



Metamorphosis
TEACHING LEARNING COMMUNITIES

A Model For All Seasons:

Developing
Computational Fluency
with the Bead String

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A Model For All Seasons: Developing Computational Fluency with the Bead String

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Developing Computational Fluency with the Bead String

Framing Question:

What do we mean by computational fluency?



Addition/Subtraction Fluency and the CCSS

K.OA.5	Fluently add and subtract within 5.
1.OA.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.
2.OA.2 2.NBT.5	<p>Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.</p> <p>Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>
3.NBT.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.



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Framing Question:

How do we develop fluency in addition and subtraction?



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Our question:

How do kids come to structure the number system so that they deeply understand the relationship between addition and subtraction?



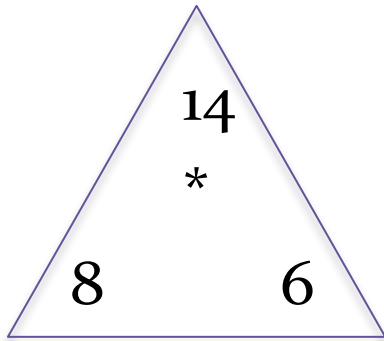
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Think about your own curriculum materials.

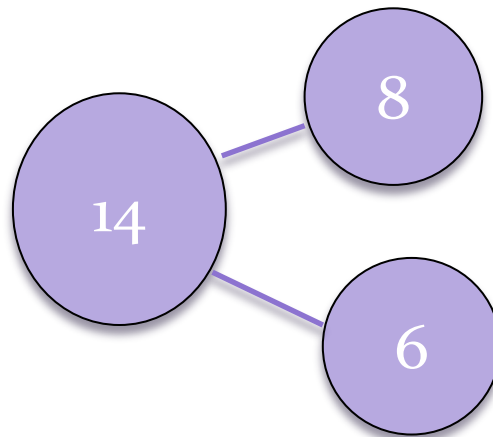
What mathematical models does the curriculum use to develop fluency in addition and subtraction?



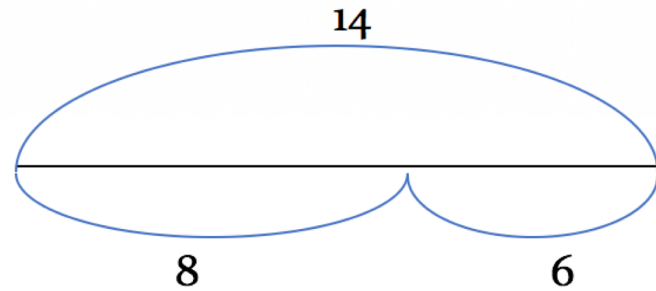
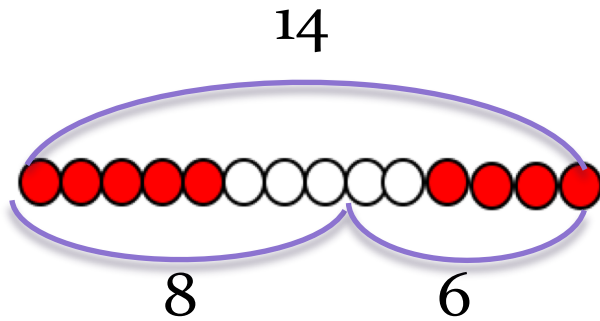
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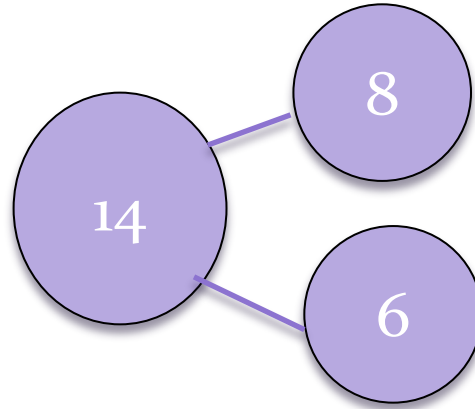
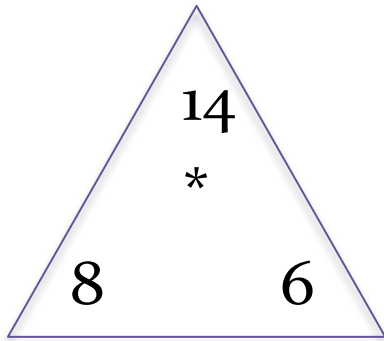
14	
8	6



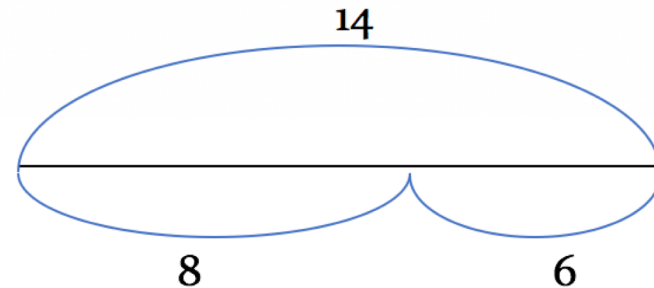
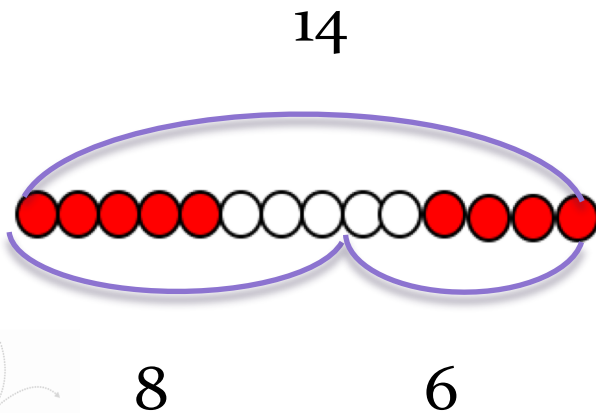
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14	
8	6



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Dynamic Models vs. Static Models

*A **dynamic model** accounts for time-dependent changes in the state of the system, while a **static** (or steady-state) **model** calculates the system in equilibrium, and thus is time-invariant.*



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Our question:

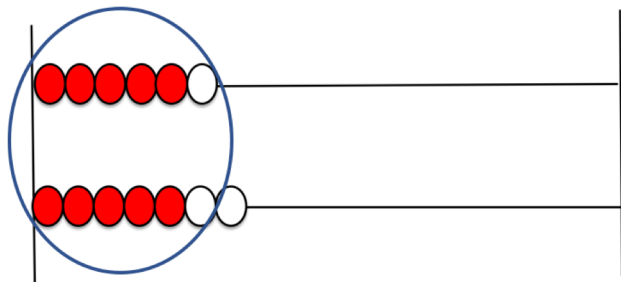
How do kids come to structure the number system so that they deeply understand the relationship between addition and subtraction?



Mathematical Models to Support the Development of Addition and Subtraction

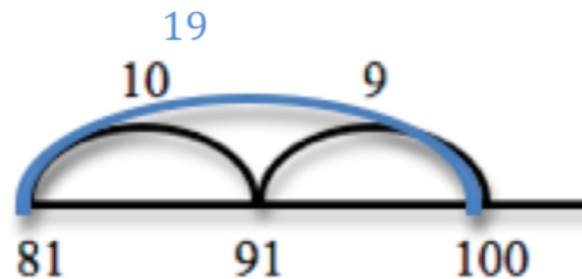
Rekenrek

$$6+7 = 6 + (6 + 1) = (6 + 6) + 1$$



Open Number Line

$$81+19 = 81+(10+9) = (81+10)+9$$



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The Bead String



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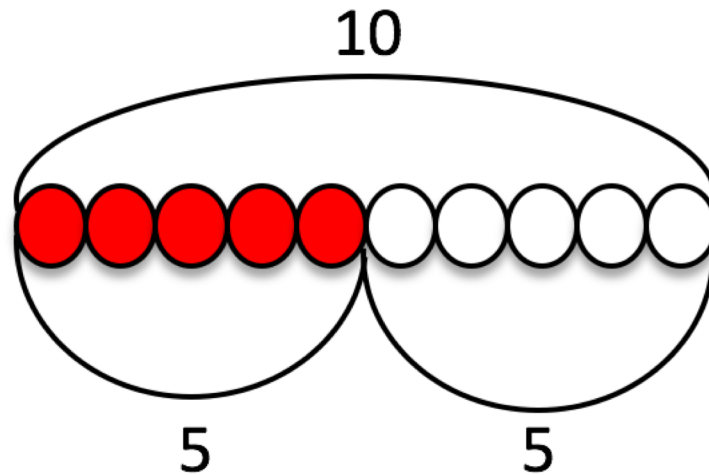
The Bead String



How might the bead string support the development of place value big ideas and part whole relationships?



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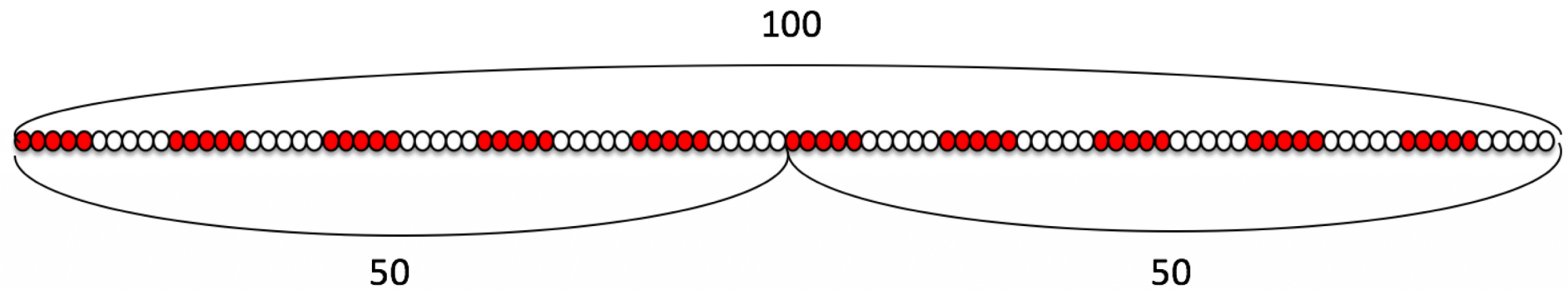
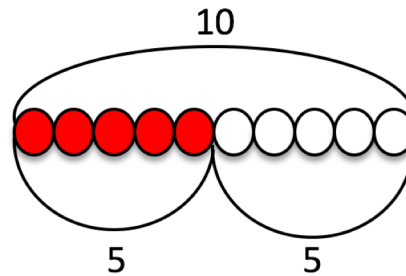
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How do students come to understand the relationship between

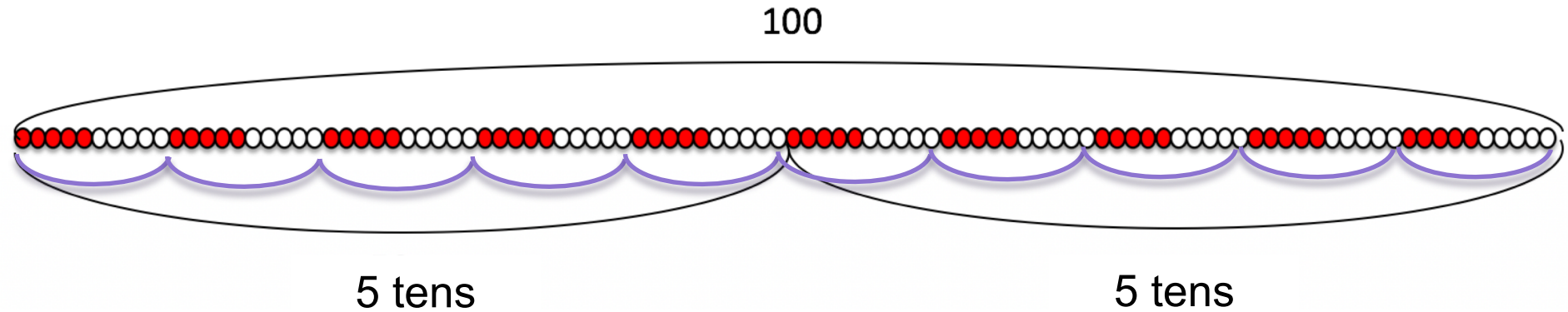
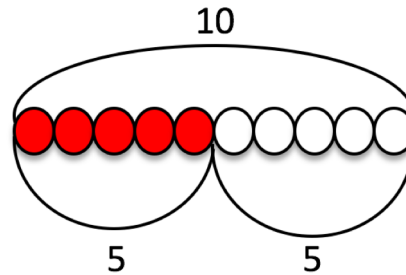
$$5 + 5 \quad \& \quad 50 + 50?$$



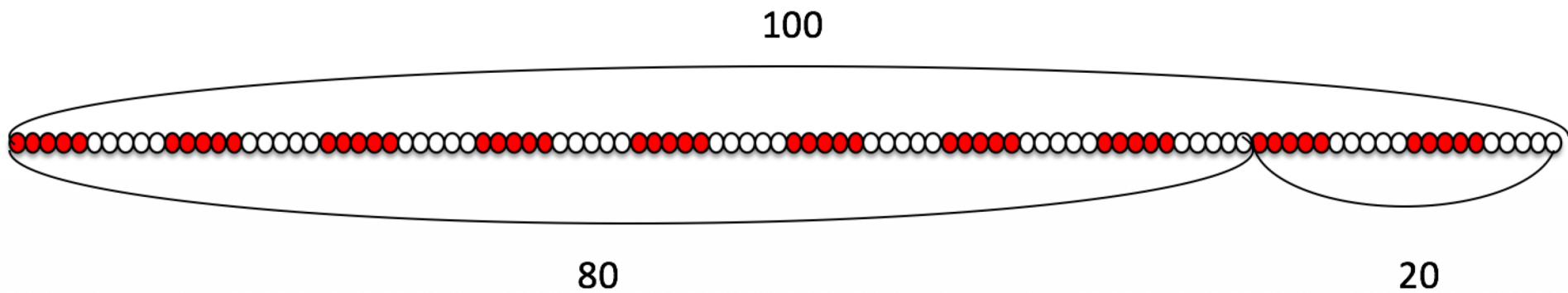
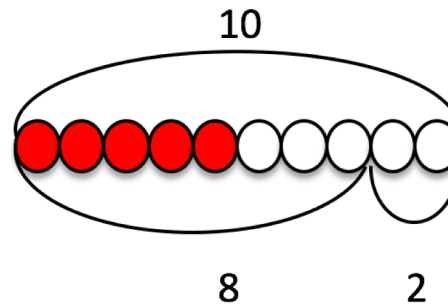
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How do students come to understand the relationship between

$$5 + 5 \quad \& \quad 6 + 4?$$



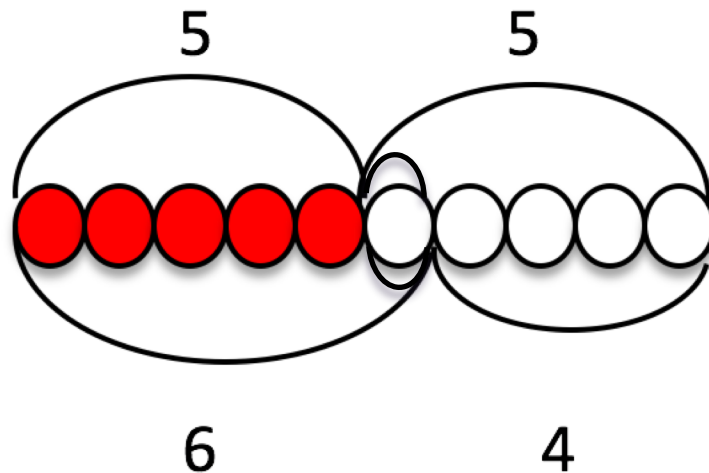
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$$5 + 5 = 6 + 4$$

$$5 + (1 + 4) = (5 + 1) + 4$$



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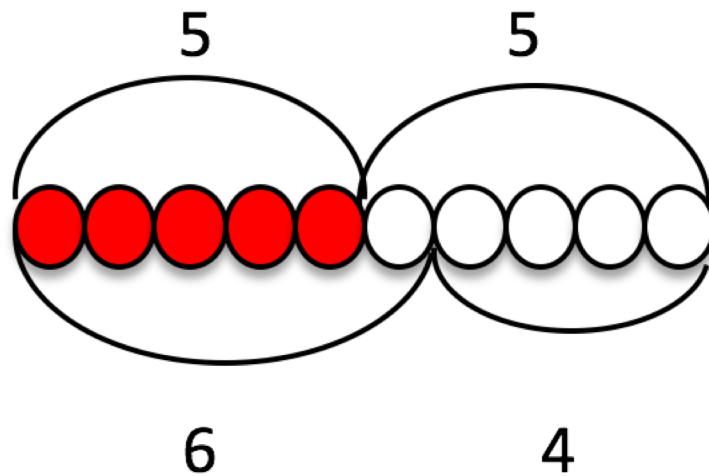


$$5 + 5 = 6 + 4$$

$$5 + (1 + 4) = (5 + 1) + 4$$



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Structuring within the base 10 system



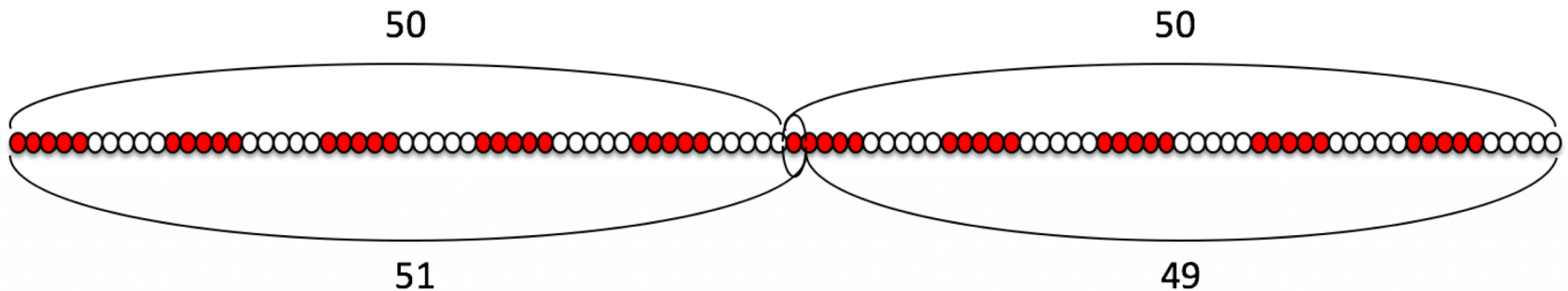
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How do students come to understand the relationship between

$$50 + 50 \quad \& \quad 51 + 49?$$



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$$50 + 50 = 51 + 49$$

$$50 + (1 + 49) = (50 + 1) + 49$$



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Structuring from the Building Blocks



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Using Tens: A Guided Mini-lesson

(students name combos of ten they know)

$$17+3$$

$$17+4$$

$$18+2$$

$$18+12$$

$$89+7$$



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How does knowing $7 + 3$ help
students think about $17 + 3$?



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As you watch a clip of this mini-lesson designed to develop students understanding of how to use their “tens” facts as a tool when adding two digit numbers, take notes on:

- How the bead string supports student sense making
- How Toni models student thinking



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1.	<u>Meechi</u> : $7 + 3 = 10$.
2.	T: Yes. $7 + 3$ equals 10, but what does it have to do with $17 + 3$?
3.	Luigi: Be--because if you know $7 + 3$ is 10, $10 + 10$ [inaudible]
4.	M: You put the one to the, to the, in the front.
5.	T: Okay, <u>Meechi</u> . I love that, I love that you're speaking, but wait—I love that you want to share—wait until he's done. Okay?
6.	Luigi: $7 + 3 = 10$. That's how you get the 10, in, in the 17.
7.	T: <u>So</u> here's what I hear you saying. [models with the bead string] Here's our 17 and here's our 3, and I hear you saying that inside 17 is 10 and 7 [models with the bead string] and then when you're going to add 3 to it ... and $7 + 3$...
8.	S: Is 10.
9.	T: ... there's your 10. Now altogether we have ...
10.	Lesley: 20.
11.	T: 20. How cool is that! <u>So</u> let's write this up here using numbers. [represents the problem numerically] So $17 + 3$, we can think about as being $10 + 7 + 3$, that's where you broke the 17 into a 10 and 7, and then we have, $10 + 7 + 3$. Those are our addends ... You with us? You might want to look at what I'm writing. And then you said, here's a 10 and guess what? Here's another 10. <u>So</u> 10 and 10 is ...
12.	S: 20.
13.	T: 20. Cool! So inside 17, you pulled out that 7 and you made a 10. That's pretty cool. And you used what you knew. All right. Here's another problem ...



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Building Structures to Connect Addition and Subtraction



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How Many Am I Hiding?

Materials:

10 bead rekenrek string or rekenrek with bottom row covered

Directions:

Player one moves some beads to the left, and hides the remaining beads on the right under his/her hand. Partner two says how many they see and how many are hiding.

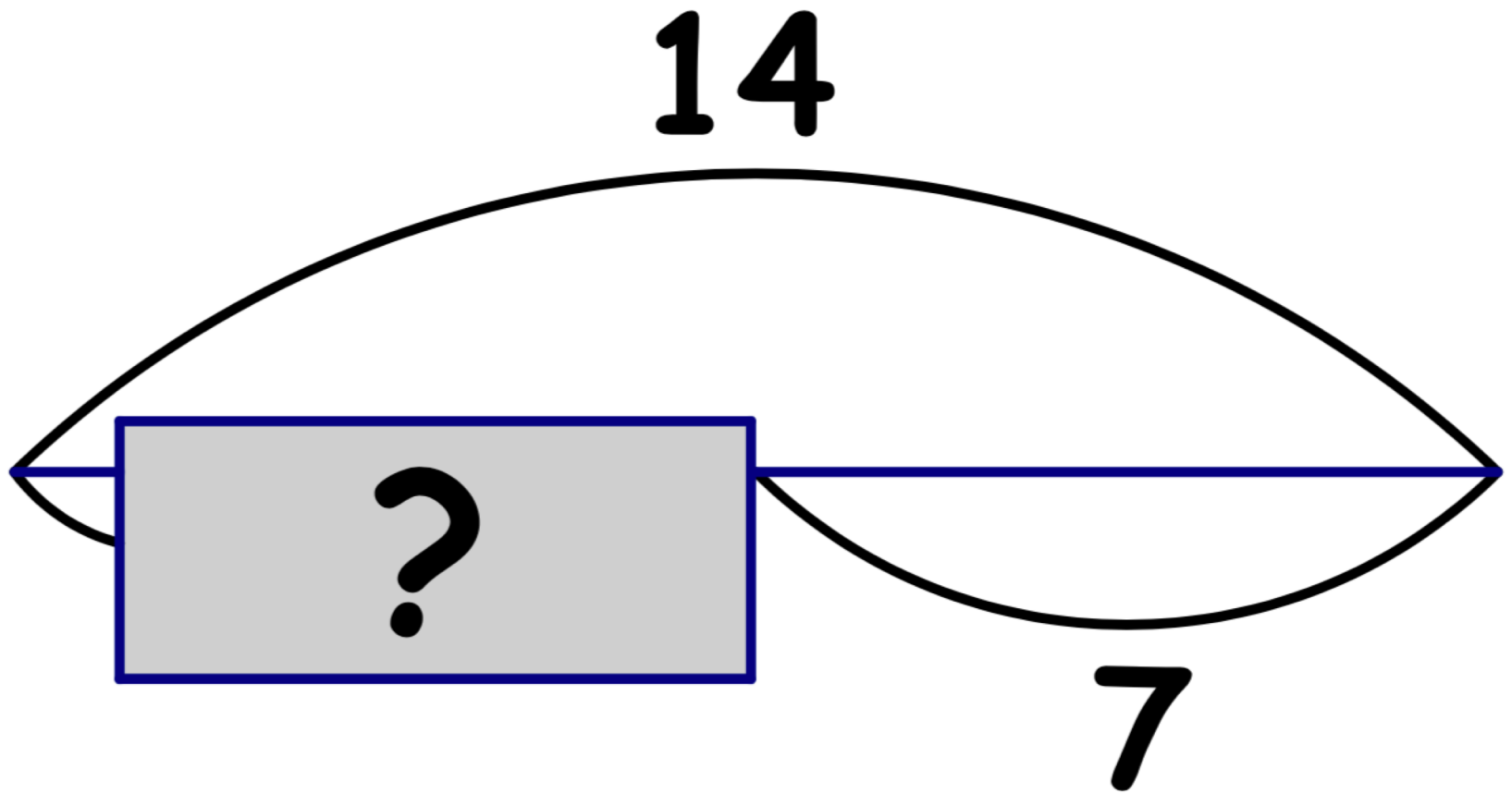


Example of student dialogue:

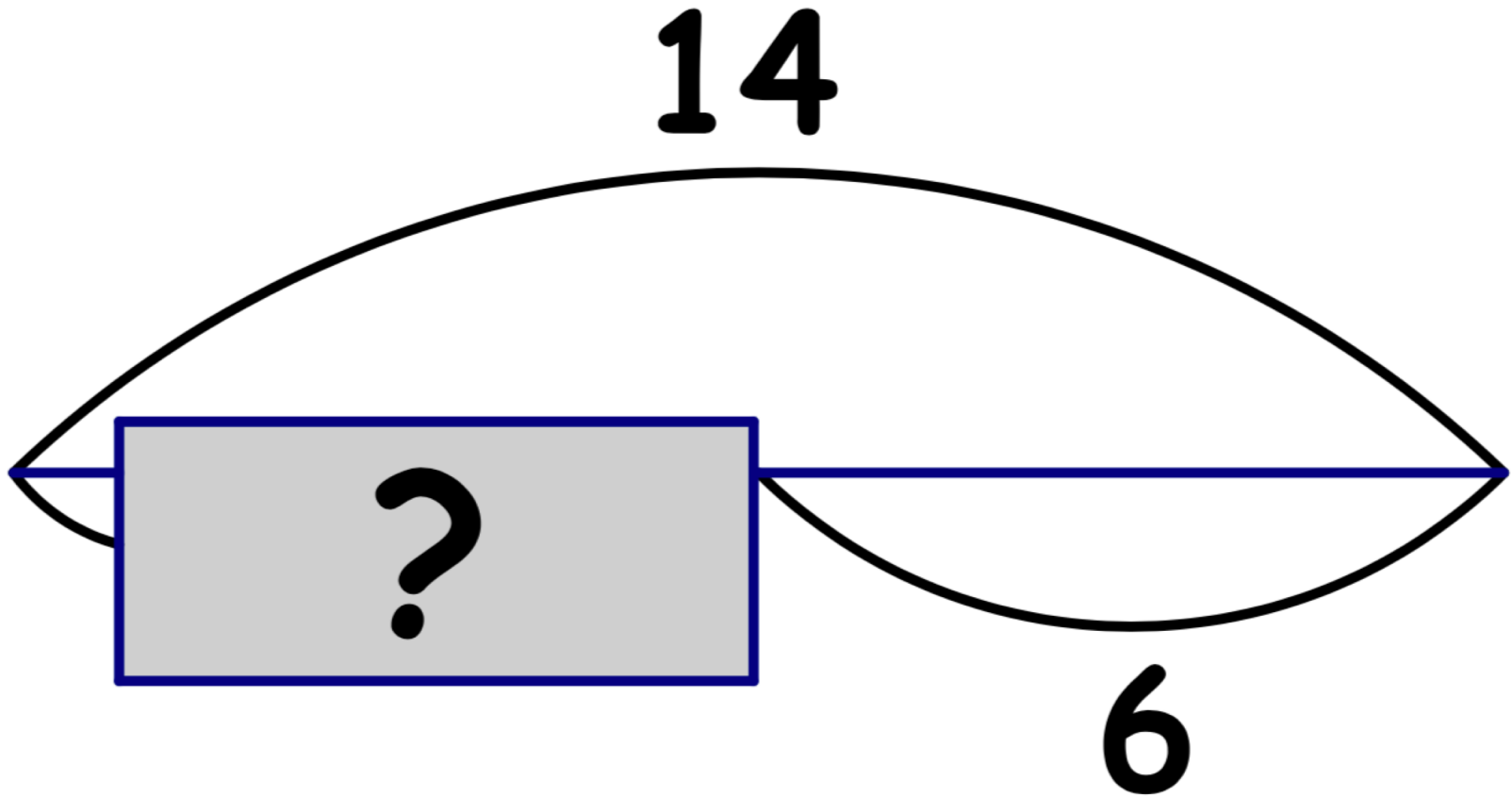
Player one says, "How many beads do you see? How many am I hiding?"



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Developing Computational Fluency with the Bead String



Developing Computational Fluency with the Bead String

As you watch a clip of this mini-lesson designed to develop students understanding of part/whole relations, take notes on:

- How the bead string supports student sense making
- How Toni models student thinking



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In Summary:

- The primary goal of teaching is to help kids structure mathematical big ideas.
- Models as tools to think with are critical to students' development of reasoning
- There's a big difference between teaching kids how to “do strategies” and how to use relationships.
- The bead string is a dynamic model that can help students structure the number system and reason with relationships.
- The bead string is a mathematical model that students come to *think with*



Developing Computational Fluency with the Bead String

- Thank you for coming! Join us at our other sessions:

Session Title	Time/Room
Routines to Grow Problem-Solving Strategies in Early Childhood	Friday, 8:00-9:15 Walter E. Washington Convention Center, 144 ABC
Interactive Early Algebra Puzzles for Young Learners: Free Web-Based Activities for Your Classroom	Friday, 9:45-11:00 Marriott Marquis, Independence Ballroom E (Level M4)
Enticing All Students to Contribute to Rich Math Discussions	Friday, 1:30-2:30 Marriott Marquis, Marquis Ballroom Salon 6 (Level M2)
Harnessing the Power of Mathematical Models to Re-Envision Early Childhood Routines	Friday, 3:15-4:30 Marriott Marquis, Marquis Ballroom Salon 9&10 (Level M2)
The Power of Mathematical Models: How to Build Multiplicative Reasoning in Diverse Populations	Friday, 3:15-4:30 Walter E. Washington Convention Center, 204 AB
Ignite Session	Friday, 6:00-7:00 Walter E. Washington Convention Center, Ballroom B

