From Hand Holding to Problem Solving:

Learning How to Teach Students to Grapple with Uncertainty

Eddie Fletcher, Kevin Ehly, & Alona Cohen

The Bartok Problem

When I was in Prague in 1975, Bartok told me that he was born on a very hot summer Sunday and that on his seventh birthday his father took him to the circus, which was visiting the city for the weekend. How old was Bartok?

Note: Please don’t share your answers

Brooklyn Frontiers High School

- New school model - opened in 2011
- 180 students
- Target students entering high school 2 years over-age (16 years old at beginning of 9th grade)
- ~50% Students with Disabilities
- ~90% of students well below proficiency in math (NYS Assessment)
- All students have a history of past failure in school

Lesson Study Conditions

- 55 minutes of weekly of full department meeting time (6 teachers- Math + SpEd)
- 1 additional hour of full department meeting time every other week
- Weekly 1:1 planning meetings with the dept facilitator and principal
- Monthly meetings of all department facilitators
- Monthly cross-dept sharing meetings
- Administrators are members of Lesson Study teams

Basic Lesson Study Structure

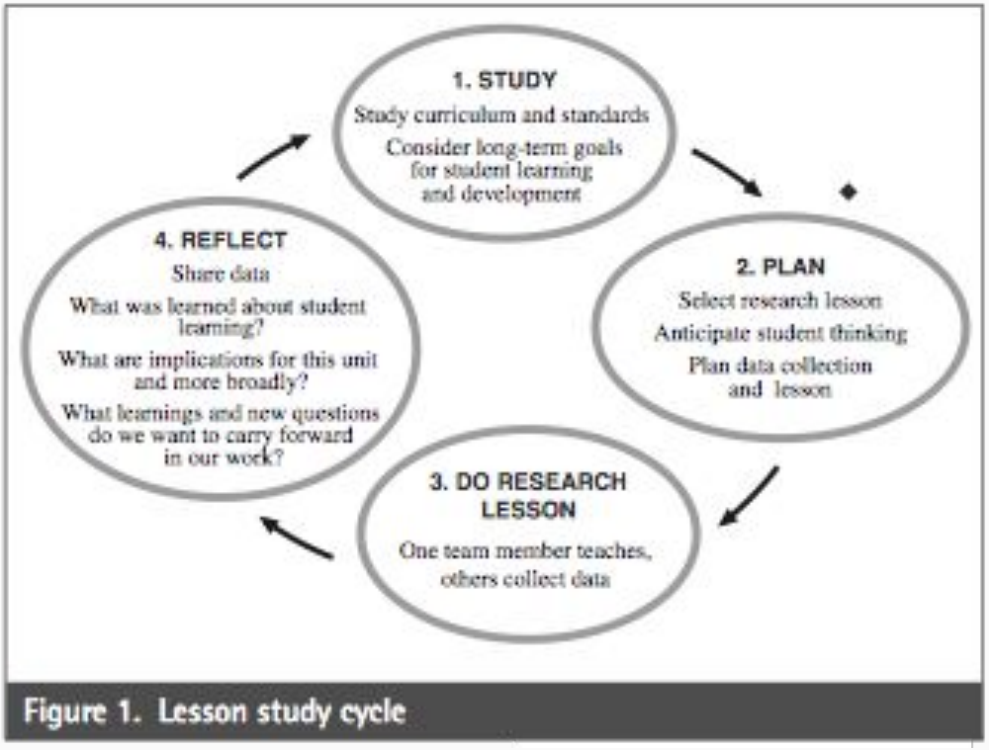

Figure 1. Lesson study cycle

Hurd & Licciardo-Musso (2005)

Challenge: What order would you use?

A) Create Lesson Materials

B) Determine LS Date/Class

C) Perform dry run of Lesson

D) Write the Lesson Plan

E) Debrief Lesson Observation

F) Discuss Class Norms & Students

G) Determine Lesson Objective

H) **Observe the Lesson**

I) Determine Observation Protocol

J) Bring in outside Research

K) Reflect on Learning

L) Determine the LS Topic

M) Implement in individual classes

N) Reflect on Lesson Study Process

This was the order we used in the math department

B-L-G-A-F-J-M-K-D-C-I-H-E-M-K-N

O/F L/G D B I H E K N M

L G F D A C I B M/H E K J N

A) Create Lesson Materials

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How we used lesson study to transform our department

Before

- Teachers over scaffolded problems
- Students were able to complete work
- Students constantly asked teachers for help or affirmation
- Students didn't recognize material out of context
- Students wouldn't attempt new problems

Typical responses on state tests

25 The function, $t(x)$, is shown in the table below.

x	t(x)
–3	10
–1	7.5
1	5
3	2.5
5	0

Determine whether $t(x)$ is linear or exponential. Explain your answer.

29 The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where $C(m)$ is the total cost for m months of membership.

State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

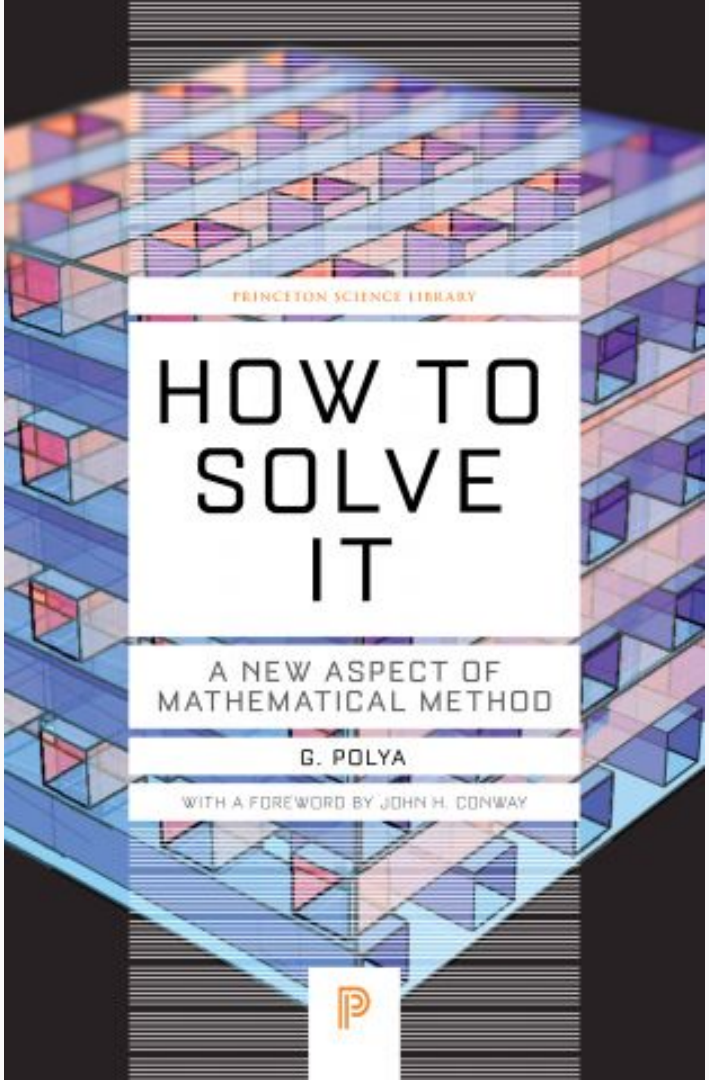
The relationship between
is 129.5

George Polya

1887 - 1985

How to Solve It

1945



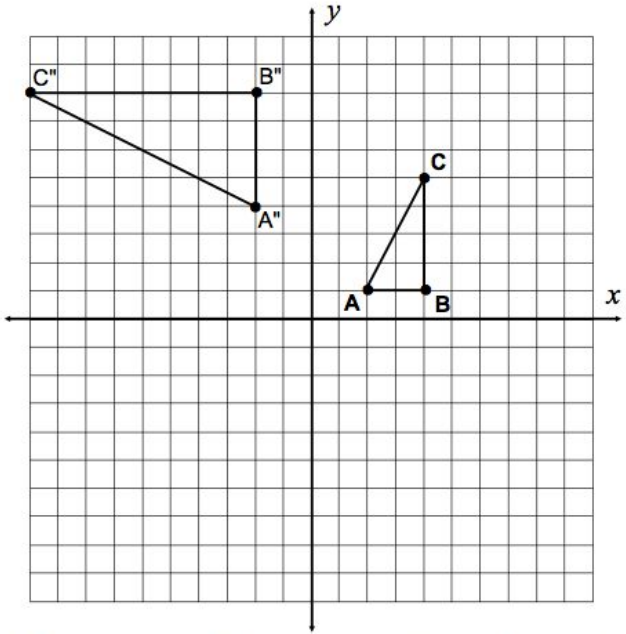
Polya’s 4 Stages of Problem Solving

- Understand the problem
- Make a plan
- Carry out the plan
- Look back, Review/Extend

Problems vs Exercises

Lesson Study Example (Geometry)

Triangle ABC has coordinates A(2, 1), B(4, 1) and C(4, 5). The image of ΔABC has coordinates A"(-2, 4), B"(-2, 8) and C"(-10, 8).



Determine one possible *composite transformation* that would send ΔABC to ΔA"B"C". _____

_____

① *I notice...* → *I can try...*

② First, I'm going to try:

_____

③ What happened to the coordinates from ***Prime*** to *Double Prime*?

_____

_____

_____

A(2, 1)	B(4, 1)	C(4, 5)
↓	↓	↓
A'(,)	**B'(,)**	**C'(,)**
↓	↓	↓
A"(-2, 4)	B"(-2, 8)	C"(-10, 8)

④ Can you find a transformation that would send ΔA'B'C' to ΔA"B"C"? ____

If you answered *yes*, which transformation? _____

⑤ My composite transformation: *First*: _____

Second: _____

How lesson study changed individual practices

Writing the Equation of a Circle given the endpoints of the Diameter

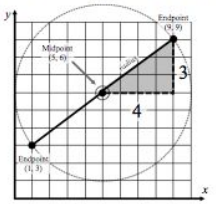
The Diameter of a circle passes through the **center** and is twice the length of the **radius**.

Step 1: Find the **Midpoint** of the diameter. This is the **center** of the circle (h, k).

Step 2: Calculate the **distance** from the center of the circle to *either* endpoint. This is the **radius** (r).

Step 3: Substitute the values of h, k, & r into the equation $(x - h)^2 + (y - k)^2 = r^2$ and simplify.

Write an equation of the circle whose diameter has endpoints (1, 3) & (9, 9).

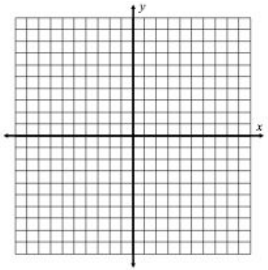


Center (5, 6) radius = 5

$(x - 5)^2 + (y - 6)^2 = 25$

Exploration

Write an equation of the circle whose diameter has endpoints located at (-5, 1) & (9, 1).



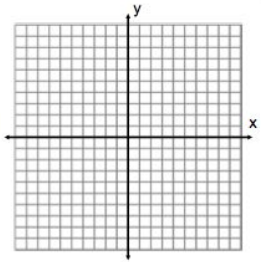
Write an equation of the circle whose diameter has endpoints (-4, 9) & (8, -7).

Step 1: x-coord. $\frac{+}{2}$ =

y-coord. $\frac{+}{2}$ =

The Midpoint/Center is (,)

Step 2: $a^2 + b^2 = c^2$



Center (,) radius = ___

Step 3: (x)² + (y)² =

Exploration Questions

1) What is the problem asking you to do?

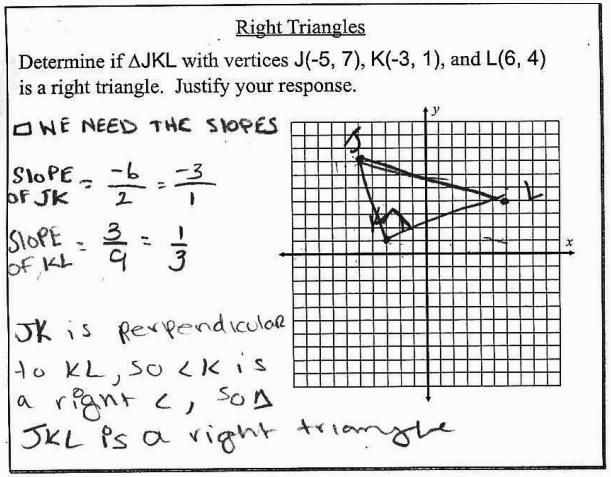
2) What information are you given?

3) What information do you still need?

4) How could you find that information?

After

- Students empowered to attempt new problems
- Fewer blanks on Regents exams
- Students take ownership of learning process- stopped calling for the teacher at every step
- Teachers developed questioning skills



Right Triangles

Determine if ΔJKL with vertices J(-5, 7), K(-3, 1), and L(6, 4) is a right triangle. Justify your response.

☐ WE NEED THE SLOPES

SLOPE OF JK = $\frac{-6}{2} = \frac{-3}{1}$

SLOPE OF KL = $\frac{3}{9} = \frac{1}{3}$

JK is perpendicular to KL, so ∠K is a right ∠, so Δ JKL is a right triangle

Exploration Questions

1) What is the problem asking you to do?

See if it's a right Δ

2) What information are you given?

Vertices

3) What information do you still need?

is there a 90° ∠?

☐ Slope flipped

☐ different signs

4) How could you find that information?

See if 2 sides are perpendicular.

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For this project, imagine that you are a lawyer who has been called in to negotiate pay raises for workers at a company as they sign their new contract. It is your job to make sure that the workers at the company get the best deal possible.

The following data shows the wages that the company pays each of its employees.

Number of people in each position	Position	Yearly salary	Total salary per position
1	President	$200,000	$200,000
3	Vice Presidents	$100,000	$300,000
5	Managers	$50,000	$250,000
10	Supervisors	$30,000	$300,000
11	Workers I	$28,000	$308,000
20	Workers II	$20,000	$400,000
22	Workers III	$18,000	$396,000
6	Workers IV	$16,000	$96,000
		Total	**$2,250,000**

Part I [*Meets*]

Show your work when possible. Feel free to attach scrap paper.

1) Determine the mean, median, and mode yearly salary at the company.

mean - 28,846.15
- 23,648.65
median - 20,000
mode - 18,000

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6	Workers IV	~~$16,000~~	$96,000
78		Total	$2,250,000

Part I [*Meets*]

Show your work when possible. Feel free to attach scrap paper.

1) Determine the mean, median, and mode yearly salary at the company.

mean:
step 1: 2,250,000
step 2: 2,250,000 ÷ 78 = 28846.15
median: 20000
mode: 18,000

ur response.

Revisiting Bartok

When I was in Prague in 1975, Bartok told me that he was born on a very hot summer Sunday and that on his seventh birthday his father took him to the circus, which was visiting the city for the weekend. How old was Bartok?

What do you notice?

What is the plan?

Math for America

NYC DOE Learning Partners Program

Q & A

No, we will not give you the answer to the Bartok problem.

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