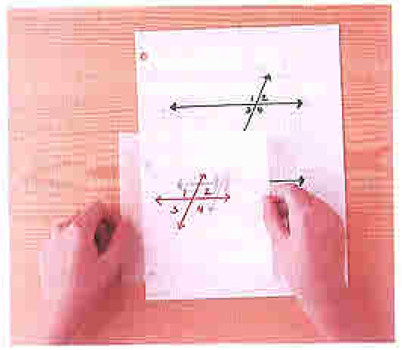
Investigation 1
Which Angles Are Congruent?



Using the lines on your paper as a guide, draw a pair of parallel lines. Or use both edges of your ruler or straightedge to create parallel lines. Label them k and ℓ. Now draw a transversal that intersects the parallel lines. Label the transversal m, and label the angles with numbers, as shown at right.

nal)

Step 1 Place a piece of patty paper over the set of angles 1, 2, 3, and 4. Copy the two intersecting lines m and ℓ and the four angles onto the patty paper.

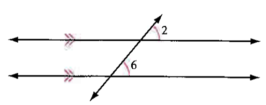


Step 2 Slide the patty paper down to the intersection of lines m and k, and compare angles 1 through 4 with each of the corresponding angles 5 through 8. What is the relationship between corresponding angles? Alternate interior angles? Alternate exterior angles?

Compare your results with the results of others in your group and complete the three conjectures below.

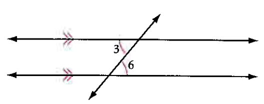
Corresponding Angles Conjecture, or CA Conjecture C-3a

If two parallel lines are cut by a transversal, then corresponding angles are _?_.



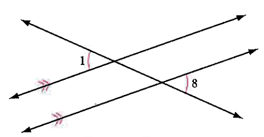
Alternate Interior Angles Conjecture, or AIA Conjecture C-3b

If two parallel lines are cut by a transversal, then alternate interior angles are _?_.



Alternate Exterior Angles Conjecture, or AEA Conjecture C-3c

If two parallel lines are cut by a transversal, then alternate exterior angles are _?_.

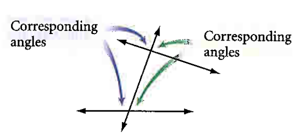


The three conjectures you wrote can all be combined to create a Parallel Lines Conjecture, which is really three conjectures in one.

Parallel Lines Conjecture C-3

If two parallel lines are cut by a transversal, then corresponding angles are _?_, alternate interior angles are _?_, and alternate exterior angles are _?_.

Step 3 What happens if the lines you start with are not parallel? Check whether your conjectures will work with nonparallel lines.

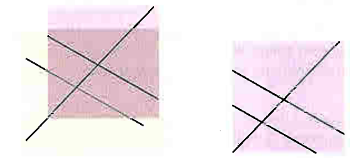


Text D

What about the converse of each of your conjectures? Suppose you know that a pair of corresponding angles, or alternate interior angles, is congruent. Will the lines be parallel? Is it possible for the angles to be congruent but for the lines not to be parallel?

Investigation 2
Is the Converse True?

You will need
- a straightedge
- patty paper
- a protractor *(optional)*

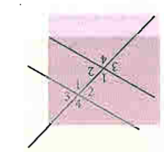


Step 1 Draw two intersecting lines on your paper. Copy these lines onto a piece of patty paper. Because you copied the angles, the two sets of angles are congruent.

Slide the top copy so that the transversal stays lined up.

Trace the lines and the angles from the bottom original onto the patty paper again. When you do this, you are constructing sets of congruent corresponding angles. Mark the congruent angles.

Are the two lines parallel? You can test to see if the distance between the two lines remains the same, which guarantees that they will never meet.

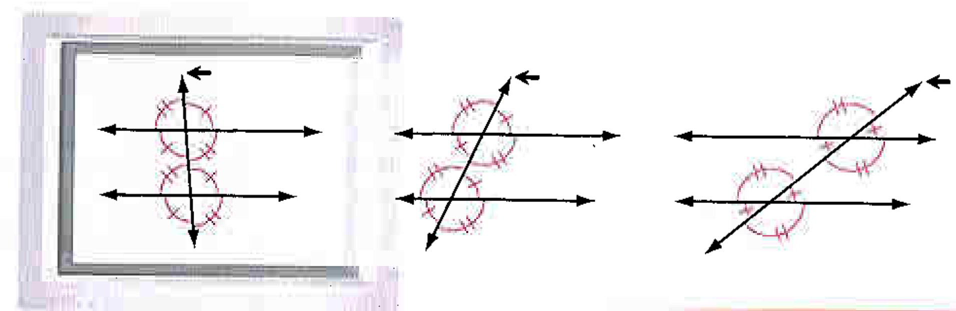


Step 2 Repeat Step 1, but this time rotate your patty paper 180° so that the transversal lines up again. What kinds of congruent angles have you created? Trace the lines and angles and mark the congruent angles. Are the lines parallel? Check them.

Step 3 Compare your results with those of your group. If your results do not agree, discuss them until you have convinced each other. Complete the conjecture below and add it to your conjecture list.

Converse of the Parallel Lines Conjecture C-4

If two lines are cut by a transversal to form pairs of congruent corresponding angles, congruent alternate interior angles, or congruent alternate exterior angles, then the lines are _?_.



[► For an interactive version of both investigations, see the **Dynamic Geometry Exploration** Special Angles on Parallel Lines at www.keymath.com/DG. ◄]

3-2 Technology LAB

Explore Parallel Lines and Transversals

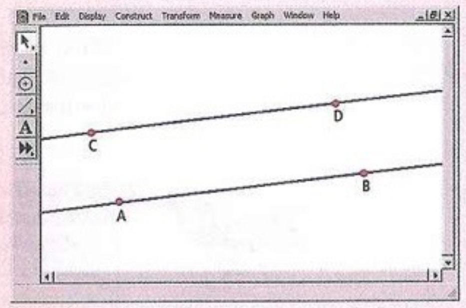
Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

Use with Lesson 3-2

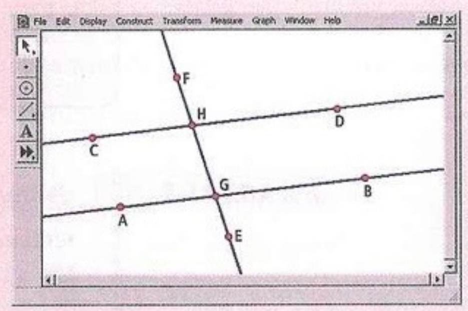
go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab3

Activity

1. Construct a line and label two points on the line A and B.



2. Create point C not on $\overleftrightarrow{AB}$. Construct a line parallel to $\overleftrightarrow{AB}$ through point C. Create another point on this line and label it D.



3. Create two points outside the two parallel lines and label them E and F. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection G and H.

4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point E or F and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

Angle	$\angle AGE$	$\angle BGE$	$\angle AGH$	$\angle BGH$	$\angle CHG$	$\angle DHG$	$\angle CHF$	$\angle DHF$
Measure								
Measure								

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.
2. Repeat steps in the previous problem for alternate interior angles, alternate exterior angles, and same-side interior angles.
3. Try dragging point C to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?

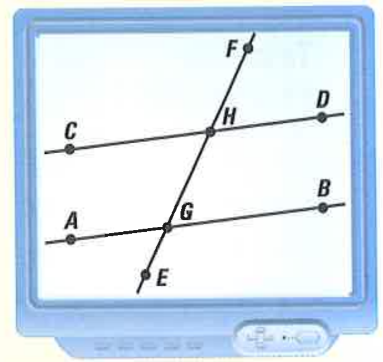
Text E

Parallel Lines and Angles

You can use geometry software to explore the properties of parallel lines.

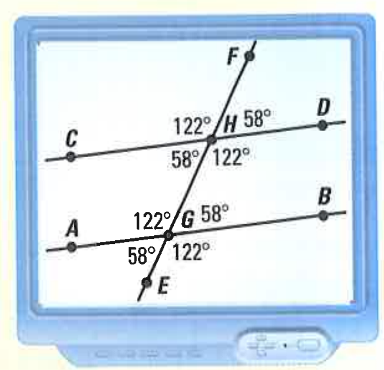
STUDENT HELP
SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

CONSTRUCT



1. Draw two points. Label them A and B. Draw $\overleftrightarrow{AB}$.
2. Draw a point not on $\overleftrightarrow{AB}$. Label it C.
3. Use your software's *construct parallel line* feature to construct a line through C parallel to $\overleftrightarrow{AB}$.
4. Draw a point on the line you constructed. Label it D. Move A, B, C, and D to the edges of the screen, as shown.
5. Draw two points outside the parallel lines. Label them E and F. Draw transversal $\overleftrightarrow{EF}$.
6. Find the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$. Label it G. Find the intersection of $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$. Label it H.

INVESTIGATE



1. Measure all eight angles formed by the three lines. What do you notice?

2. Drag point E or F to change the angle the transversal makes with the parallel lines. Be sure E and F stay outside the parallel lines. What do you notice?

MAKE A CONJECTURE

3. Make a conjecture about the measures of corresponding angles when two parallel lines are cut by a transversal.

4. Make a conjecture about the measures of alternate interior angles when two parallel lines are cut by a transversal.

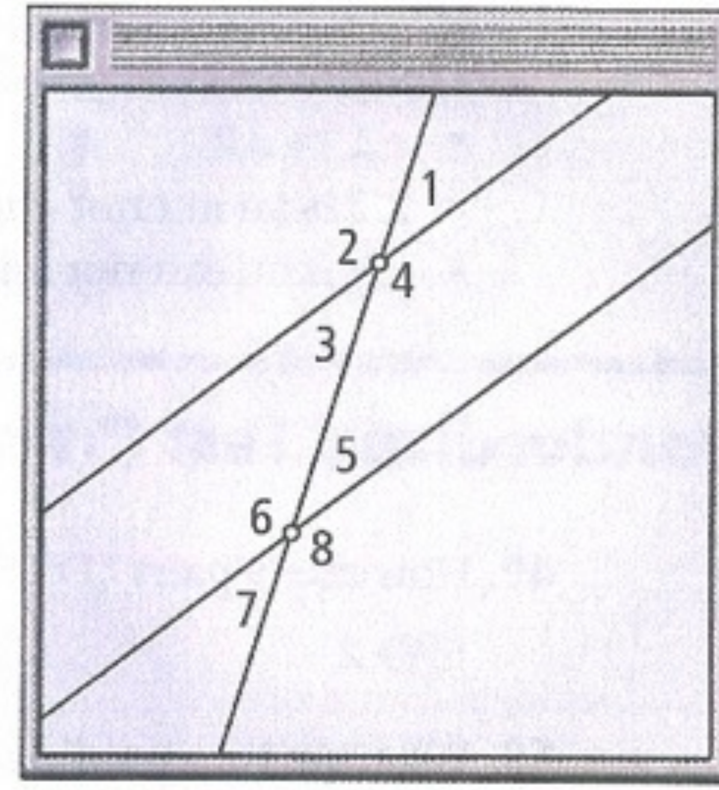
EXTENSION

CRITICAL THINKING Calculate the sum of two consecutive interior angles. Make and test a conjecture about the sum.

Text F

Activity

Use geometry software to construct two parallel lines. Check that the lines remain parallel as you manipulate them. Construct a point on each line. Then construct the transversal through these two points.



1. Measure each of the eight angles formed by the parallel lines and the transversal. Record the measurements.

2. Manipulate the lines. Record the new measurements.

3. When a transversal intersects parallel lines, what are the relationships among the angle pairs formed? Make as many conjectures as possible.

Text G

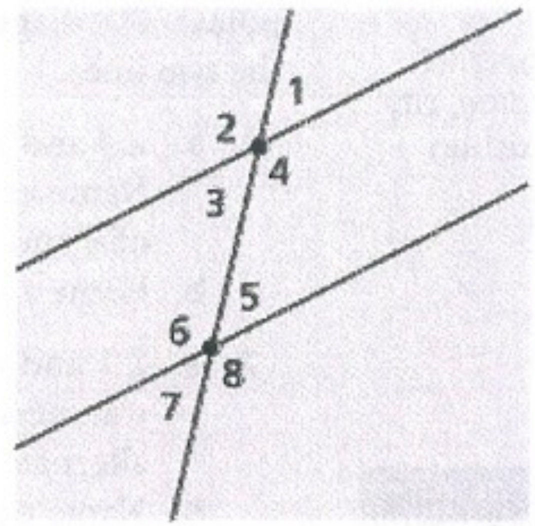
Exploring Parallel Lines and Related Angles

Before Lesson 7-1

Work in pairs or small groups.

Construct

Use geometry software to construct two parallel lines. Make sure that the lines remain parallel when you manipulate them. Construct a point on each line. Then construct the line through these two points. This line is called a *transversal.*



Investigate

Measure each of the eight angles formed by the parallel lines and the transversal and record the measurements. Manipulate the lines and record the new measurements. What relationships do you notice?

Conjecture

When two parallel lines are intersected by a transversal, what are the relationships among the angles formed? Make as many **conjectures** as possible.

Text H

Lesson 3-6 Parallel Lines

Vocabulary

transversal

corresponding angles

BIG IDEA When a line intersects two or more parallel lines, the angles that are formed have equal measures.

Lines have *tilt*. The tilt of a line can be measured by the angle it makes with some line of reference. It is often convenient to make this reference line horizontal. A road might be described as having a *grade* of 7°.

Mental Math

What is the measure of

a. the complement of a 36° angle?

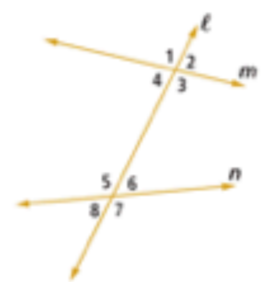
b. the supplement of a 36° angle?

c. the complement of a $p°$ angle?

d. the supplement of a $p°$ angle?

Corresponding Angles

Consider the angles formed when two lines m and n are intersected by a third line ℓ, called a **transversal**. We say that the transversal "cuts" the lines. Eight angles are formed: four by m and the transversal, four by n and the transversal. Any pair of angles in similar locations with respect to the transversal and each line is called a pair of **corresponding angles**. In the drawing, angles 1 and 5 are corresponding angles because both angles are to the left of the transversal and above lines m and line n, respectively. The pairs of angles 2 and 6, 3 and 7, and 4 and 8 are also corresponding angles.



Recall the definition of parallel lines: Two coplanar lines are *parallel* if and only if they are the same line or they do not intersect. In symbols, $(\ell \parallel m) \Leftrightarrow (\ell = m \text{ or } \ell \cap m = \emptyset)$. We say that segments or rays are parallel if the lines containing them are parallel.

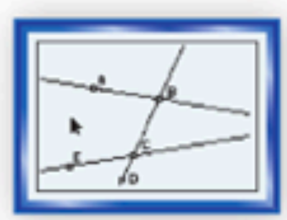
Activity 1

MATERIALS DGS

Step 1 Start with a clear DGS screen. Construct two nonparallel lines that do not intersect on the screen.

Step 2 Construct a transversal and its points of intersection with the two lines in Step 1.

(continued on next page)



Step 3 Label the points A, B, C, D, and E as in the figure at the bottom of previous page.

Step 4 Measure the corresponding angles $\angle ABC$ and $\angle ECD$.

Step 5 Drag the line $\overleftrightarrow{AB}$, the line $\overleftrightarrow{EC}$, or the transversal $\overleftrightarrow{DC}$ to change the tilt of each of these. Try to drag the lines so that measured angles have equal measurement.

Step 6 Make a conjecture about the measures of the angles $\angle ABC$ and $\angle ECD$ and the relationship between lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{EC}$.

The results of Activity 1 might make you think that if two lines have the same tilt, that is, if they make corresponding angles with equal measures, then the lines are parallel. Yet, on Earth, two north-south streets make the same angle with any east-west street, but they would intersect at the North Pole (and South Pole!) if extended. Assumptions about parallel lines are needed to ensure that the geometry you are studying is not the geometry of the curved surface of Earth. We make the following assumption.

Corresponding Angles Postulate

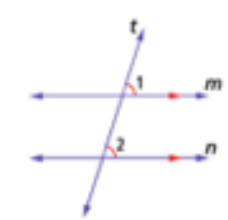
Suppose two coplanar lines are cut by a transversal.

a. If two corresponding angles have the same measure, then the lines are parallel.
Abbreviation: (measures of corr. ∠s =) ⇒ (∥ lines)

b. If the lines are parallel, then corresponding angles have the same measure.
Abbreviation: (∥ lines) ⇒ (measures of corr. ∠s =)

Segments or rays are considered to be parallel if the lines containing them are parallel. For instance, by Part a of the Corresponding Angles Postulate, if segments are drawn that, together with line ℓ, create angles with equal measures, then the segments are parallel. Palm fronds show many parallel segments as shown at the right.

In the figure at the right, the small red arrows on lines m and n indicate that m and n are parallel. By Part b of the Corresponding Angles Postulate, if $m \parallel n$, then $m\angle 1 = m\angle 2$.



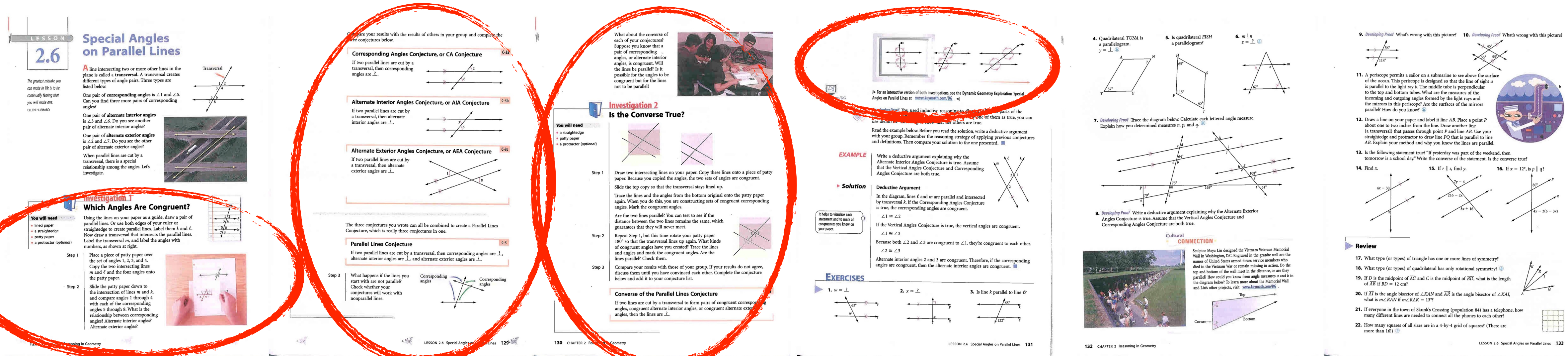
Properties of linear pairs and vertical angles can be used with the Corresponding Angles Postulate to determine the measures of angles formed by parallel lines. In the solution to Example 1, we mention the geometric justifications.

Text I

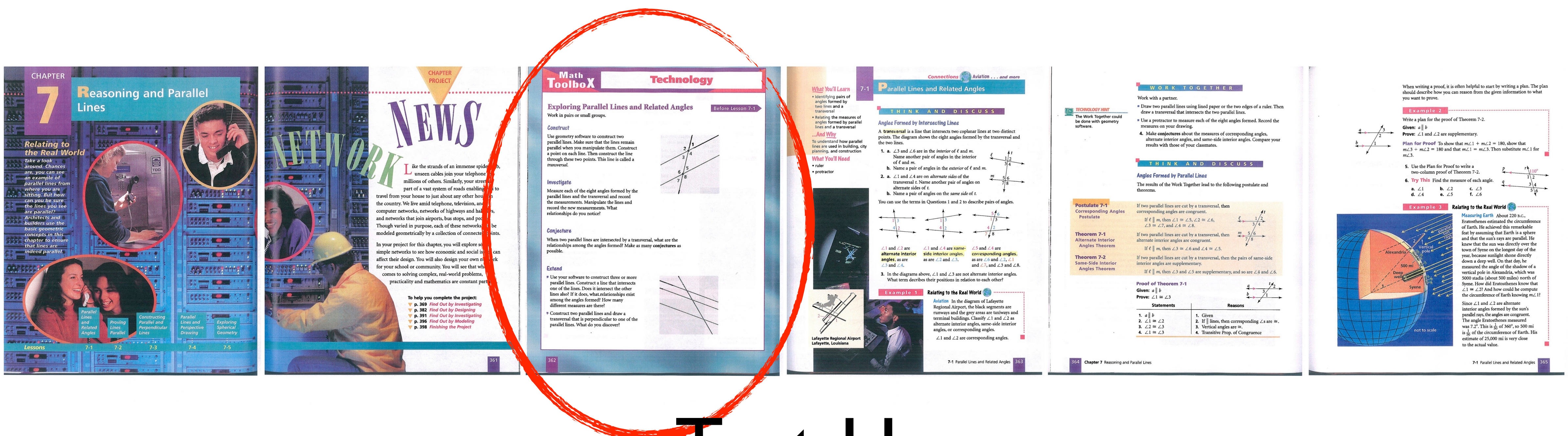
Embedded/ Supplemental

- Embedded - Activity occurs within the lesson of a textbook.
- Supplemental - Activity occurs outside of the lesson of a textbook.

Embedded/Supplemental



Text D



Text H

Essential/Not Essential

- Essential - The mathematical content of the activity is only described within the particular activity
- Not Essential - The mathematical content of the activity is described again outside of the activity

Embedded/Essential

	Embedded	Supplemental
Essential		
Not Essential		

Technology Required/ Technology Not Required

- Technology Required - According to the directions of the activity, the activity is to be completed with interactive geometry software.
- Technology Not Required - According to the directions of the activity, the activity may be completed with or without interactive geometry software.

Technology Required

For You to Do

1. Use geometry software to construct two parallel lines and a moveable transversal. Measure all the angles and note any two angles that have the same measure and any two angles with measures that add to 180°.

   Move the transversal. Check which of the equalities found above is invariant. Is there a state of the sketch in which all the angles have the same measure?

Text B

EXPLORE: PARALLEL CROSSING

Use geometry software or the opposite edges of a straightedge to draw two parallel lines. Draw a transversal through the two lines.

Measure the various angles, and make as many conjectures as you can about the angles that are congruent and supplementary in your figure. (You may want to check to be sure that your conjectures hold for the calligraphy guidesheet at the right.)

MATERIALS

Protractor, Straightedge
Geometry software (optional)

Text A

Focused/Open

- Focused - Students are expected to notice specific patterns to complete the activity.
- Open - Students are expected to make observations about any patterns to complete the activity.

Ends Specified/ Ends Not Specified

Activity

Use geometry software to construct two parallel lines. Check that the lines remain parallel as you manipulate them. Construct a point on each line. Then construct the transversal through these two points.

1. Measure each of the eight angles formed by the parallel lines and the transversal. Record the measurements.
2. Manipulate the lines. Record the new measurements.
3. When a transversal intersects parallel lines, what are the relationships among the angle pairs formed? Make as many conjectures as possible.

Text G

3-2 Technology LAB

Explore Parallel Lines and Transversals

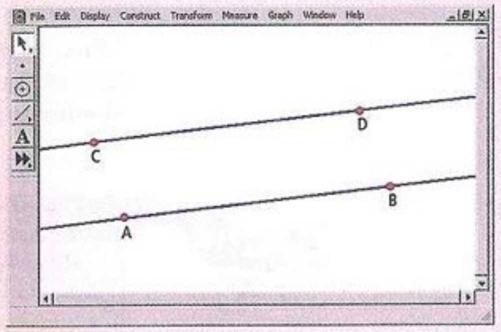
Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

Use with Lesson 3-2

go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab3

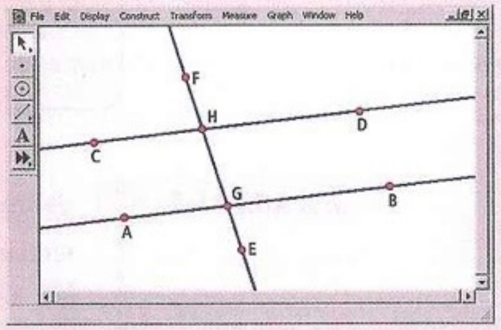
Activity

1. Construct a line and label two points on the line A and B.



2. Create point C not on $\overleftrightarrow{AB}$. Construct a line parallel to $\overleftrightarrow{AB}$ through point C. Create another point on this line and label it D.

3. Create two points outside the two parallel lines and label them E and F. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection G and H.



4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point E or F and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

Angle	$\angle AGE$	$\angle BGE$	$\angle AGH$	$\angle BGH$	$\angle CHG$	$\angle DHG$	$\angle CHF$	$\angle DHF$
Measure								
Measure								

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.
2. Repeat steps in the previous problem for alternate interior angles, alternate exterior angles, and same-side interior angles.
3. Try dragging point C to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?

Text E

Details of Construction

Activity

Use geometry software to construct two parallel lines. Check that the lines remain parallel as you manipulate them. Construct a point on each line. Then construct the transversal through these two points.

1. Measure each of the eight angles formed by the parallel lines and the transversal. Record the measurements.
2. Manipulate the lines. Record the new measurements.
3. When a transversal intersects parallel lines, what are the relationships among the angle pairs formed? Make as many conjectures as possible.

Text G

Text E

3-2 Technology LAB

Use with Lesson 3-2

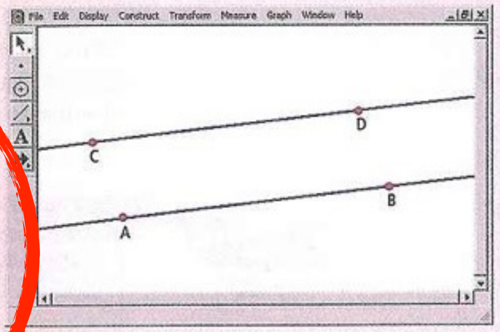
Explore Parallel Lines and Transversals

Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

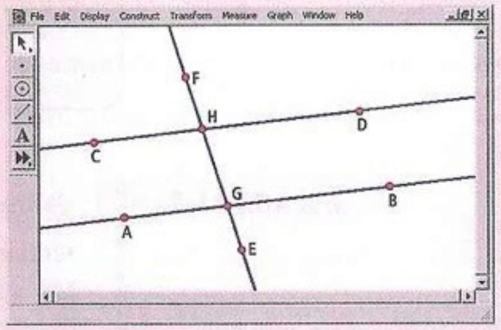
go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab3

Activity

1. Construct a line and label two points on the line A and B.



2. Create point C not on $\overleftrightarrow{AB}$. Construct a line parallel to $\overleftrightarrow{AB}$ through point C. Create another point on this line and label it D.
3. Create two points outside the two parallel lines and label them E and F. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection G and H.



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Angle	$\angle AGE$	$\angle BGE$	$\angle AGH$	$\angle BGH$	$\angle CHG$	$\angle DHG$	$\angle CHF$	$\angle DHF$
Measure								
Measure								

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.
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3. Try dragging point C to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?

A Stated Purpose of the Activity

3-2 Technology LAB

Explore Parallel Lines and Transversals

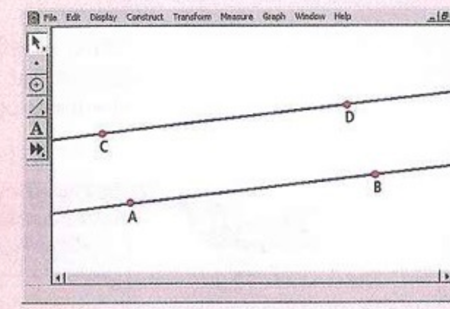
Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

Use with Lesson 3-2

go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab3

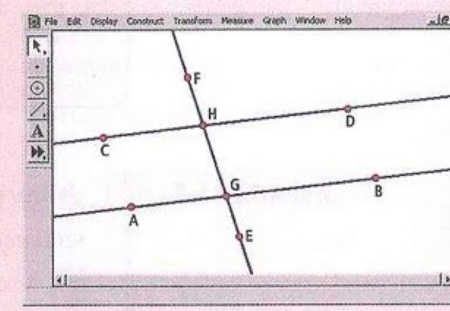
Activity

1. Construct a line and label two points on the line *A* and *B*.



2. Create point *C* not on $\overleftrightarrow{AB}$. Construct a line parallel to $\overleftrightarrow{AB}$ through point *C*. Create another point on this line and label it *D*.

3. Create two points outside the two parallel lines and label them *E* and *F*. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection *G* and *H*.



4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point *E* or *F* and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

Angle	∠AGE	∠BGE	∠AGH	∠BGH	∠CHG	∠DHG	∠CHF	∠DHF
Measure								
Measure								

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.
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Text E

Evaluating and Writing Dynamic Geometry Tasks

- The Mathematics Teacher, Vol. 107, No. 9 (May 2014), pp. 701-705
- Frameworks for evaluating the <u>mathematical depth</u> and <u>technological action</u> of dynamic geometry tasks

CONNECTING RESEARCH to teaching

Aaron Trocki

Evaluating and Writing Dynamic Geometry Tasks

The advent of dynamic geometry software has changed the way students draw, construct, and measure by using virtual tools instead of or along with physical tools. Use of technology in general and of dynamic geometry in particular has gained traction in mathematics education, as evidenced in the Common Core State Standards for Mathematics (CCSSI 2010).

Research has shown the potential benefit of using technology, particularly dynamic geometry tasks, to promote mathematical reasoning. The Common Core State Standards for Mathematics (CCSSM) high school geometry standards require students to "make formal geometric constructions with a variety of tools and methods" (p. 76), and dynamic geometry software is among the tools listed. Further, Hollebrands (2003) has noted that within such dynamic environments students have opportunities to consider invariant relationships through dragging as well as make corresponding conjectures and conclusions.

However, little guidance is provided to teachers for evaluating the quality of dynamic geometry tasks, much less for writing their own. The purpose of this article is to introduce a framework for analyzing and writing dynamic geometry tasks that are designed to engage students in mathematical reasoning. We begin by asking readers to compare two sample tasks, each of which is designed to engage students in developing and testing conjectures about parallelograms. We then introduce the framework and illustrate how it can be used to evaluate the potential of each task in accomplishing the desired result.

Connecting Research to Teaching appears in alternate issues of *Mathematics Teacher* and brings research insights and findings to the journal's readers. Manuscripts for the department should be submitted via http://mt.msubmit.net. For more information on the department and guidelines for submitting a manuscript, please visit http://www.nctm.org/publications/content.aspx?id=10440#connecting.

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TWO TASKS TO COMPARE

Consider parallelogram tasks 1 and 2 (see **figs. 1** and **2**), where students are provided with a prepared sketch in the dynamic geometry environment. Quadrilateral *ABCD* is constructed to be a parallelogram—that is, no matter what students do by dragging edges or vertices, *ABCD* will always be a parallelogram. Each task therefore possesses *mathematical fidelity*, a guarantee that the objects in the sketch are what they are claimed to be. Without mathematical fidelity, the remainder of the task may become meaningless.

Read each task and consider this question: Which task contains prompts that encourage students to experiment and build their mathematical understanding? Which task would you consider using in the classroom?

A PROFESSIONAL DEVELOPMENT PROJECT

The impetus for this work originated in a professional development project in which the following research question was addressed: How do the design and use of tasks that incorporate dynamic

Mathematical Depth

Allowance for Mathematical Depth

Hierarchical Levels	Descriptions
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a mathematics fact, rule, formula, or definition.
2	Prompt requires student to report information from the construction. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.

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- 4 - Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
- 5 - Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.

Technological Action

- N/A - Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
- A - Prompt requires drawing within current sketch.
- B - Prompt requires measurement within current sketch.
- C - Prompt requires construction within current sketch.
- D - Prompt requires dragging or use of other dynamic aspects of the sketch.
- E - Prompt requires the creation or consideration of multiple examples from which one can generalize.
- F - Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
- G - Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise. (Adapted from Sinclair [2003, p. 312])

Types of Technological Action

Affordances	Descriptions
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
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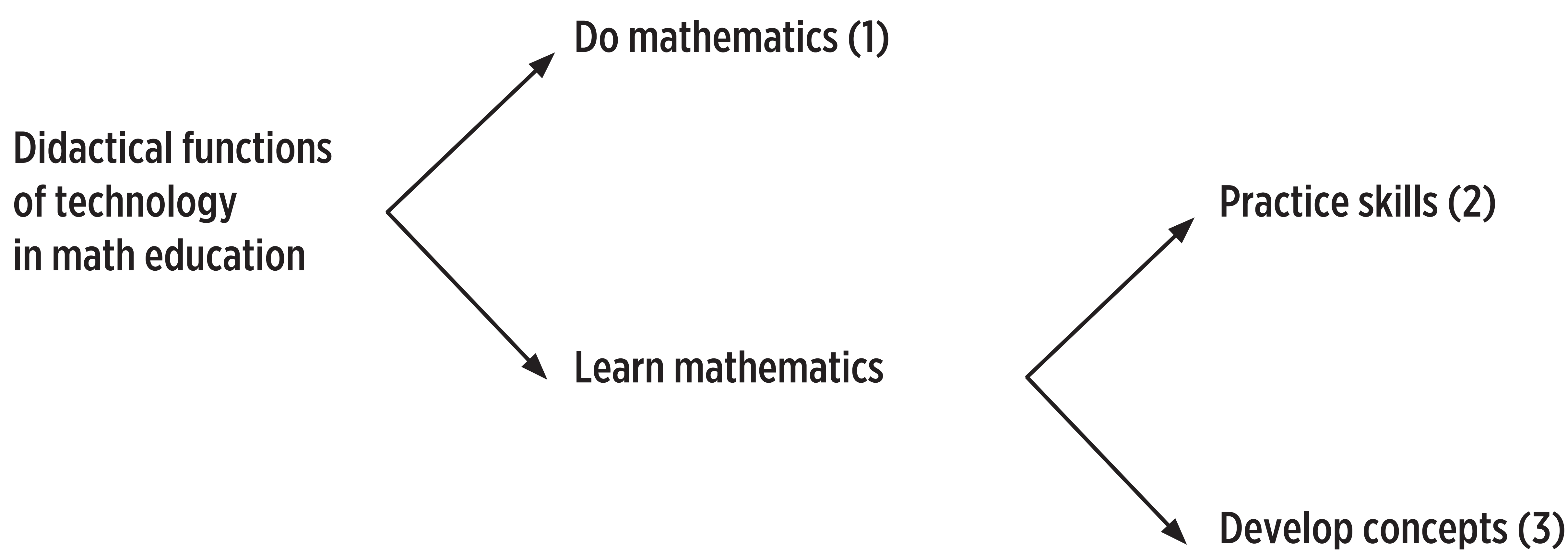
Replacing, Amplifying, Transforming (RAT) Framework

- Focuses on whether the technology….
  - *replaces* a similar presentation without the technology
  - *amplifies* the learning process that was present in the non-technology version
  - *transforms* the learning experiences to provide possibilities that were otherwise not possible without the technology
- From Hughes, Thomas, & Scharber (2006)



Three purposes for technology in mathematics education.
(Drijvers, 2012)

Questions to Consider

- As teachers, what impacts your decisions as to whether to use technology in your classroom?
- What do you look for in a "good" technology task/activity?
- How do you think about designing and evaluating technology tasks for your students?
- How could textbooks/curriculum better support teachers and to use technological tools?
- How do you help students recognize the affordances and limitations of technology?

Research on Technology in Mathematics Education

Zbiek, Heid, Blume, & Dick (2006)

- “major limitations of computer use in the coming decades are likely to be less a result of technology limitations than a result of limited human imagination and the constraints of old habits and social structures” (Kaput, 1992)

Thank You!

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