

Teaching Mathematics Online Using Inquiry Methods

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**How do you get from
here to there?**









My Classroom

AUDIO & VIDEO

TalkVideo

PARTICIPANTS

MAIN ROOM (10)

Participant#2
Moderator

Participant#10

Participant#11

Participant#3

Participant#4

Participant#5

Participant#6

Participant#7

Participant#8

Participant#9

CHAT - Supervised

i actually did the same thing
elianna

Participant#88:09 AM

32 males and females, 12 of which are males

Participant#48:09 AM

nah that was a good warm up one. wasnt too logn but still required you to use your brain

- Participant#11 joined the Main Room. (4:11 PM) -

K12_2841809

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As we have seen, the law of cosines is an important tool in solving problems involving non-right triangles. A second law, known as the law of sines, will also prove useful.

When a (not-necessarily right) triangle is labeled as the one above, the law of sines states the following:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To put it another way, the ratio of the sine of an angle to the length of the opposite side is the same for all parts of the triangle.

What is Inquiry Based Learning (IBL)?

Inquiry-Based Learning (IBL) is an approach to teaching and learning in which the classroom environment is characterized by the student being the active participant while the teacher's role is decentralized.

<https://www.artofmathematics.org/blogs/jfleron/what-is-inquiry-based-learning>

Dr. Phil Hotchkiss and Dr. Julian Fleron

My Path to IBL

High School Geometry - The Choice



My Path to IBL

Topology - The Problems



Definition 13. A *topological space* (X, τ) is a set X and a family τ of subsets of X satisfying the following conditions:

1. the empty set $\emptyset$ and X are members of τ ;
2. if A and B are in τ , then $A \cap B$ is in τ ; and
3. if I is an indexing set, and A_δ is in τ for each $\delta \in I$, then $\bigcup_{\delta \in I} A_\delta$ is in τ .

The members of τ are called *open sets* and τ is called a *topology* on X .

Exercise 14. Find all the possible topologies on the set

$$X := \{a, b, c\}.$$

Exercise 15. Let X be any set. Show that the power set of X is a topology on X . We call this topology the *discrete topology* on X .

Exercise 16. Consider the collection

$$\tau := \{U \subseteq \mathbb{R} \mid U = \emptyset \text{ or } \mathbb{R} \setminus U \text{ is finite}\}$$

where $\mathbb{R}$ denotes the set of real numbers. Show τ defines a topology on $\mathbb{R}$. This is called the *finite complement topology*.

Definition 17. Let (X, τ) be a topological space. A subset A of X is called a *closed set* iff $X \setminus A$ is open.

Exercise 18. Let (X, τ) be a topological space. Suppose $A \subseteq X$.

My Path to IBL

$$2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}} - \text{The Moment}$$

$2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}}$
 $= \sqrt{\frac{8}{3}}$
 $= \frac{2\sqrt{2}}{\sqrt{3}}$ cool it works
 $= 2\sqrt{\frac{2}{3}}$

Are there more?

$3\sqrt{\frac{3}{5}} \neq \sqrt{3\frac{3}{5}} = \sqrt{\frac{18}{5}} = \frac{3\sqrt{2}}{\sqrt{5}}$
 $3\sqrt{\frac{4}{5}} \neq \sqrt{3\frac{4}{5}} = \sqrt{\frac{24}{5}} = \frac{2\sqrt{6}}{\sqrt{5}}$ too long

~~$\frac{2\sqrt{3}}{\sqrt{5}} = \sqrt{\frac{27}{5}} = \sqrt{\frac{27}{5}} = \sqrt{3\frac{3}{5}}$~~
 always relatively prime? do these need to be equal?

$3\sqrt{\frac{2}{5}} \neq \sqrt{3\frac{2}{5}} = \sqrt{\frac{14}{5}} = \frac{\sqrt{14}}{\sqrt{5}}$ NO
 $3\sqrt{\frac{3}{5}} \neq \sqrt{3\frac{3}{5}} = \sqrt{\frac{18}{5}} = \frac{3\sqrt{2}}{\sqrt{5}}$ NO
 $3\sqrt{\frac{4}{5}} = \sqrt{3\frac{4}{5}} = \sqrt{\frac{24}{5}} = \frac{2\sqrt{6}}{\sqrt{5}}$ NO

$a\sqrt{\frac{b}{c}} = \sqrt{a\frac{b}{c}} = \sqrt{\frac{ac+b}{c}}$
 $a\sqrt{\frac{b}{c}} = \sqrt{\frac{ac+b}{c}} = \sqrt{\frac{ac+b}{c}}$ NO

$ac+b = a^2b$
 $ac = a^2b - b$
 $ac = b(a^2 - 1)$

$\frac{2\sqrt{4}}{5} = 3\sqrt{\frac{4}{5}}$
 $a=3, b=4, c=5$
 $\sqrt{\frac{b(a^2-1)+b}{c}} = \sqrt{\frac{4(9-1)+4}{5}} = \sqrt{\frac{40}{5}} = 2\sqrt{\frac{4}{5}}$

$a\sqrt{\frac{b}{c}} = \sqrt{a\frac{b}{c}} \neq \sqrt{\frac{ac+b}{c}}$
 $= \sqrt{\frac{ac+b}{c}} = \sqrt{\frac{ac+b}{c}} = \sqrt{\frac{ac+b}{c}} = \sqrt{\frac{ac+b}{c}}$

$a^2 + 4b^2 = \text{perfect square}$
 $b=2, 9+4(4)=25$
 $b=3, 9+4(9)=45$ NO
 $b=4, 9+4(16)=73$ NO
 $b=5, 9+4(25)=109$ NO
 $b=6, 9+4(36)=153$ NO
 $b=7, 9+4(49)=205$ NO
 $b=8, 9+4(64)=265$ NO
 $b=9, 9+4(81)=333$ NO
 $b=10, 9+4(100)=409$ NO

$a = \frac{c \pm \sqrt{c^2 - 4b(-b)}}{2}$
 $a = \frac{c \pm \sqrt{c^2 + 4b^2}}{2}$

$3\sqrt{\frac{3}{5}} = \sqrt{3\frac{3}{5}} = \sqrt{\frac{27}{5}} = \frac{3\sqrt{3}}{\sqrt{5}}$

$a\sqrt{\frac{b}{c}} = \sqrt{a\frac{b}{c}}$ when
 $a = \frac{c \pm \sqrt{c^2 + 4b^2}}{2b}$

$2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}}$ $a=2, b=2, c=3$
 $3\sqrt{\frac{3}{5}} = \sqrt{3\frac{3}{5}}$ $a=3, b=3, c=5$
 $4\sqrt{\frac{4}{5}} = \sqrt{4\frac{4}{5}}$ $a=4, b=4, c=5$

$64 = 4c + 4$
 $60 = 4c$
 $15 = c$

$4\sqrt{\frac{4}{15}} \neq \sqrt{4\frac{4}{15}} = \sqrt{\frac{64}{15}} = \frac{8}{\sqrt{15}}$

$\text{so } a\sqrt{\frac{b}{c}} = \sqrt{a\frac{b}{c}}$ $2b = a^2$ and $a^2 = a^2$
 when $a=b$ and $ac+b = a^3$
 $ac+a = a^3$
 $a(c+1) = a^3$
 $c+1 = a^2$
 $a=b$ and $c = a^2 - 1$

$\sqrt{\frac{a^2}{a^2-1}} = \sqrt{\frac{a^2}{a^2-1}}$
 $\sqrt{\frac{a^2}{a^2-1}} = \sqrt{\frac{a^2}{a^2-1}}$

$a \neq b$ $a\sqrt{\frac{b}{c}} = \sqrt{a\frac{b}{c}} \Rightarrow a^2 = a^2 \Rightarrow a=1, 0$ but $a \neq 0$

proof let $a \neq 1$ and $a \neq 0$
 $\text{func } \sqrt{\frac{a^2}{a^2-1}} = \sqrt{a^2 \frac{a^2}{a^2-1}} = \sqrt{a^2 \frac{a^2}{a^2-1}} = \sqrt{a^2 \frac{a^2}{a^2-1}}$

check $100\sqrt{\frac{100}{9999}} = \sqrt{100\frac{100}{9999}} = \sqrt{\frac{10000}{9999}} = \frac{100}{\sqrt{9999}}$

notice denominators are
 $3, 5, 15, 24, 35, \dots$
 $\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$
 $\frac{1}{5} + \frac{1}{7} = \frac{12}{35}$
 $\frac{1}{7} + \frac{1}{9} = \frac{16}{63}$ etc.

know $k^2 = 1+3+5+7+\dots+(2k-1)$
 $2k^2 = 3+5+7+\dots+(2k+1)$

proof $a\sqrt{\frac{b}{c}} = \sqrt{a\frac{b}{c}}$
 $a \neq 1$ $2\sqrt{\frac{2}{3}} = 2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}} = \sqrt{\frac{8}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$
 assume true for $a=k$



Inquiry Maths

www.inquirymaths.org

Andrew Blair

I. Self-Regulation Cards - CHOICE

www.inquirymaths.org

Find more
examples.

Practise a
procedure.

Change the
prompt.

Inquire with
another student.

Ask the teacher or
a student to
explain.

Decide on the aim
of the inquiry.

I. Adapting Self Regulation Cards to the Online Environment

Use polls!

- **Analytics**
- **Flexibility**

Let's Pretend...

A. I want to work on this problem by myself.

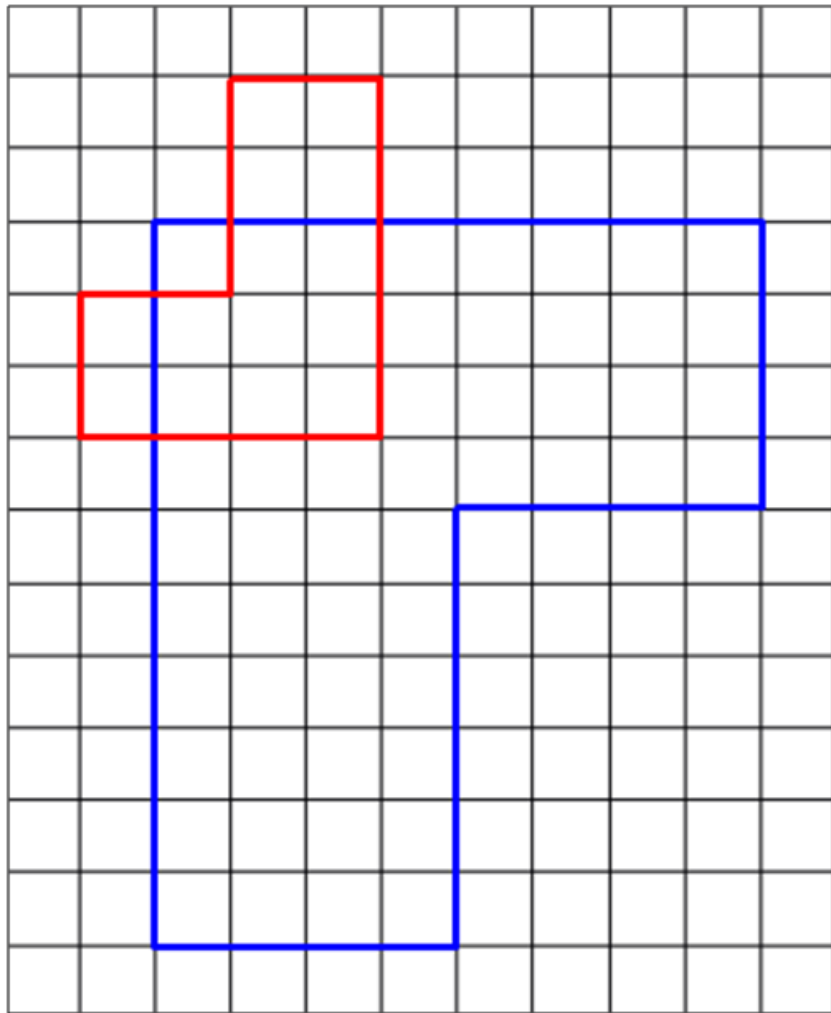
B. I want to work on this problem with a few other students.

C. I want to work on this problem with the whole class.

D. I want to work on this problem with guidance from the teacher

II. Prompts - Problems

$$24 \times 21 = 42 \times 12$$



There are four pairs of integers which have a highest common factor of 6 and a lowest common multiple of 180..

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 6 & 8 \\ -1 & -5 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 6 & 0 \\ -3 & 2 & -3 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 8 & 3 \\ -2 & 12 & 8 \\ -4 & -3 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & -2 & 7 \\ -2 & 0 & 8 \\ 2 & -7 & 6 \end{bmatrix}$$

II. Adapting Prompts to the Online Environment

Error Analysis Questions!

- **Practice at finding their errors**
- **Using writing to justify thinking**
- **Often Accompanied by a Poll**

Find the error(s), correct the error(s), and give an explanation of how to avoid the error.

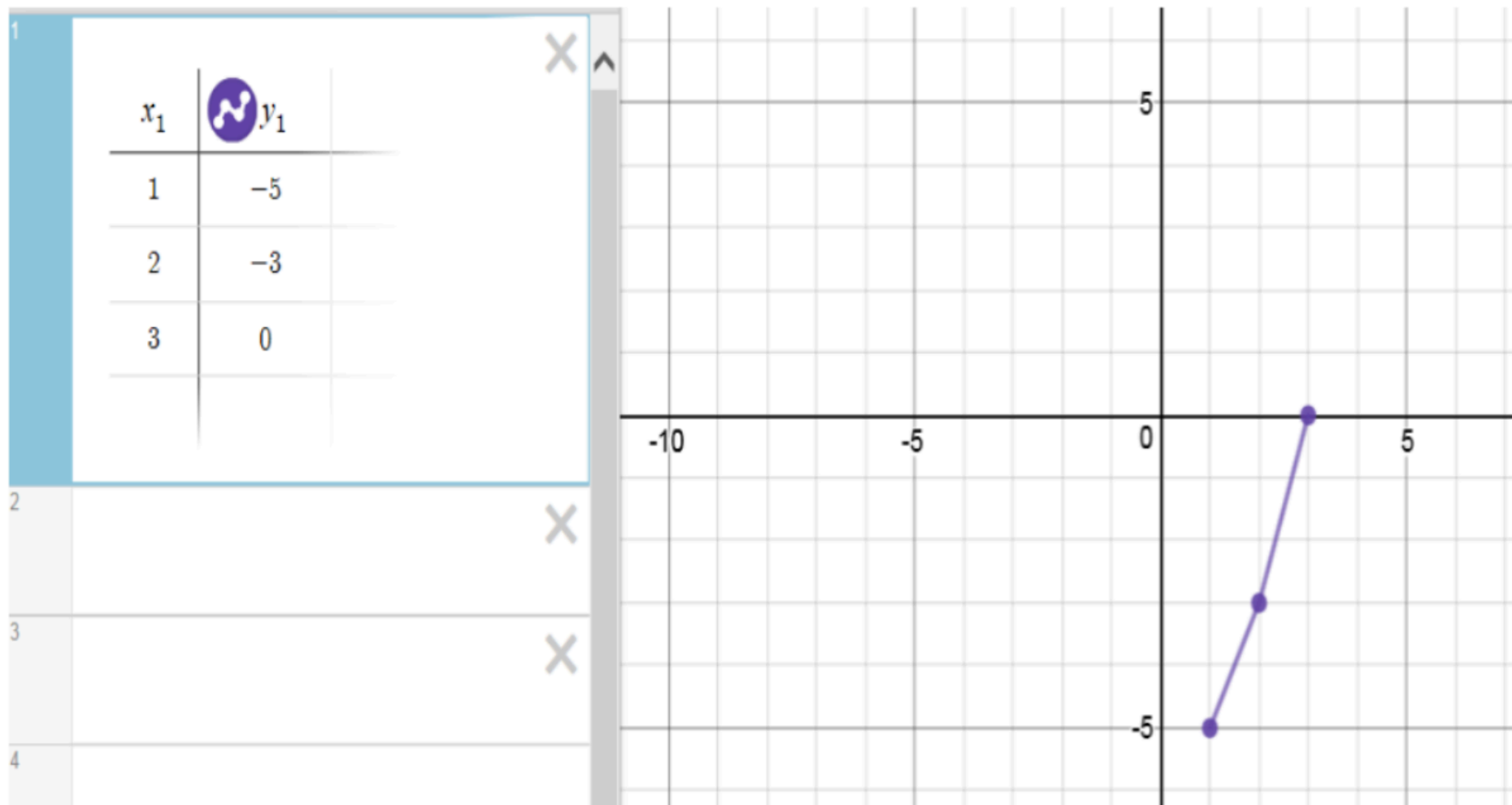
Brian's Original Work:

$$\sqrt{550x^{16}y^{81}} = \sqrt{25x^{16}y^{81}} \sqrt{22} = 5x^4y^9\sqrt{22}$$

Find the error(s), correct the error(s), and give an explanation of how to avoid the error.

A student is asked to graph: $f(x) = x^2 - 2x - 3$

She does the following work:



II. Adapting Prompts to the Online Environment

Directed/Guided Inquiry Questions

$$f(x) = x^2 - 4x - 5, \quad g(x) = x + 1$$

$$f(x) + g(x)$$

$$f(x) - g(x)$$

$$f(x) \bullet g(x)$$

$$f(x) \div g(x)$$

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 6 & 8 \\ -1 & -5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 6 & 0 \\ -3 & 2 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 8 & 3 \\ -2 & 12 & 8 \\ -4 & -3 & 0 \end{bmatrix}$$

What are these? Why so many numbers?

**I notice that $1 + 3 = 4$ and
that those numbers are all
in the same spot in the**

My Dan Meyer Slide - Open Inquiry



What I Learned

- **I love IBL!**
- **Practice makes proficient!**
- **Social Loafing**
- **Atychiphobia**
- **Research says ...**
- **Students can teach!**
- **Assessment - True/False**

The Modified Moore Method

Questions?

