# Social Networks
# to Teach Graph Theory
## Lessons from the Classroom


UNIVERSITY OF CENTRAL ARKANSAS

Todd Abel
University of Central Arkansas

ASHE COUNTY schools

Christina Pennington
Ashe County (NC) High School

# How the Math Teacher Network Facilitates Collaboration

(i.e, how Dan Meyer brought us here)

2016 NCTM ANNUAL MEETING & EXPOSITION
April 13–16 • San Francisco

**130**

**Social Networks for Teaching Graph Theory**

10–12 Session

Graph theory is a important topic in discrete mathematics and an excellent low-threshold topic for mathematical reasoning and problem-solving. In this presentation, we discuss the use of social networks for introducing and teaching graph theory. Social network theory is relatable and rich in context. Key ideas and activities will be discussed.

**Todd A. Abel**
Appalachian State University, Boone, North Carolina
**Mary E. Searcy**
Appalachian State University, Boone, North Carolina

**2005 (MOSCONE WEST)**

# How the Math Teacher Network Facilitates Collaboration

(i.e, how Dan Meyer brought us here)

## Every Handout & Presentation From NCSM & NCTM 2016

*By Dan Meyer • April 17, 2016 • 15 Comments*

It will probably take more than one post t
sent a script through the conference prog

- NCSM Annual Meeting [huge zip file
- NCTM Research Preconference
- NCTM Annual Meeting

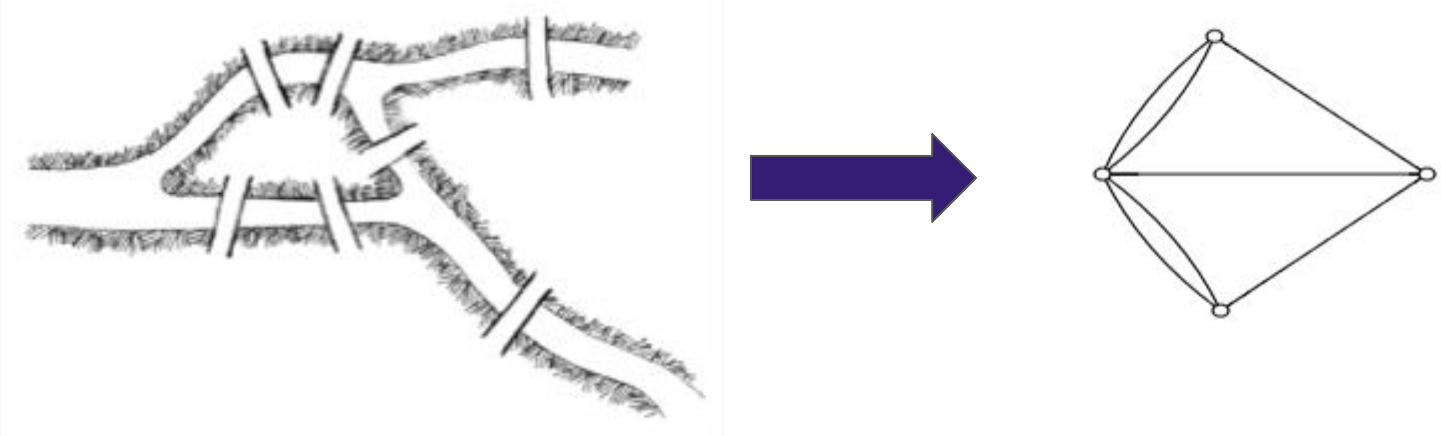
After skimming through every file, I'll note
looking for those resources, you're in luck

In case your interests follow my own, her
reading:

- *Abel & Searcy*. Using Social Networks to Teach Graph Theory.
- *Boakes*. Seeing Is Believing. Using Video Reflection Techniques to Strengthen Instruction.
- *Carroll*. Learning from Research: Using Worked Examples in Math Class.
- *Cooperman*. Challenging Precalculus Alternative Assessments Using the Free Online Desmos Calculator.
- *Creagar & Daiga*. Providing Students with the Power to Prove.
- *DenBesten & Oswalt*. Functions for ALL: Toward a Rigorous and Thorough Understanding.
- *Franklin, et al*. Essential Knowledge for Effective Teaching and Learning of Statistics.
- *Gann*. Building Proficiency in Mathematical Modeling

# One Possible Trajectory

Bridges of Konigsberg - Can you cross every bridge exactly once?



# One Possible Trajectory

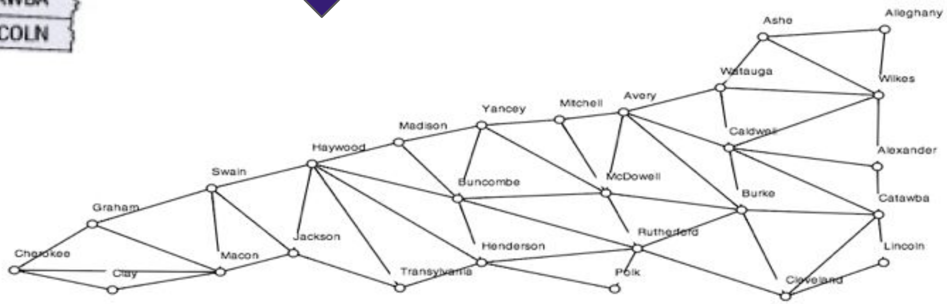
Define terms: *vertex; edge; graph; incident; adjacent; connected; degree; subgraph; paths and circuits; adjacency matrix.*

Now consider: “What additional information can be captured in the graph?”

# One Possible Trajectory

## Graph Coloring

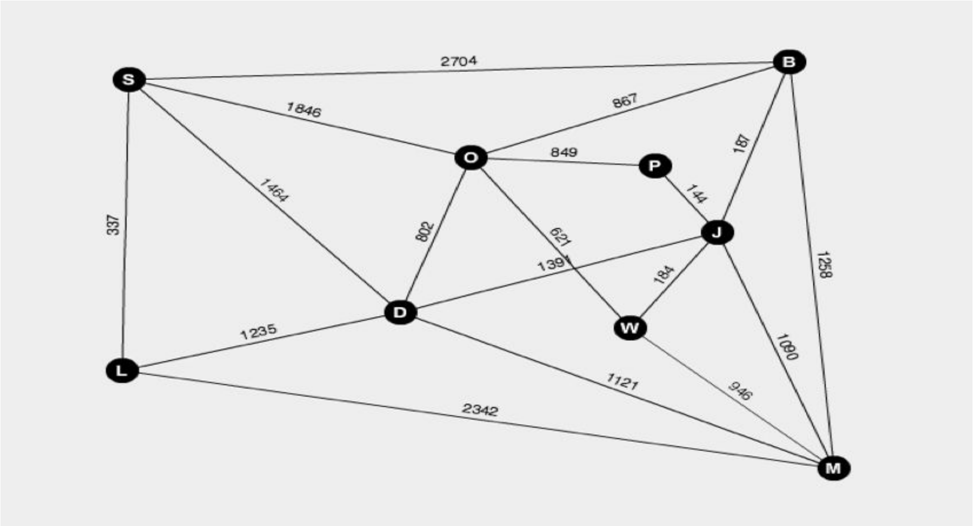
How many colors do we need to color a map if no two adjacent regions share a color?

# One Possible Trajectory

## Edge Weights

What's the shortest path from *B* to *L*?



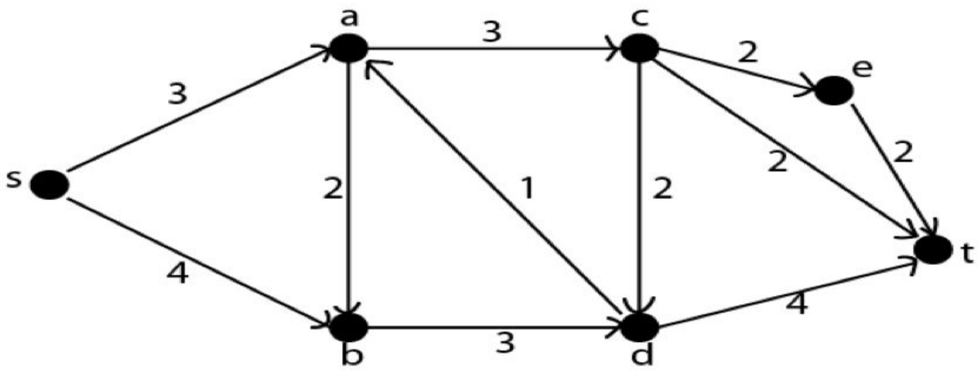

Now we can also motivate things like:

- Shortest path algorithm
- Minimum spanning trees
- Traveling Salesman Problem
- Discussions about efficiency of algorithms

# One Possible Trajectory

## Directed Edges

What if information only flows in one direction?




We can ask some of the same questions we did with undirected graphs, and also consider

- Do our previous algorithms still work, or can they be altered?
- Maximum Flow
- Matching Problems

# How Mathematical Modeling Is Often Taught

"Learn" (or be told about) a new concept

Practice with it on a bunch of simple problems

Now apply it to something that may or may not be interesting

Student: why should I care about this?
Teacher: You can do really cool stuff with it, trust me!

Student: why should I do this?
Teacher: You can do really cool stuff with it, trust me!

Teacher: Isn't this cool?
Student 1: why don't we always do this?
Student 2: ...

# An Improvement

Start with a "real world" problem that may or may not be of any interest or relevance to your students

Use it to define or introduce some relevant concepts

Now apply it to something else

Student: why should I care about this?
Teacher: Engineers do it!

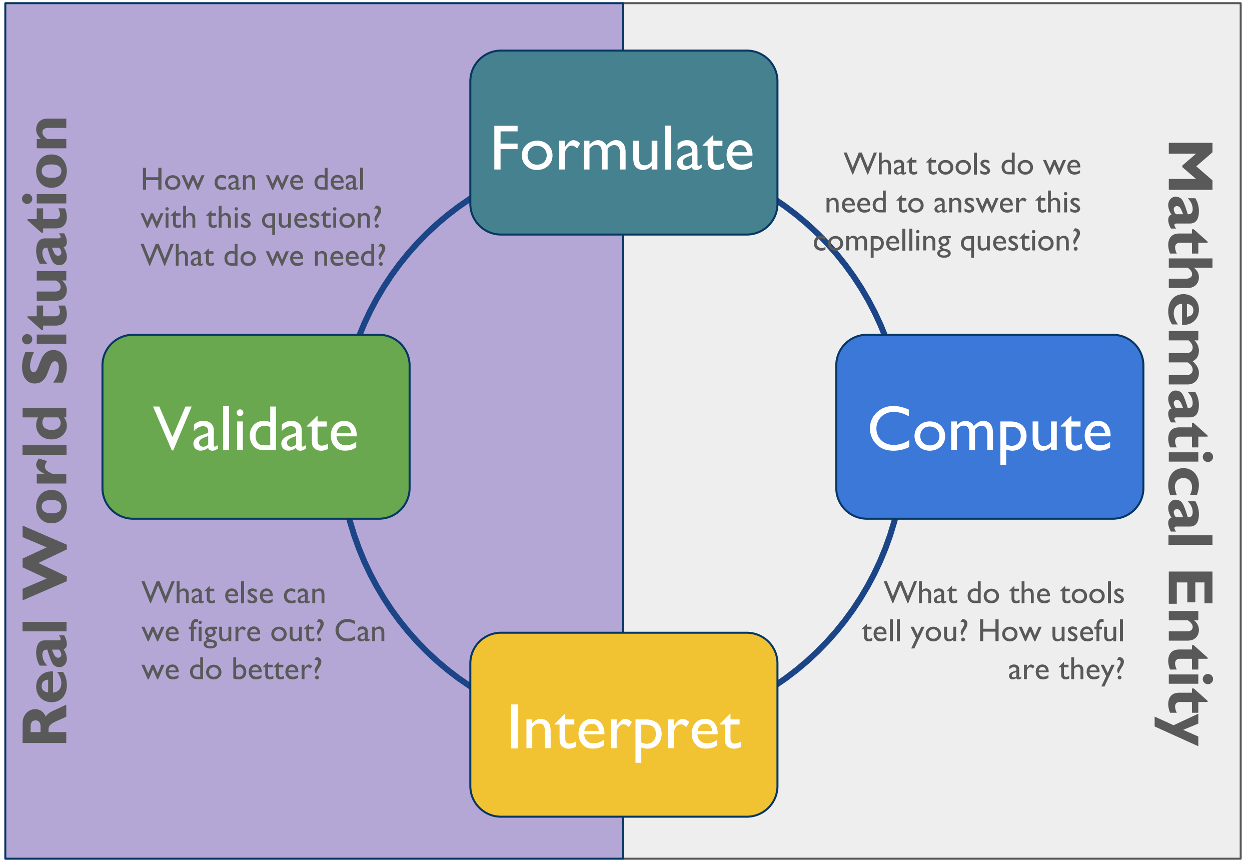
Student: why didn't you just tell me this?
Teacher: Now you know why it's important

Teacher: Isn't this cool?
Student 1: I see the pattern you want me to see
Student 2: ...

# How Mathematical Modeling Could Work

Based on CCSSM diagram of the modeling process.

See GAIMME report for lots of information and insights on teaching mathematical modeling



We need rich and compelling situations that motivate useful, interesting, and accessible mathematics.

Our previous approaches motivated interesting and accessible mathematics, but weren’t very compelling or often very useful.

# What We'll Do From Here …

1) An example of one particular approach to using social network analysis to teach graph theory

2) Other interesting things we can do with social networks and graph theory in the classroom

# Topics in Ashe County High School's Discrete Math Class

**Day 1:** How can I go "viral?"; define adjacency matrix and vertex-edge graph

**Day 2:** Basic vocabulary: *adjacent, subgraph, connected, complete, degree, clique*

**Day 3:** Hamiltonian circuits, paths, tournaments

**Day 4:** Euler circuits and paths

**Days 5-6:** Critical paths

**Days 7-8:** Conflict graphs and graph coloring

**Day 9:** Traveling Salesman Problem

**Day 10:** Shortest path

**Day 11:** Minimum spanning trees

**Day 12-13:** Binary trees and traversals

# How do you go viral? - we used to use this:


Katheryn Deprill was abandoned as a newborn in a Burger King in 1986. She posted this picture to Facebook on March 7, 2014.

## About

I am Katheryn (Hollis) Deprill, also known as The Burger King Baby. I have started the search for my biological mother in hope of reuniting with her!

### Biography

My birthday is September 15th, 1986 and I was left by my birth mother in the bathroom of an Allentown Burger King. Today I have launched a campaign to find my biological mother. I would love to be able to meet her, and hug her. I want her to know that I am not mad at her for what she did, however I have so many questions to ask her and also to start a relationship with my biological mother. If she comes forward please tell her to not be afraid and contact me as soon as possible.

# How do you go viral? - we used to use this:


It was shared over 30,000 times.

She met her birth mother on March 24

## About

I am Katheryn (Hollis) Deprill, also known as The Burger King Baby. I have started the search for my biological mother in hope of reuniting with her!

**Biography**

My birthday is September 15th, 1986 and I was left by my birth mother in the bathroom of an Allentown Burger King. Today I have launched a campaign to find my biological mother. I would love to be able to meet her, and hug her. I want her to know that I am not mad at her for what she did, however I have so many questions to ask her and also to start a relationship with my biological mother. If she comes forward please tell her to not be afraid and contact me as soon as possible.

Viral posts come almost exclusively from internet content providers. There is significant money tied up in "going viral".

Source: Buzzsum.com

Buzzsumo

## Most Viral Facebook Posts of 2017

| Post | Type | FB Interactions |
|---|---|---|
| Get clever with your clutter...7 organization hacks! | Video - Practical Tips | 15.6m |
| Best Video You Will See | Video<br>Animals<br>Heartwarming | 13.8m |
| People Are Awesome | Video<br>Amazing & Inspiring | 12.2m |
| This Guy Just Sang Whitney Houston Like You've Never Heard | Music Video<br>Inspiring | 10m |
| Brighten up your day with these 5 surprising hacks! | Video<br>Practical Tips | 8.6m |
| 8 ways to transform and upgrade your wardrobe | Video<br>Practical Tips<br>Clothing | 8.6m |
| Awesome ideas for a T-shirt that is toooooo old | Video<br>Practical Tips<br>Clothing | 7.6m |
| These 8 wow-worthy pie hacks are easy as .. | Video<br>Food | 7m |
| Nanny, play with me, no working! | Video<br>Cute Animal Panda<br>Heartwarming | 6.2m |
| 4 Surprising things you can make with only 2 ingredients! | Video<br>Food<br>Practical tips | 6.1m |
| The Night Was #ForcaBarca! | Image<br>Spanish<br>Tribal Football | 5.6m |
| ¿Cómo conseguir que todos coman verduras y que las disfruten? - Anna Easy Recipes | Food Recipes<br>Spanish | 5.3m |
| SHARE this with your friends because this little girl is a star! | Music Video<br>Inspiring | 5.4m |
| This is a great idea! We need this everywhere. | Video<br>Inspiring Idea<br>Environmental | 5.3m |

# And then there's this …

Up to 87 million[1] Facebook users' data were harvested and sold to the firm Cambridge Analytica.

- NY Times initially reported 50 million
- Cambridge Analytica claimed only 30 million

A personality quiz app called *This is Your Digital Life* gathered personal data from approximately 270,000 users ("for academic use").

1

## How can I tell if my info was shared with Cambridge Analytica?

Share Article

Recently, we shared information about the potential misuse of your Facebook data by apps and websites. We also shared plans for how we're taking action to prevent this from happening in the future.

Check below to see if your information may have been shared with Cambridge Analytica by the app "This Is Your Digital Life."

**Was My Information Shared?**

Based on our investigation, you don't appear to have logged into "This Is Your Digital Life" with Facebook before we removed it from our platform in 2015.

However, a friend of yours did log in.

As a result, the following information was likely shared with "This Is Your Digital Life":

- Your public profile, Page likes, birthday and current city

A small number of people who logged into "This Is Your Digital Life" also shared their own News Feed, timeline, posts and messages which may have included posts and messages from you. They may also have shared your hometown.

Review and update the information you share with apps and websites by checking your settings.

How does a post go viral? Who in our class should be targeted?

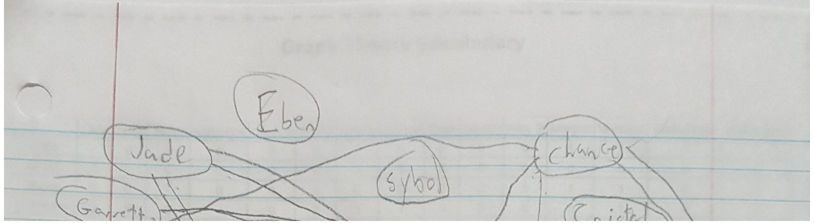
# Creating a class social network

Have students list their Facebook friends.

- As a class, discuss how to record this information
- Used a collaborative Google sheet to record information

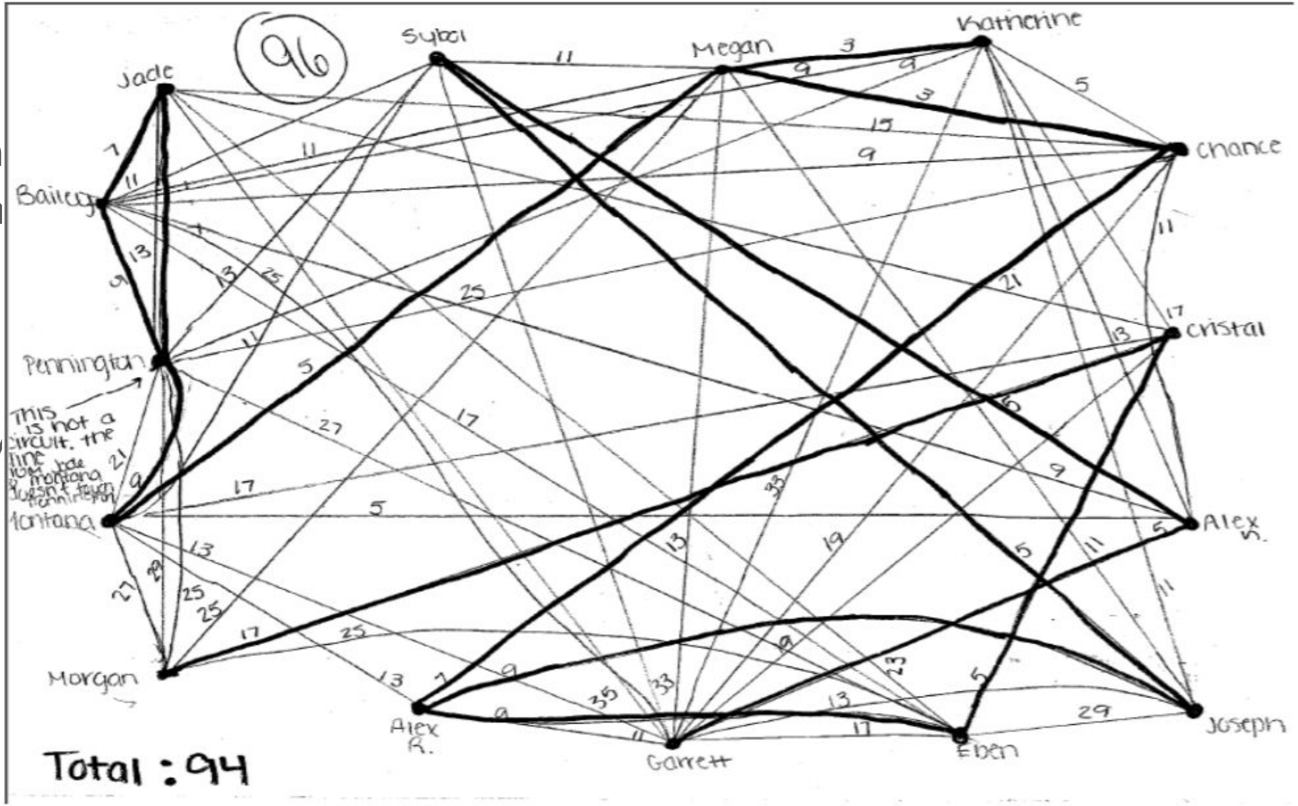
Simulate a friendship network

- Each pair of students roll one die each. If the sum is odd, they are friends (can let the sum be the edge weight if you want to use this later)
- Can tweak the probability of a friend to increase the likelihood of a useful graph




Once the friendship con
established, determine h
relationships.

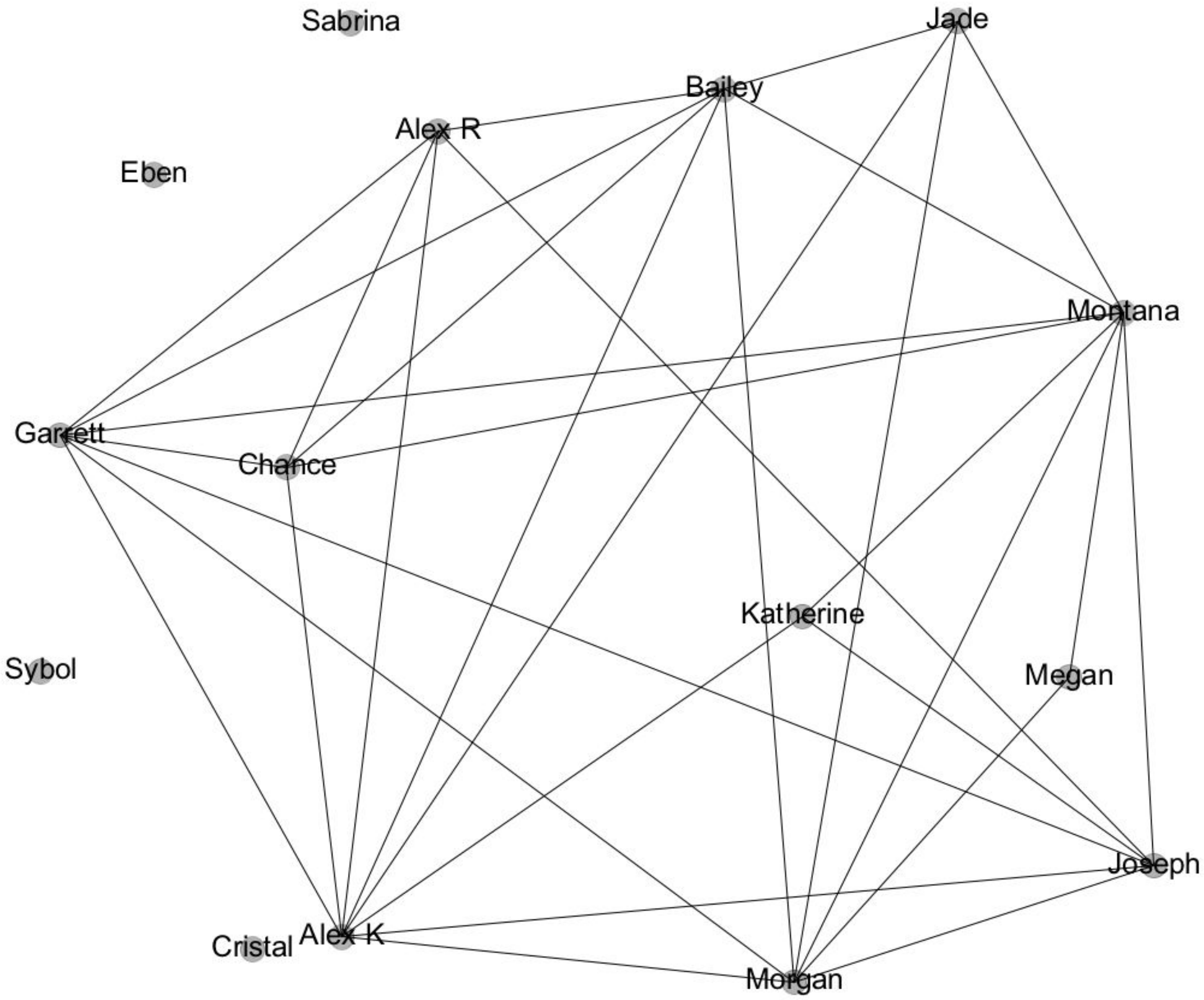
Ask students to look at
groups and make observ



# How might something go viral in this class?

Who is most important?

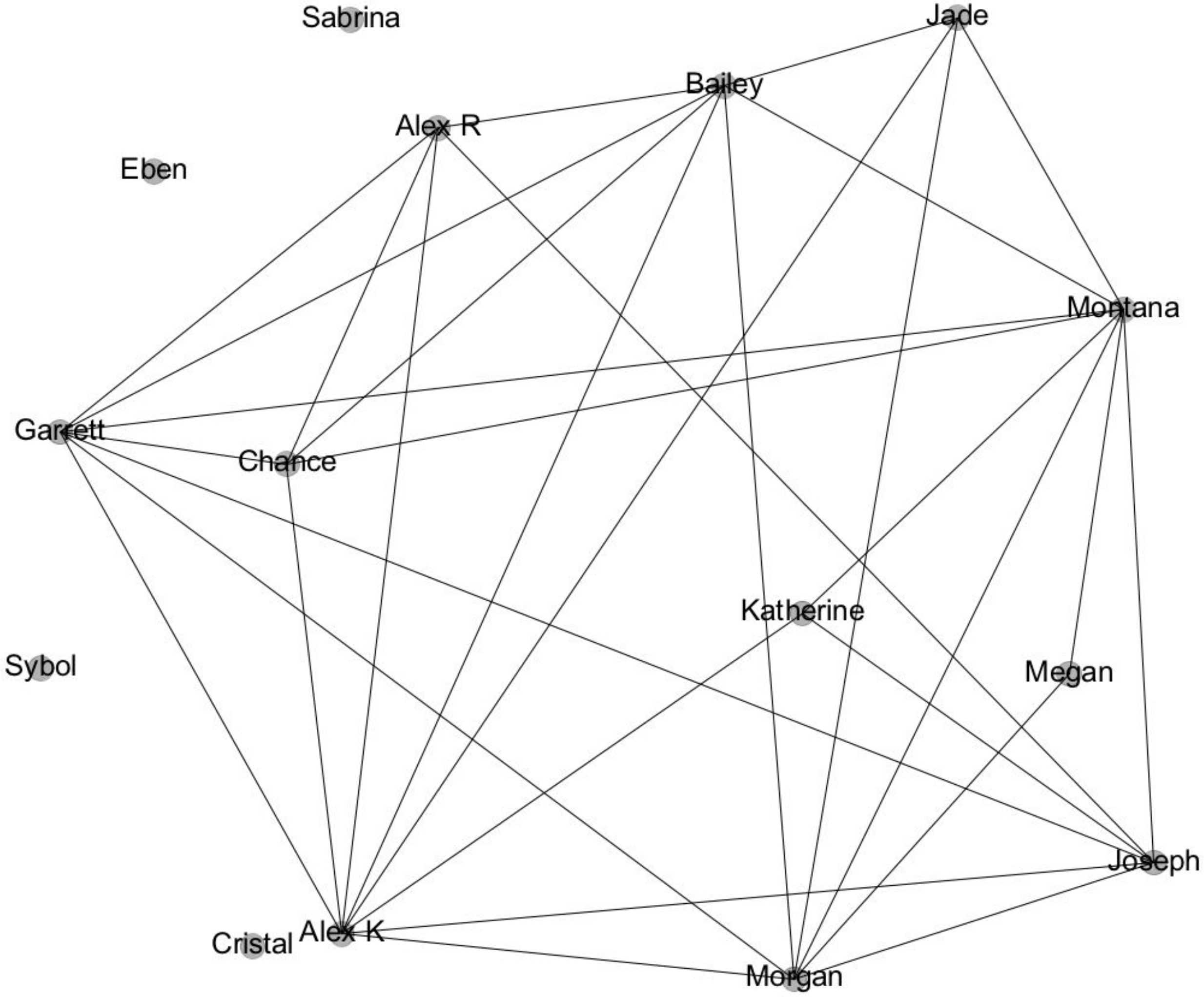
Who is least important?



# How might something go viral in this class?

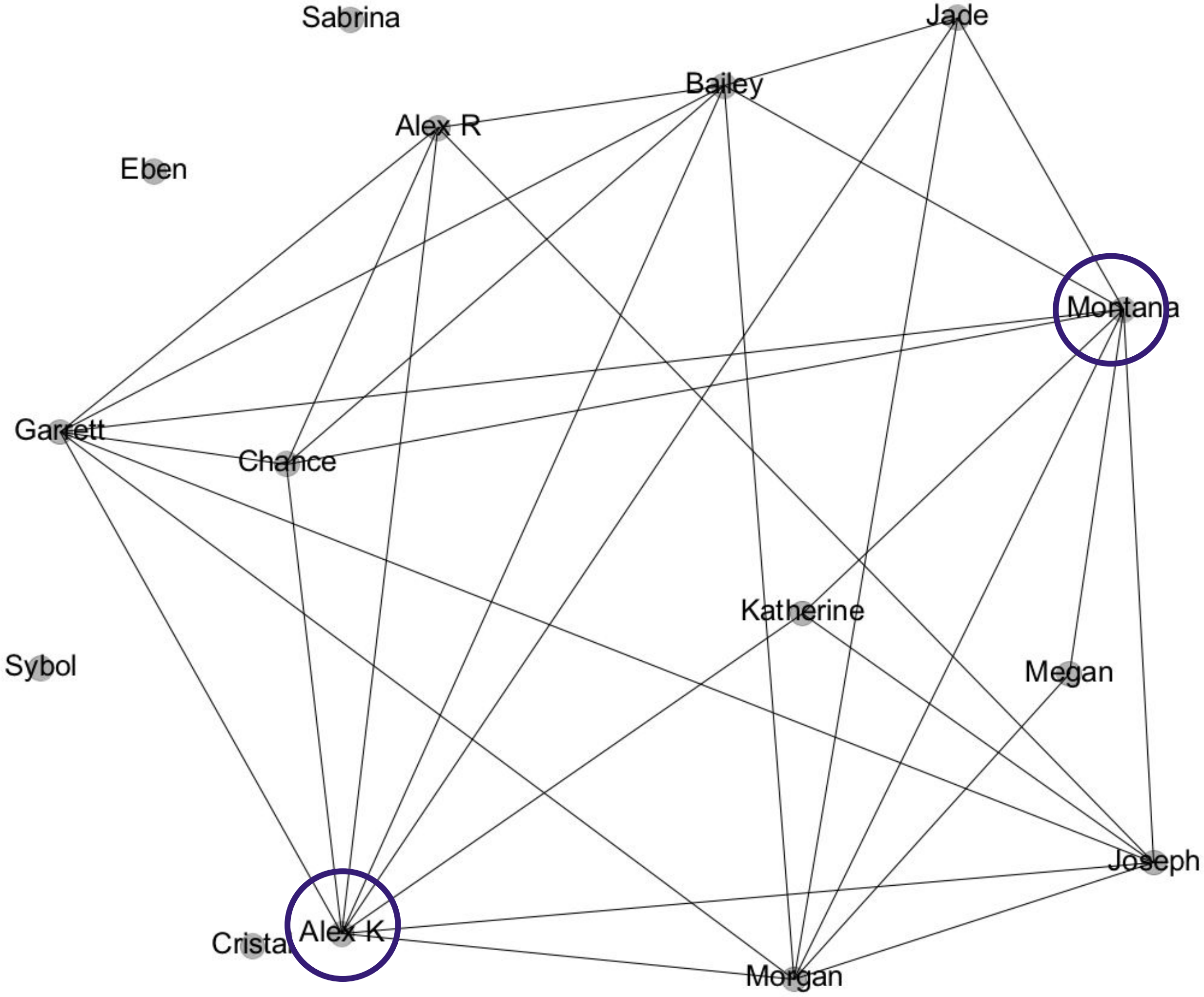
The graph is not *connected* - Sabrina, Eben, Sybol, and Cristal are not *adjacent* to any other students.

The rest of the graph comprises a *connected subgraph.*



# How might something go viral in this class?

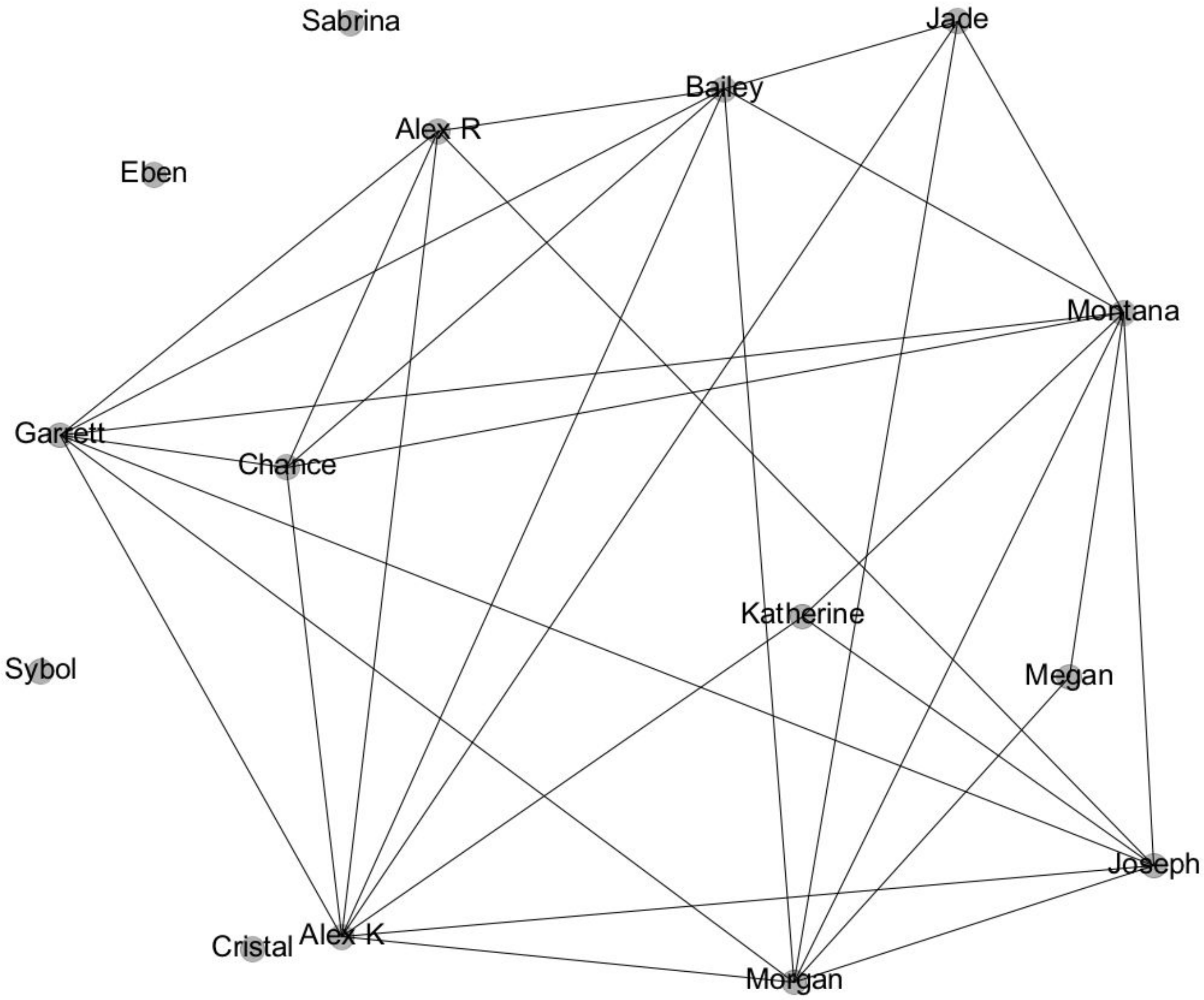
Montana and Alex K have the most connections - they both have the highest *degree* (8)



# How might something go viral in this class?

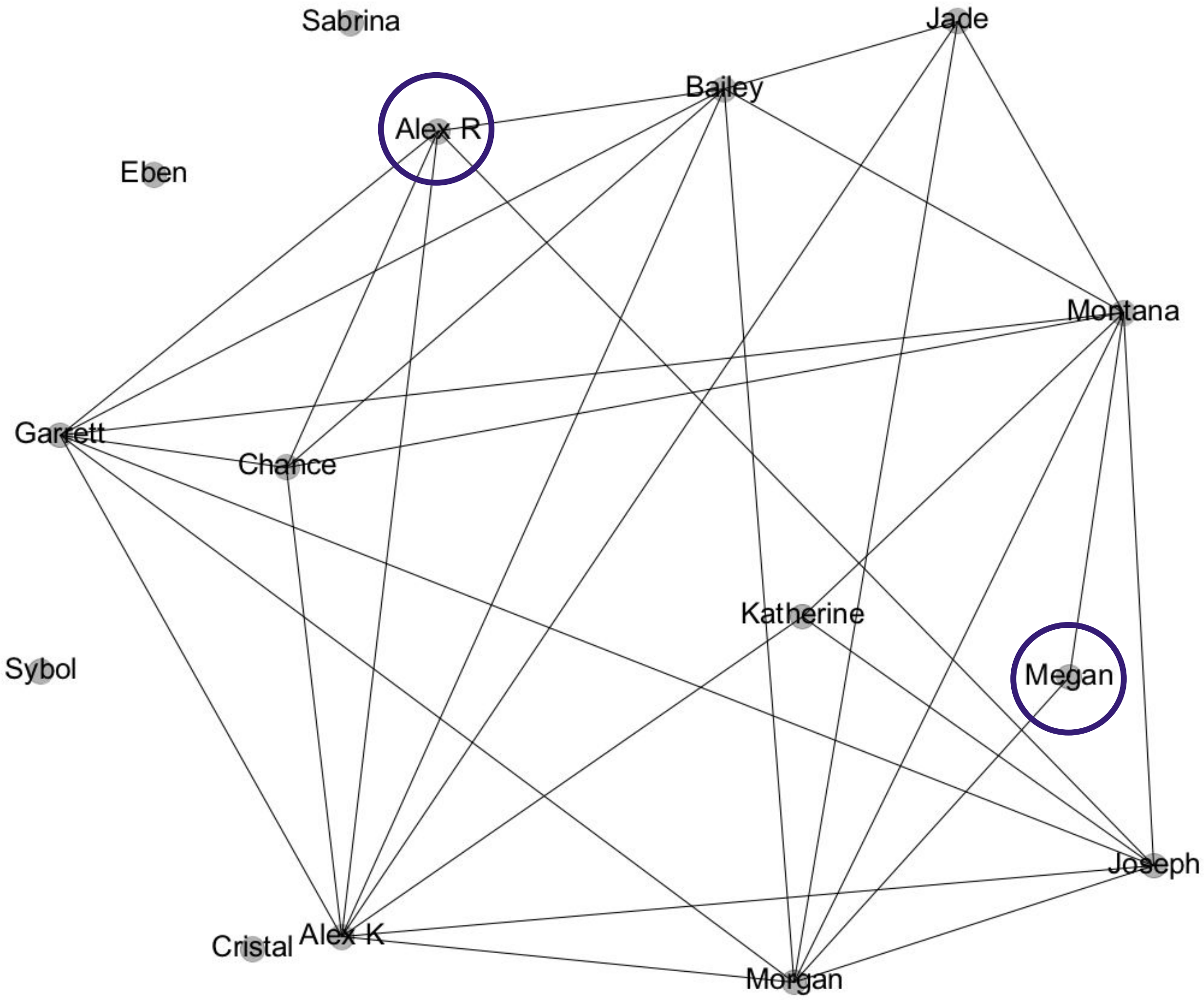
The connected subgraph is pretty well-connected, with an *average degree* of ~5.6

The *average path length* is just 1.455



# How might something go viral in this class?

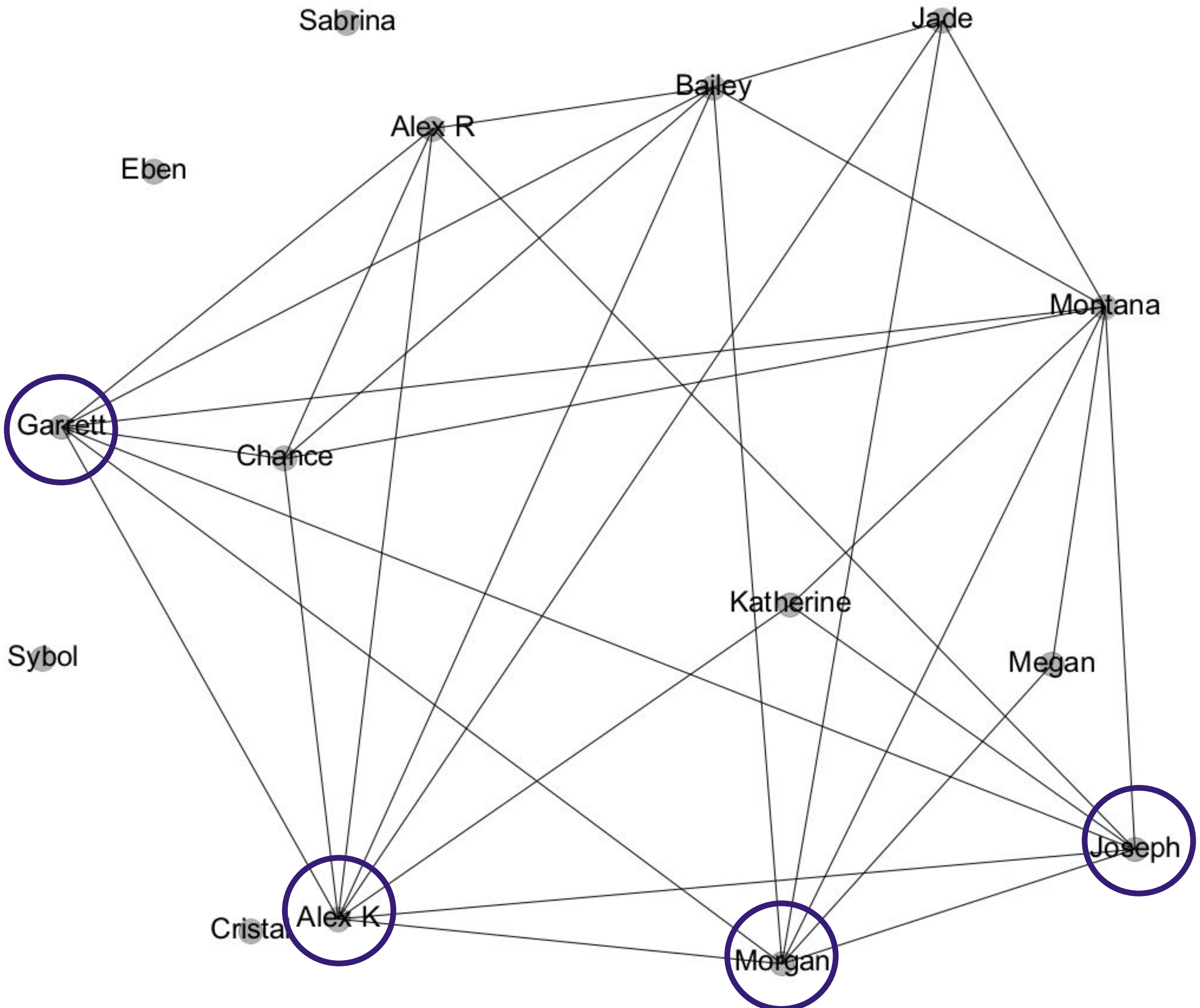
The *eccentricity* of a node is its maximum distance from another node. The *diameter* of a network is the maximum eccentricity - only 3 here



# How might something go viral in this class?

A *clique* is a complete (every pair of vertices shares an edge) subgraph.

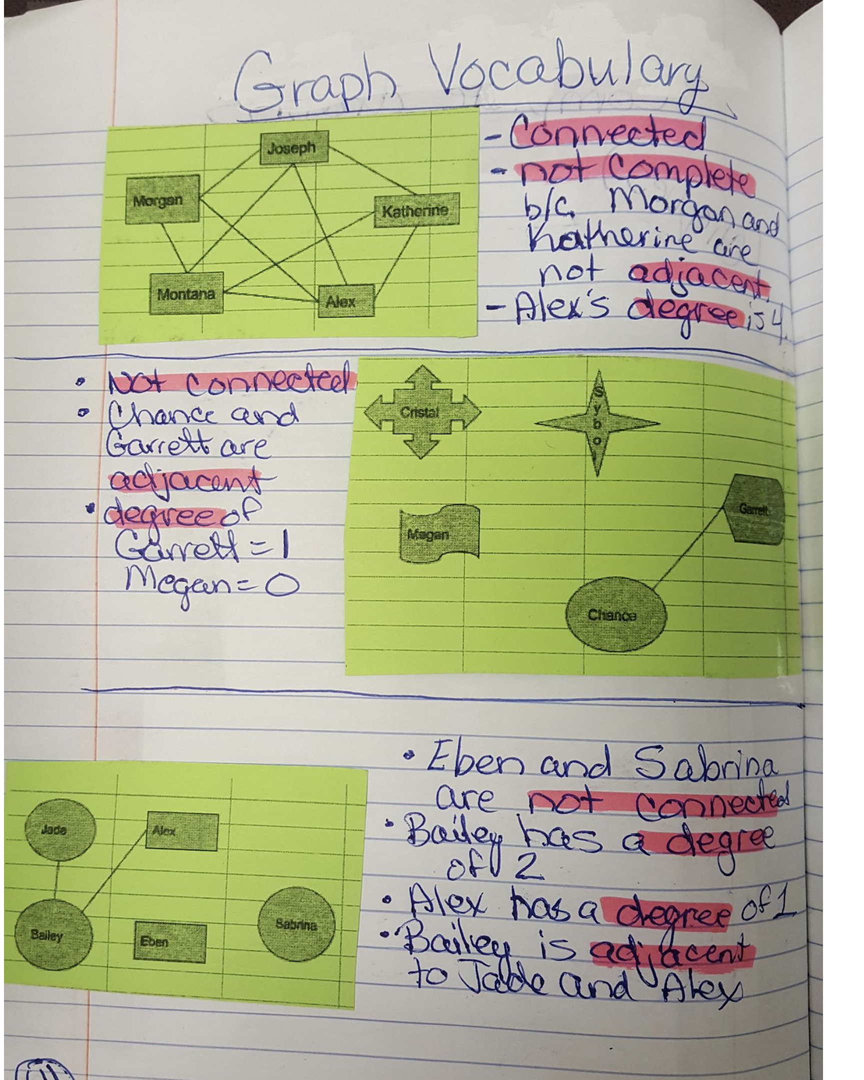
Here's one of order 4 (there are others)



Once the friendship connections are established, determine how to represent the relationships.

Ask students to look at networks in small groups and make observations

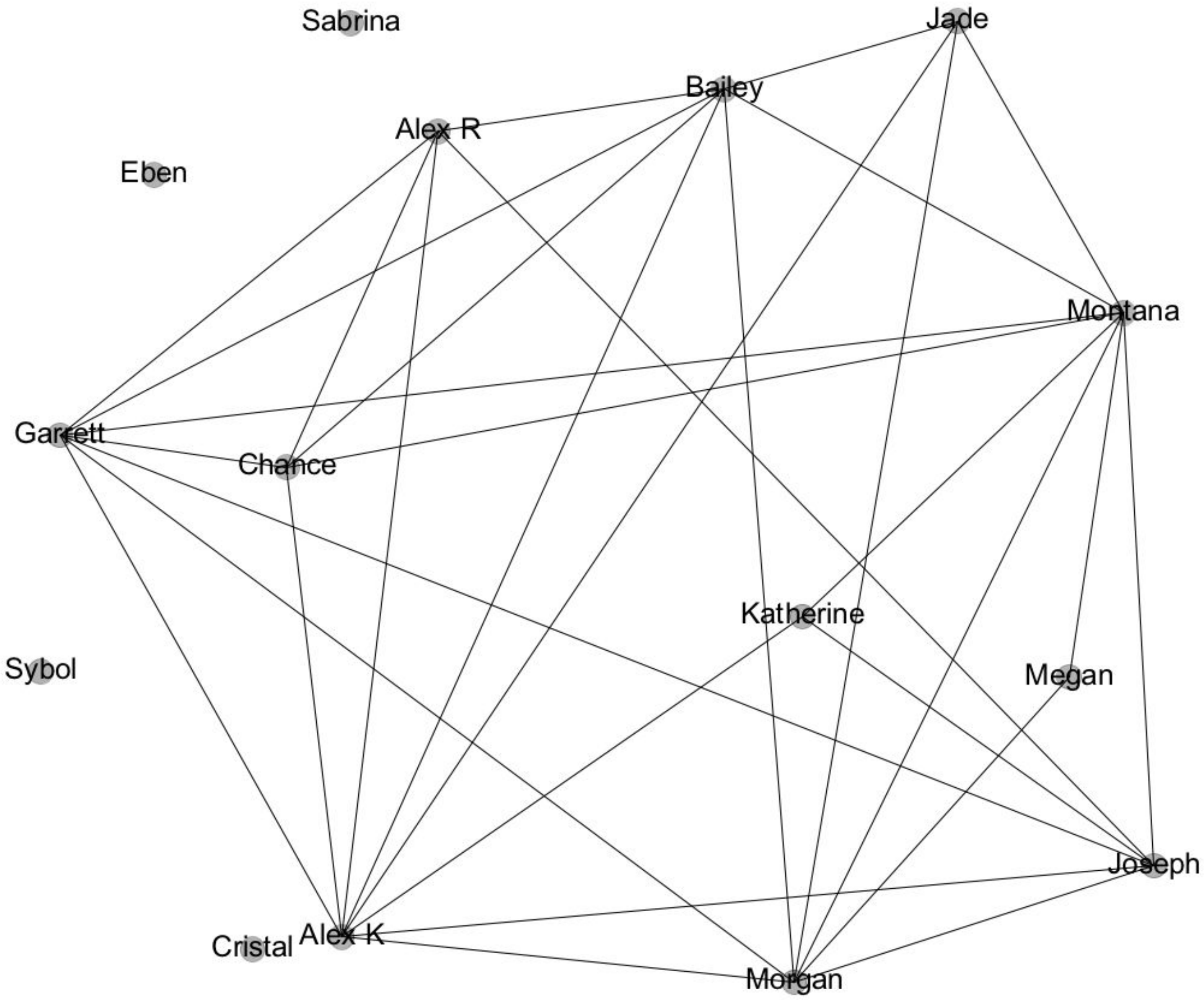
Now - wouldn't it be helpful to has some common language to discuss the things we're noticing?



# How might something go viral in this class?

*Hamiltonian circuits* were modeled as direct Snapchats - in this case passing around test solutions (something that actually happened)

*Eulerian circuits* were modeled with Konigsberg

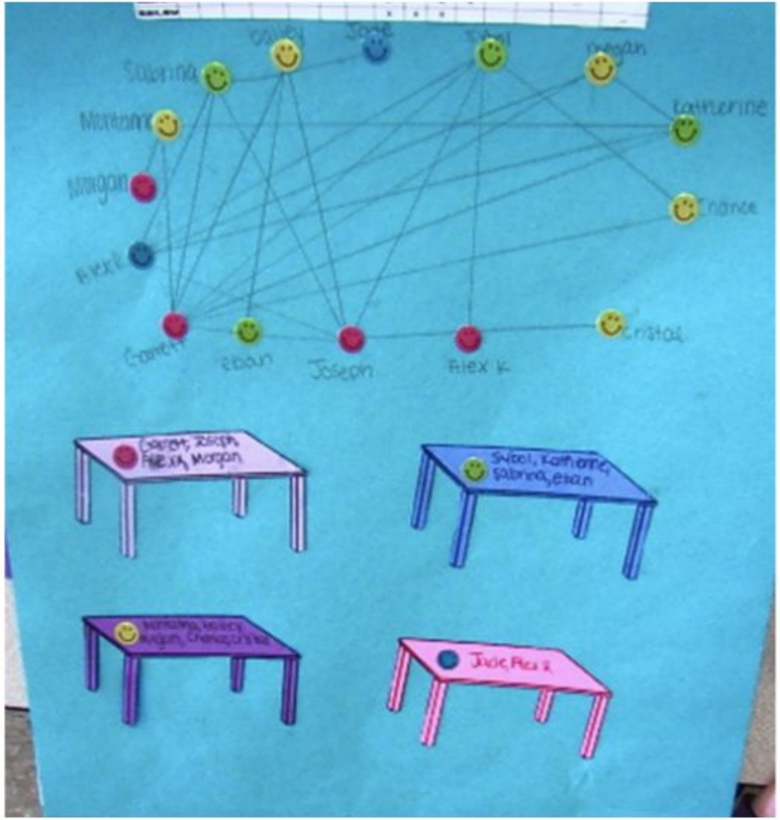


# Graph coloring

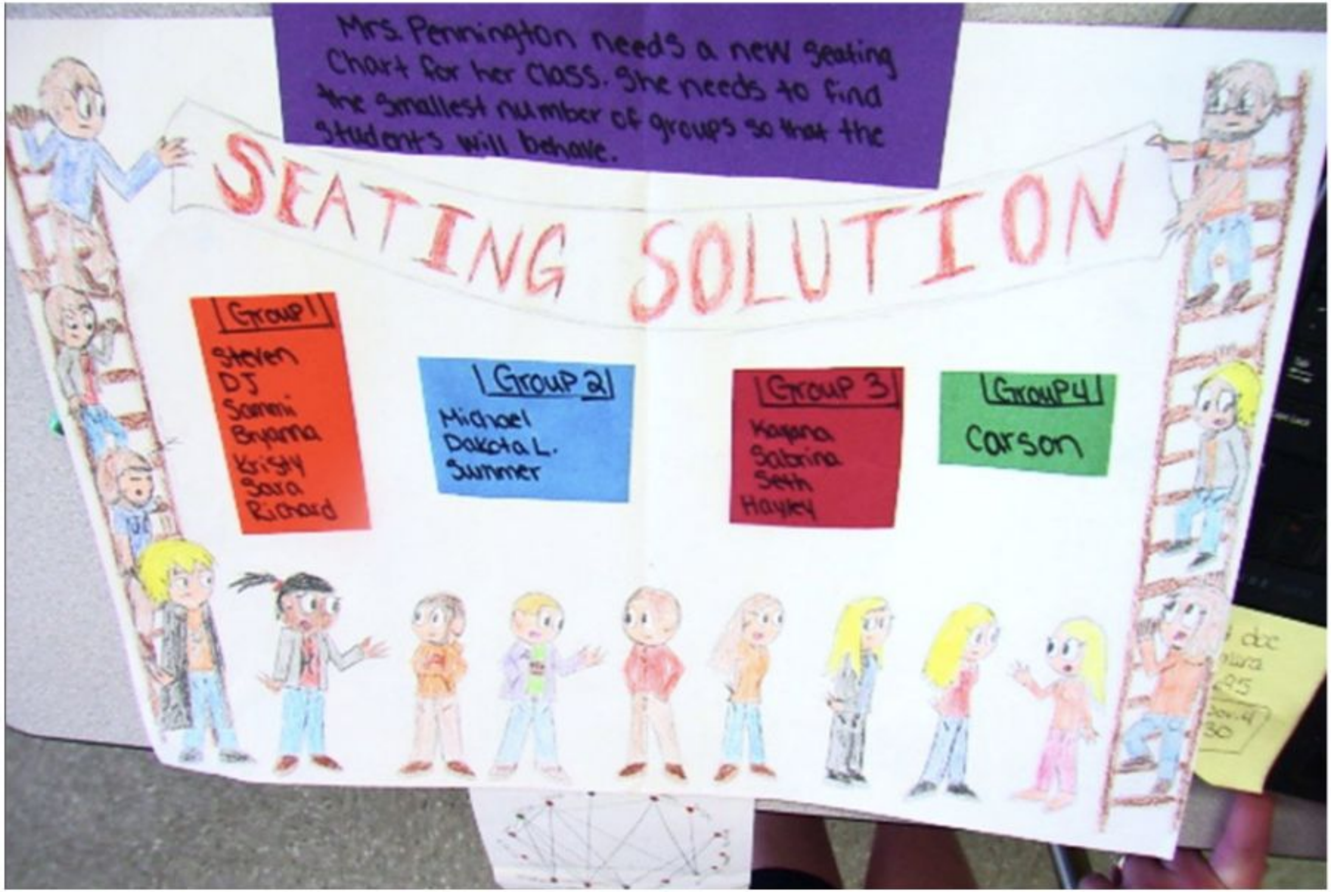
## Class Conflicts

I have enjoyed this class struggling and learning together, but it is time for a new seating chart. We have had our conflicts – whether it be not so friendly presidential candidate debates or who is "hogging" the tape. Below is a matrix that shows our class conflicts that I have observed:

| | Jade | Sybol | Megan | Katherine | Chance | Cristal | Alex K | Joseph | Eben | Garrett | Alex R | Morgan | Montana | Sabrina | Bailey |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jade | — | | | | | | | | | | | | | | |
| Sybol | | — | | | | | | | | | | | | | |
| Megan | | | — | | | | | | | | | | | | |
| Katherine | | | X | — | | | | | | | | | | | |
| Chance | | X | | | — | | | | | | | | | | |
| Cristal | | | | | | — | | | | | | | | | |
| Alex K | | X | | | | | — | | | | | | | | |
| Joseph | | X | | | | X | | — | | | | | | | |
| Eben | | | | | | | | X | — | | | | | | |
| Garrett | | X | X | X | X | | | | X | — | | | | | |
| Alex R | | X | X | X | | | | X | | | — | | | | |
| Morgan | | | | | | | | | | | | — | | | |
| Montana | | | | X | | | | | | X | | X | — | | |
| Sabrina | X | | | | | | | X | | X | X | | | — | |
| Bailey | | | | | | | | X | X | X | | | | | — |




Student solutions to the Class Conflicts problem




Student solutions to the Class Conflicts problem

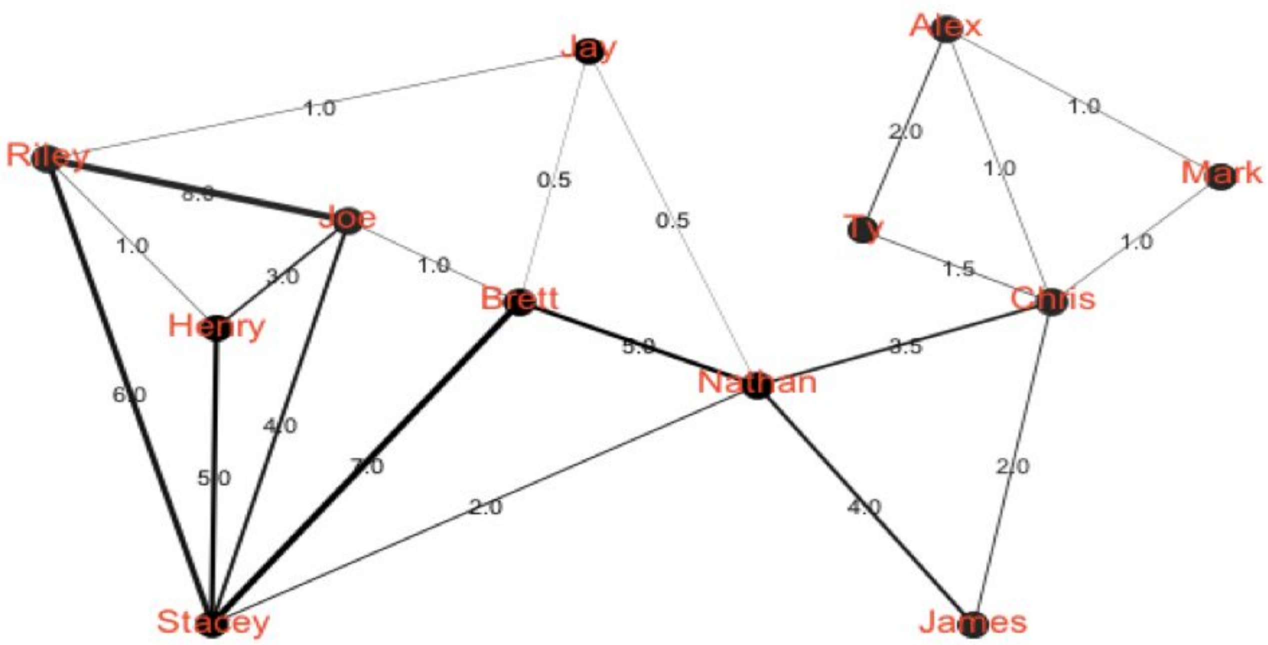
# What have we been assuming here?

- All links are equally strong
- All individuals are equally influential

This ignores that not all friendships are created equally. How could we measure the strength of a connection?

# Modeling connection strength

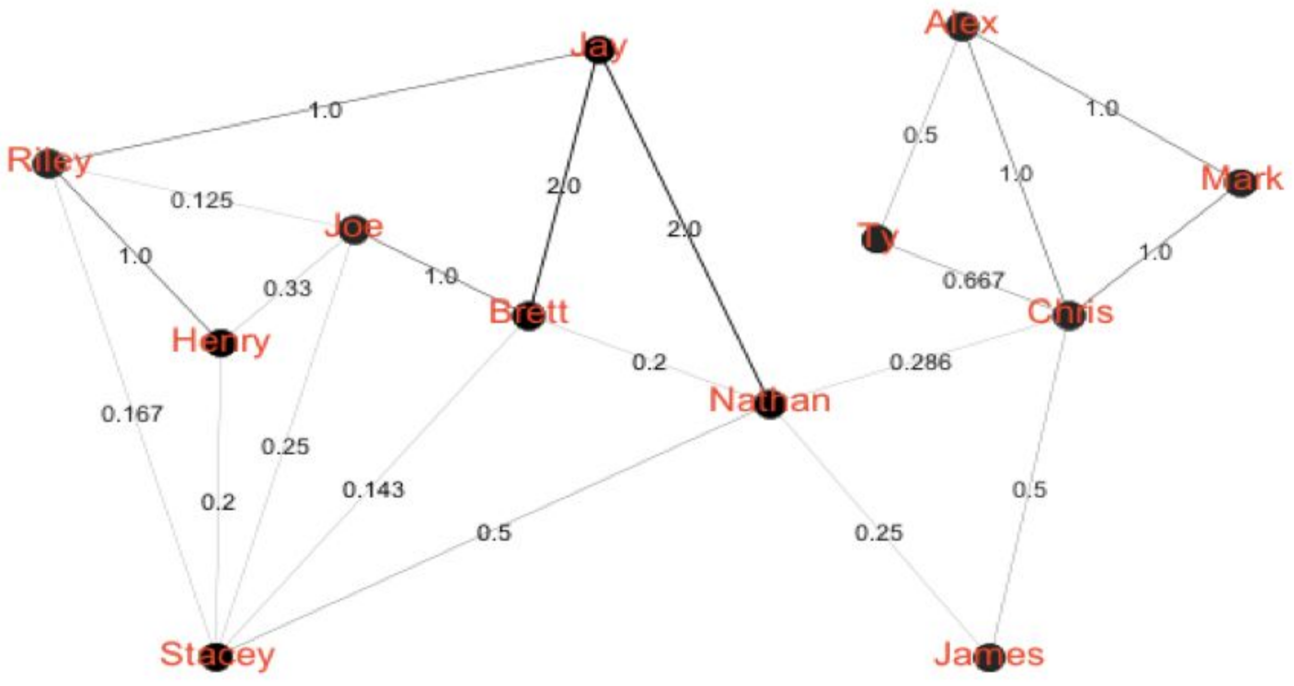
This network adds edge weights measuring how many times they interact, on average, per week



# Modeling connection strength

Maximizing path lengths is difficult, but shortest paths are easier, so we can alter our edge weights so strong relationships have smaller weights (take the reciprocal)

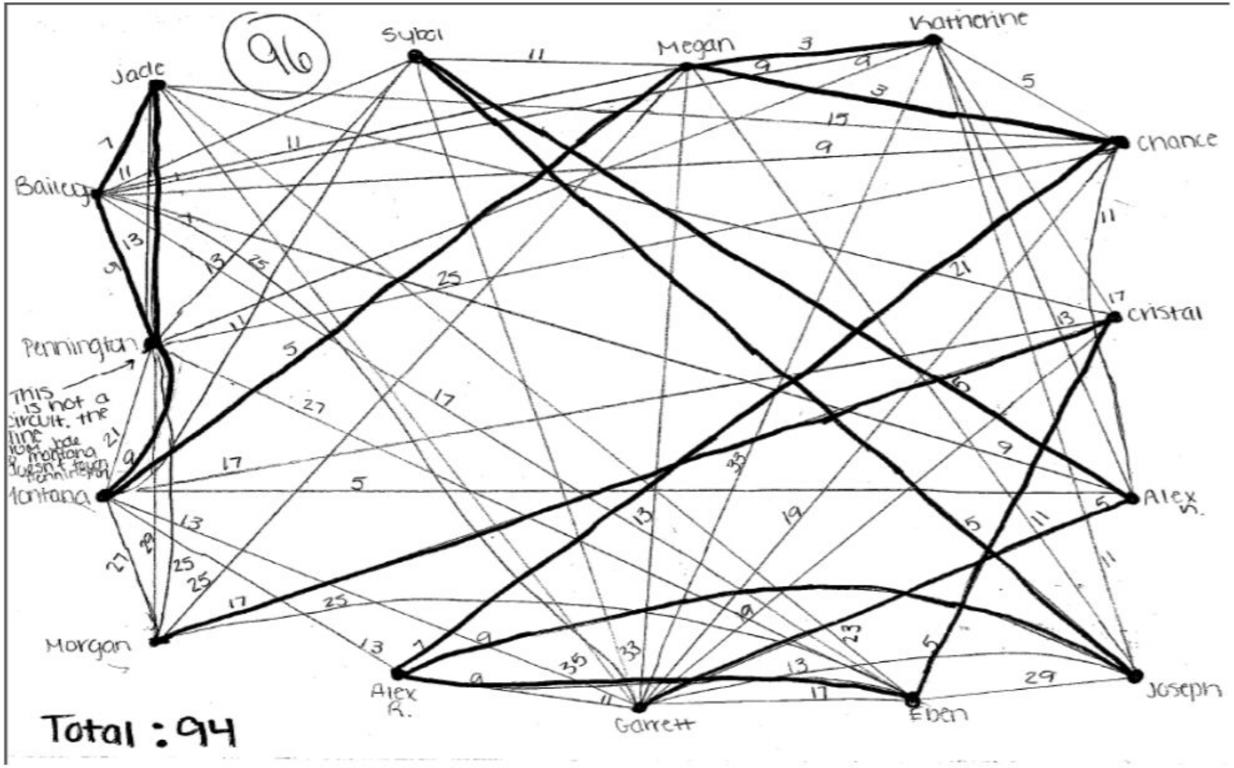
Who's most important now? Can we quantify it?



# Modeling connection strength

This is a simulated network. Edge weights were given by the sum of two dice, and we used different size dice.

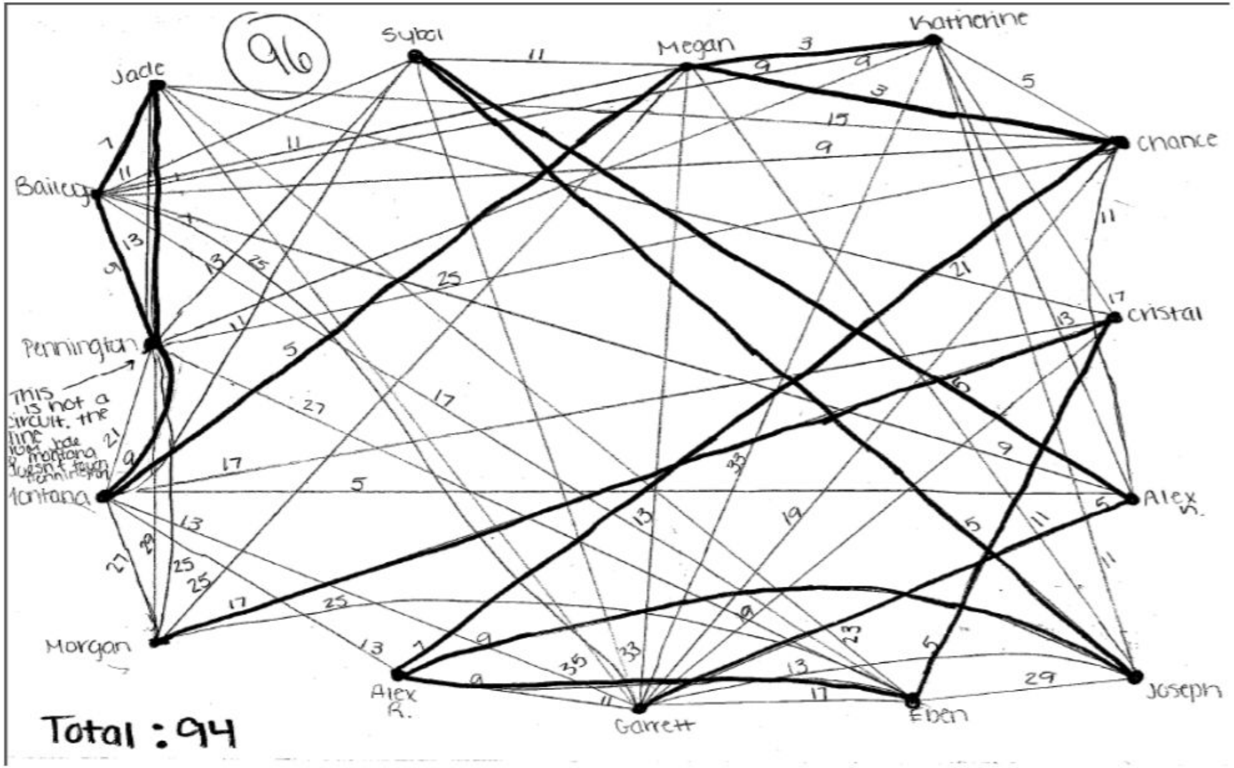
Students were asked to come up with meaning for the edge weights - they decided *the average number of hours between interactions*



# Modeling connection strength

Now we can ask - how will information spread most quickly between a pair of students?

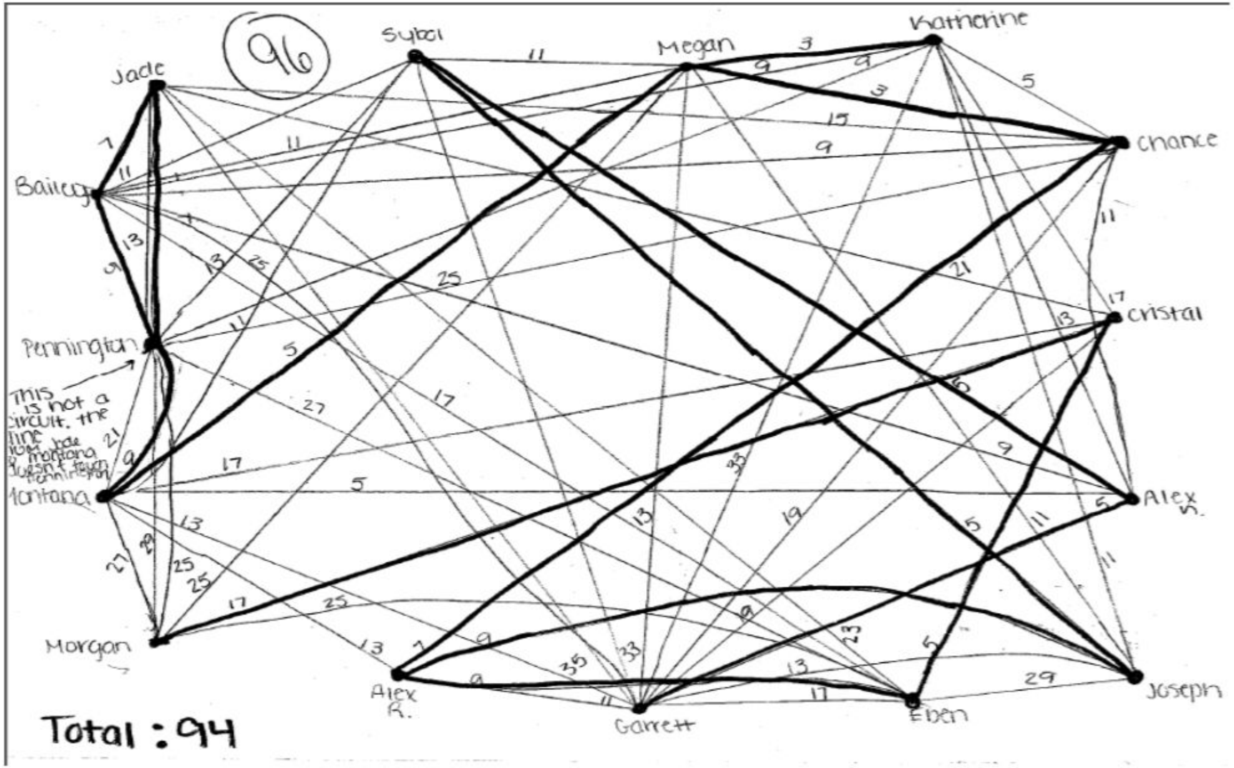
How will it spread most quickly to everyone?

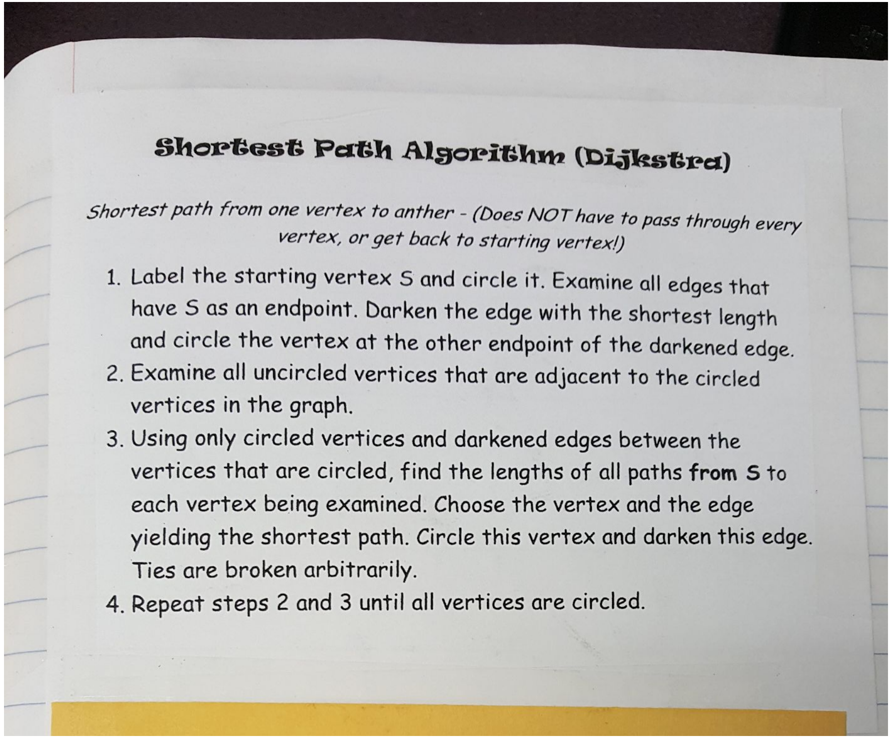


# Modeling connection strength

The *minimum spanning tree* is the cheapest set of edges that contains every vertex.

Can look at Kruskal and Prim algorithms

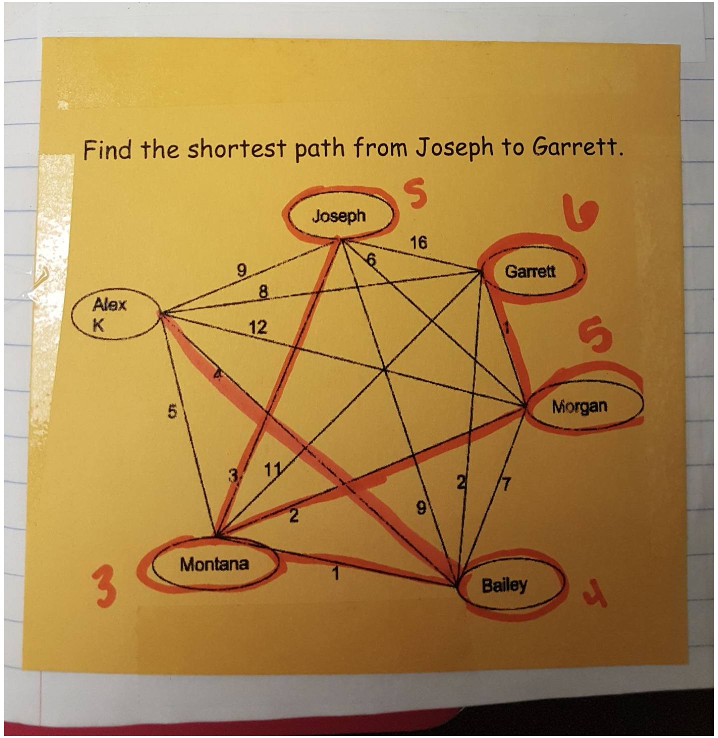




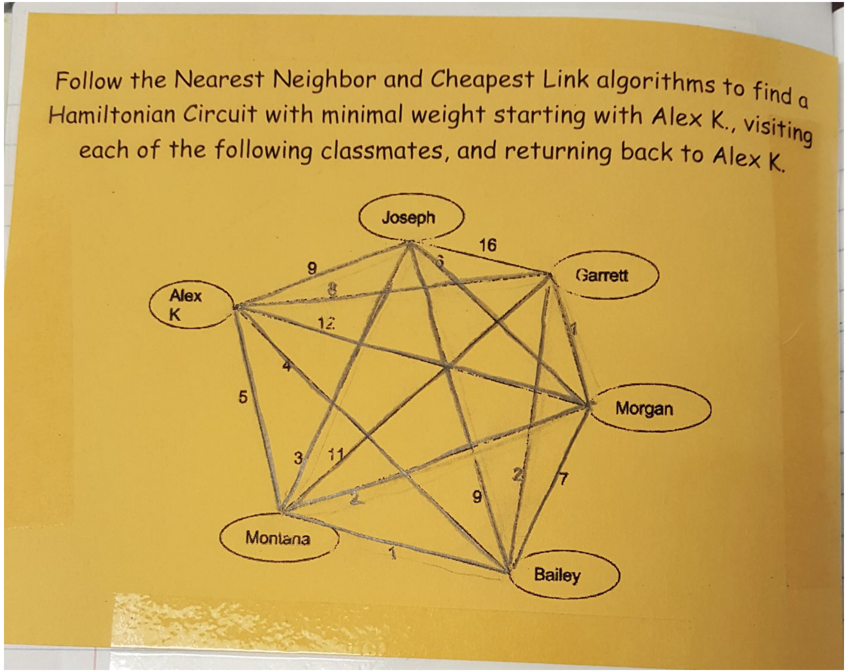
# Shortest Path Algorithm (Dijkstra)

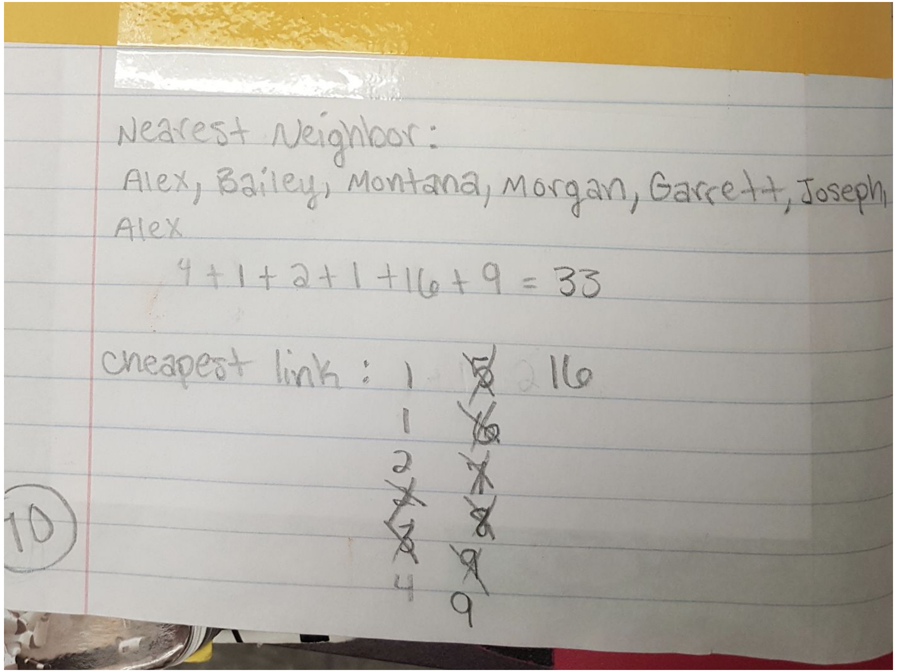
*Shortest path from one vertex to anther - (Does NOT have to pass through every vertex, or get back to starting vertex!)*

1. Label the starting vertex S and circle it. Examine all edges that have S as an endpoint. Darken the edge with the shortest length and circle the vertex at the other endpoint of the darkened edge.
2. Examine all uncircled vertices that are adjacent to the circled vertices in the graph.
3. Using only circled vertices and darkened edges between the vertices that are circled, find the lengths of all paths **from S** to each vertex being examined. Choose the vertex and the edge yielding the shortest path. Circle this vertex and darken this edge. Ties are broken arbitrarily.
4. Repeat steps 2 and 3 until all vertices are circled.

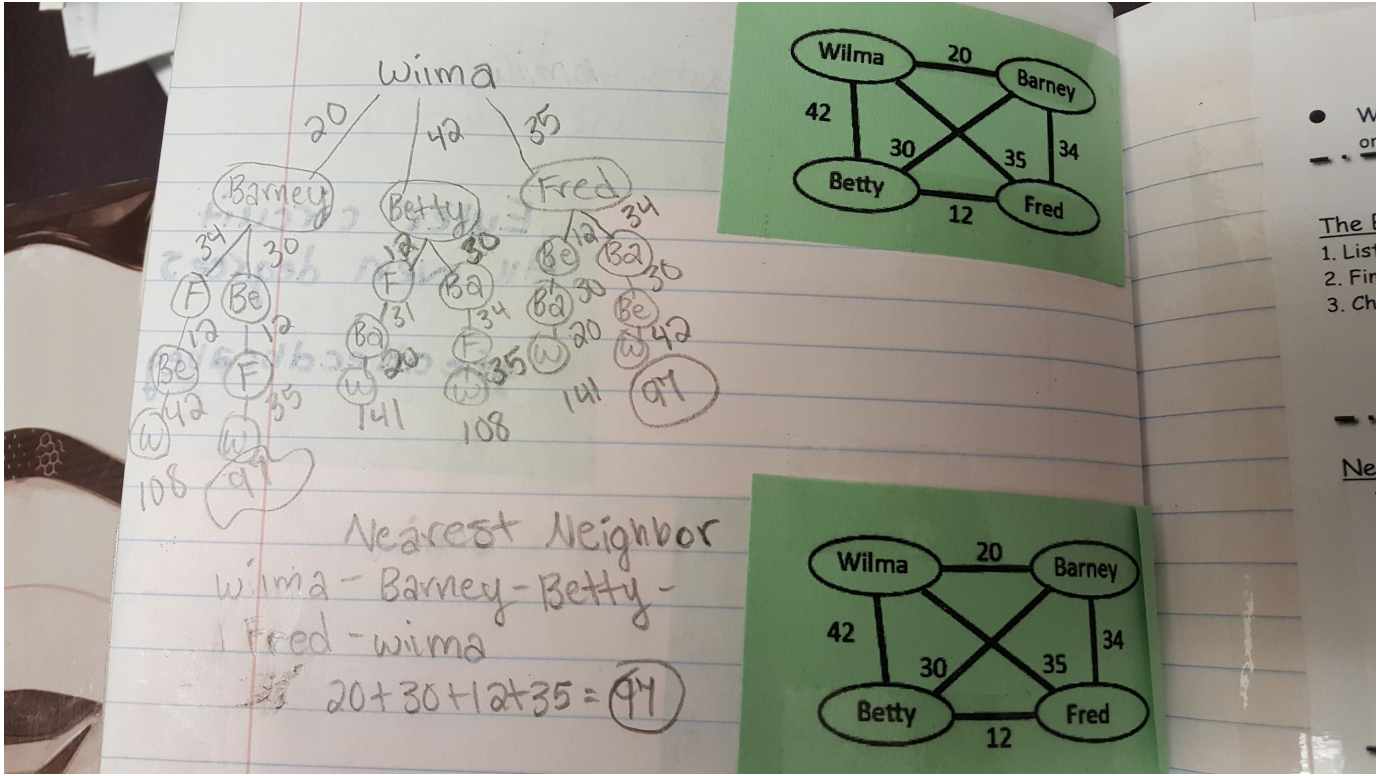



Student work for shortest path algorithm (Dijkstra's)




Student work for Nearest Neighbor and Cheapest Link algorithms for Traveling Salesman problem




Student work for Nearest Neighbor and Cheapest Link algorithms for Traveling Salesman problem

# Some Lessons Learned

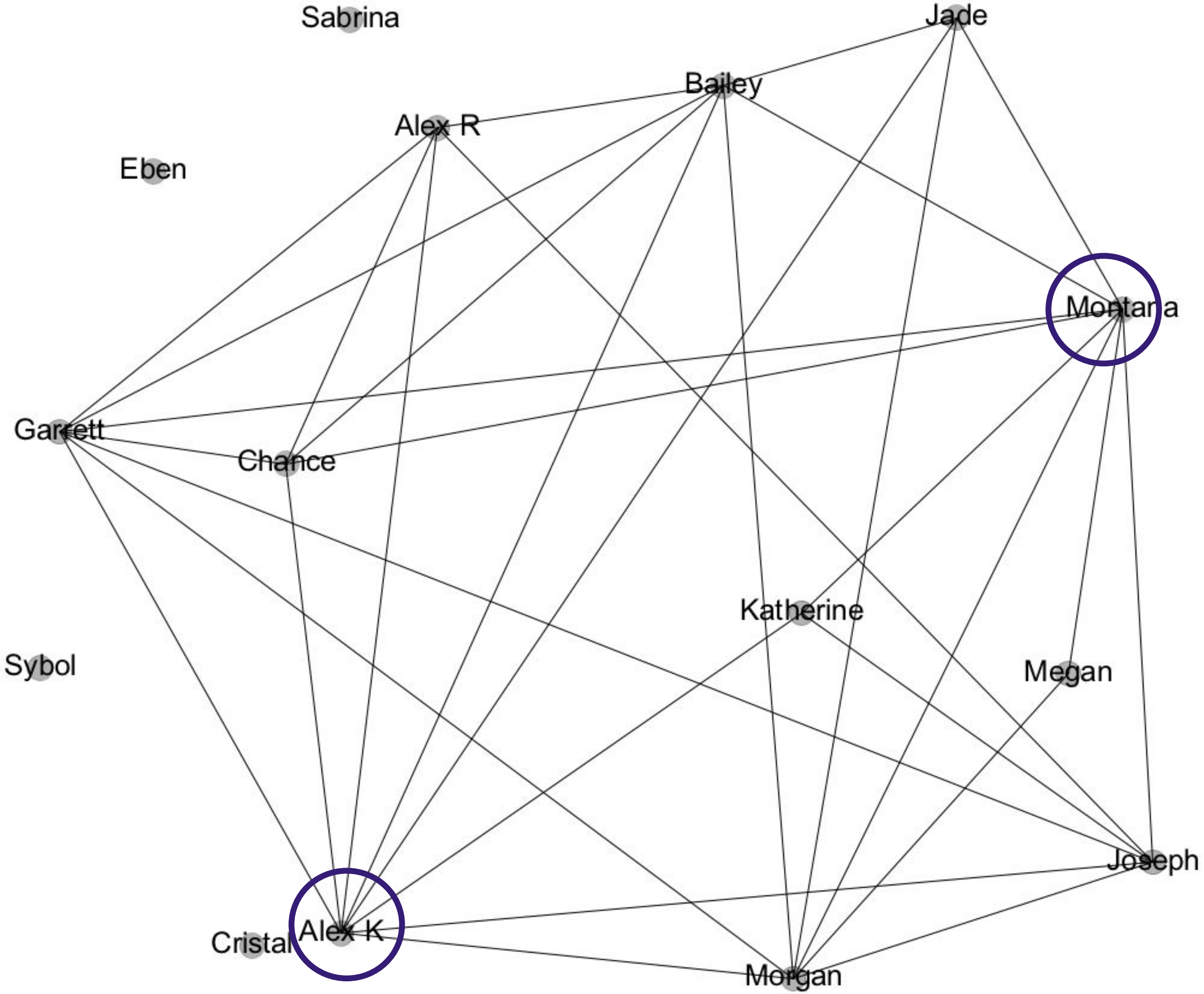
- Initial engagement was much stronger when graph theory was motivated by social networks.
  - It motivated a *need* for the terminology at the beginning
  - Students immediately saw a relevant application for the material
- Some later topics felt a bit forced for social network theory, but since students were comfortable operating in that context, they were willing to work with it.
  - They also saw connections relatively easily. When looking at other types of networks, they transferred the same language.
- Overall, this approach increased student interest and, anecdotally, understanding of the topic

# Other Cool Stuff We Can Do ...

# An Undirected Graph

What group of individuals could post something so that *everyone* saw it?

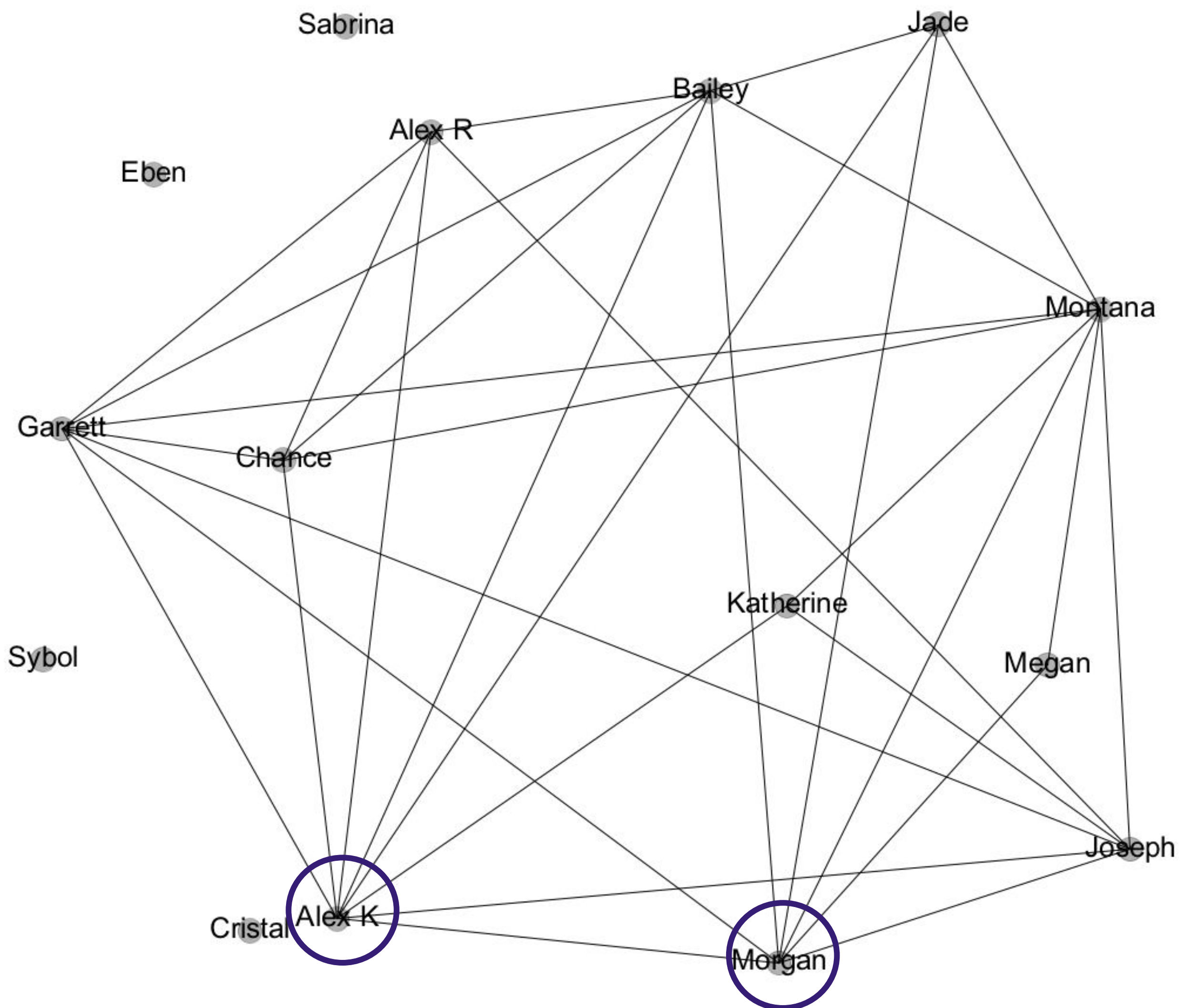
In this case, just Alex K and Montana would be enough to reach everyone



# An Undirected Graph

What group of individuals could post something so that *everyone* saw it?

So could Alex K and Morgan

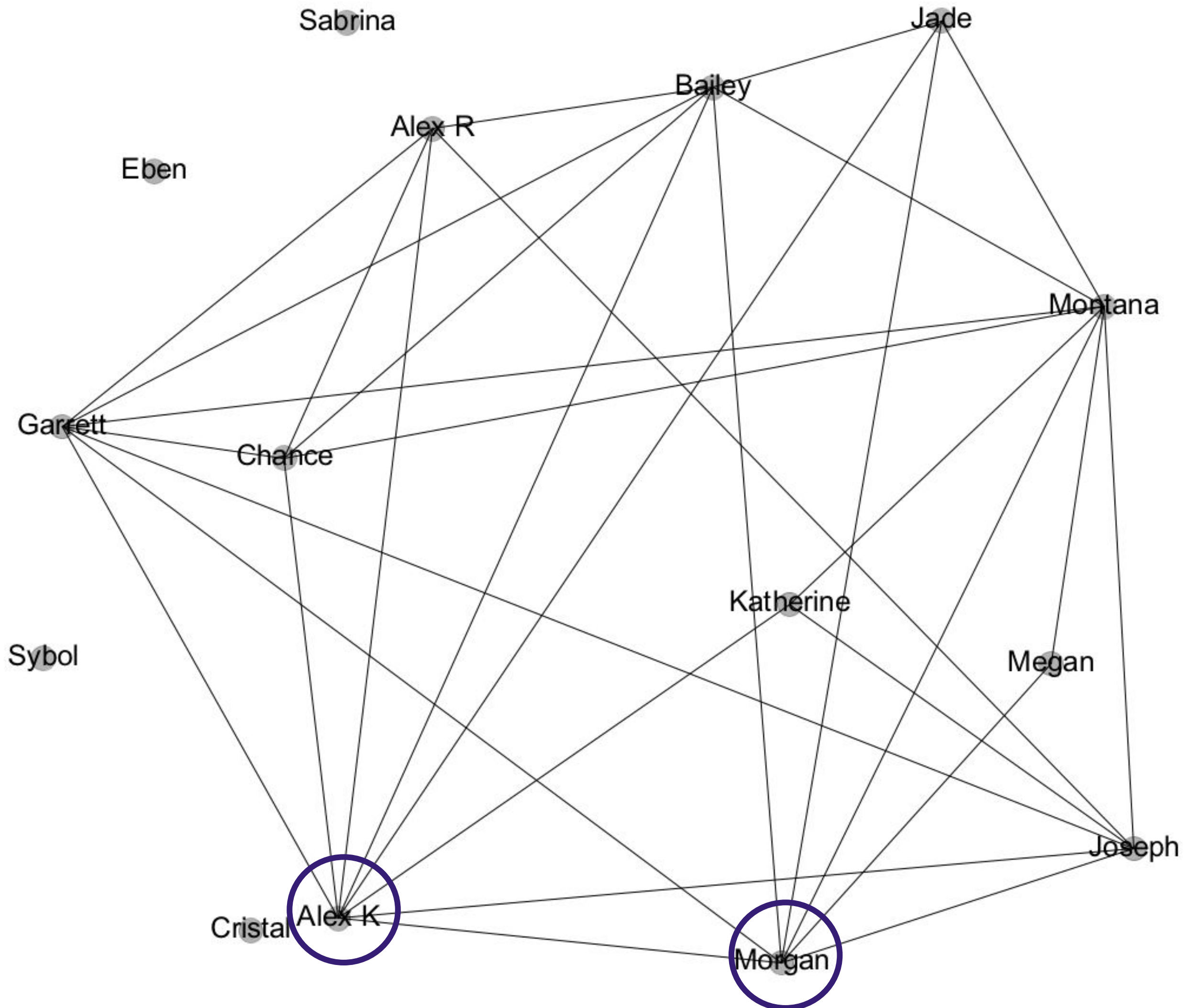


# An Undirected Graph

What group of individuals could post something so that *everyone* saw it?

So would Alex K and Morgan

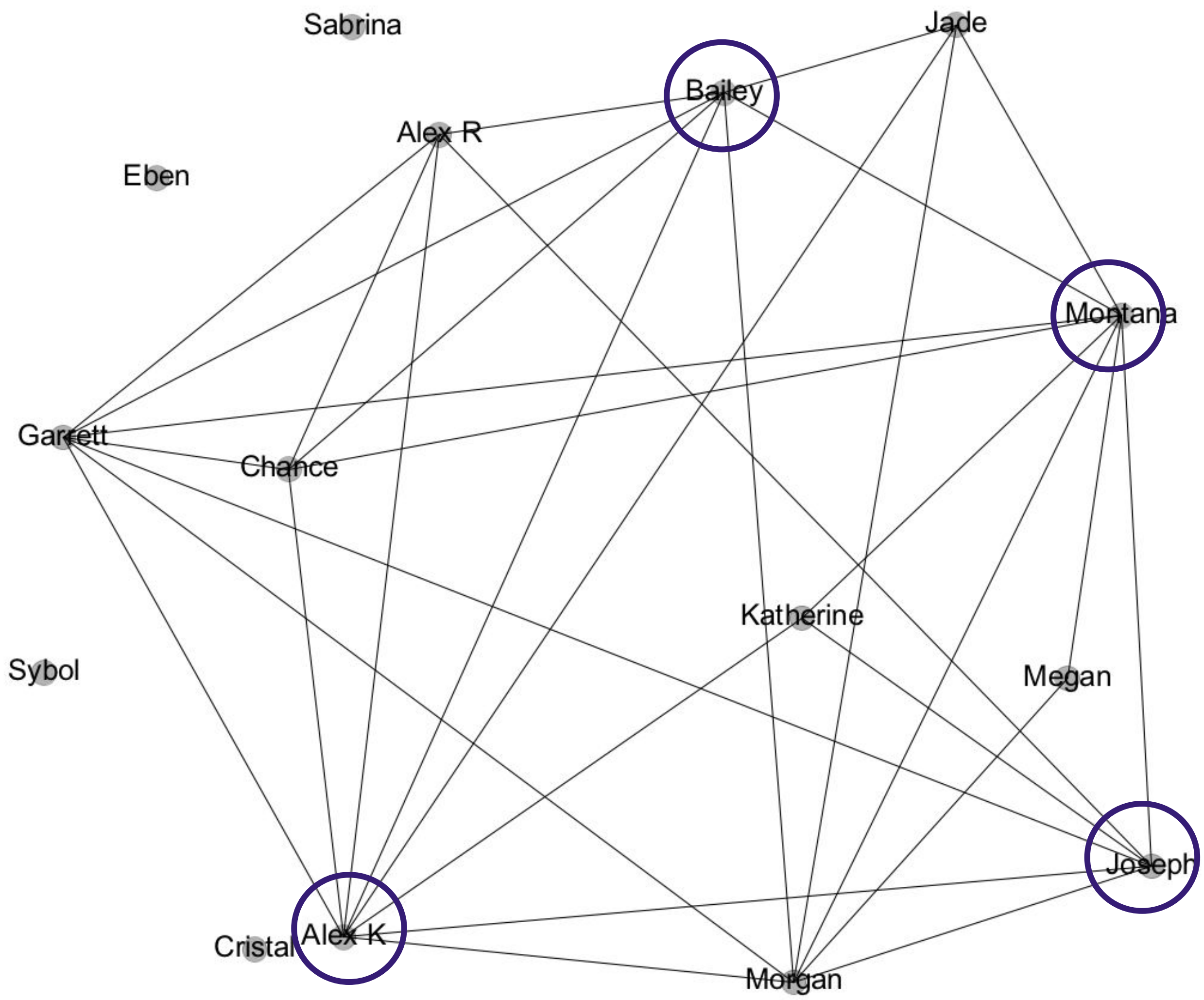
All nodes are included in the *neighborhoods* of these nodes.



# An Undirected Graph

What group of students in the connected component could make it so every student either liked a post or at least half of the student's friends like a particular post? This is the *illusion of majority*

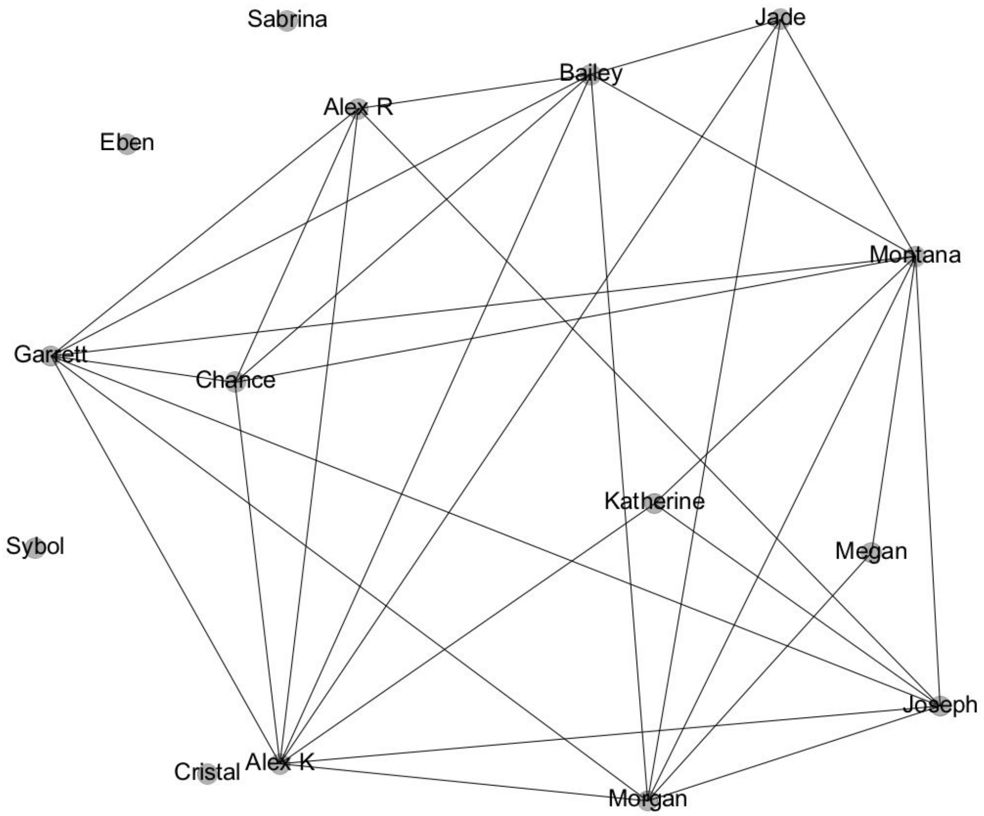
I can do it with 4. Can we do better?



# An Undirected Graph

The *clustering coefficient* of an individual node is the probability that two contacts of a randomly chosen person have contact with each other.

If $A$ is the set of nodes adjacent to $v$,



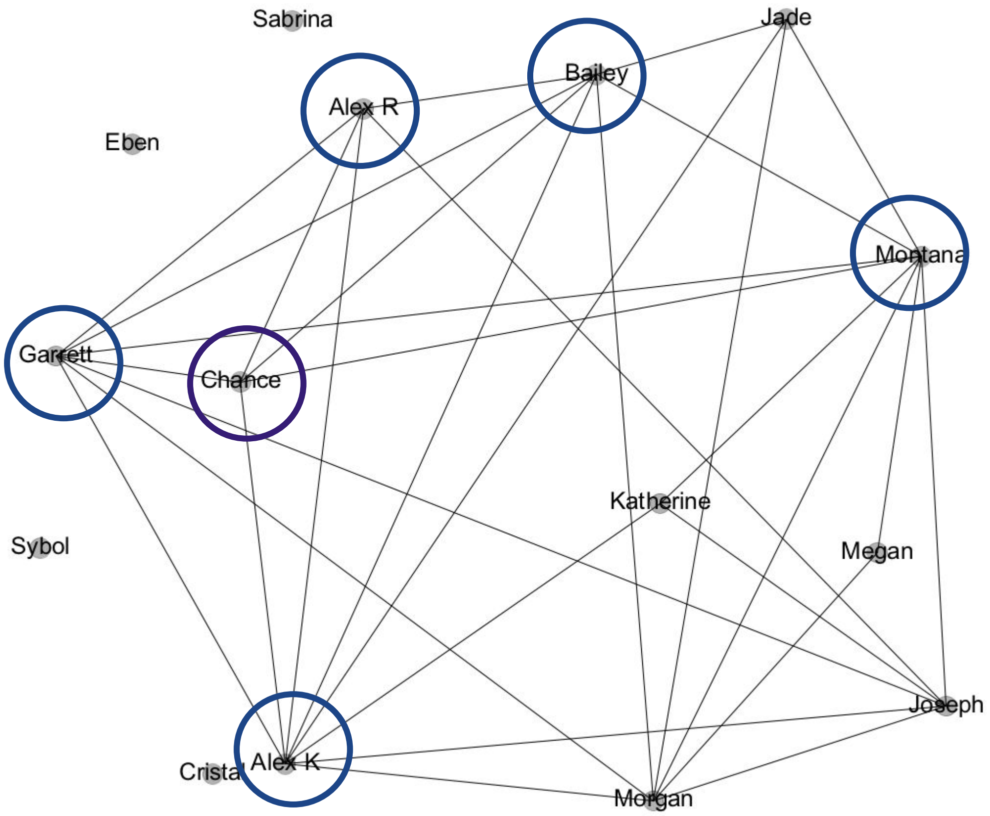

$$C(v) = \frac{\text{number of edges in subgraph made up of } A}{\text{number of edged in a complete subgraph with } n(A) \text{ nodes}}$$

# For example:

Chance has 5 neighbors, $A$={Garrett, Alex R, Alex K, Bailey, Morgan}. That subgraph has 8 edges.

A complete subgraph with 5 vertices would have 8 edges.

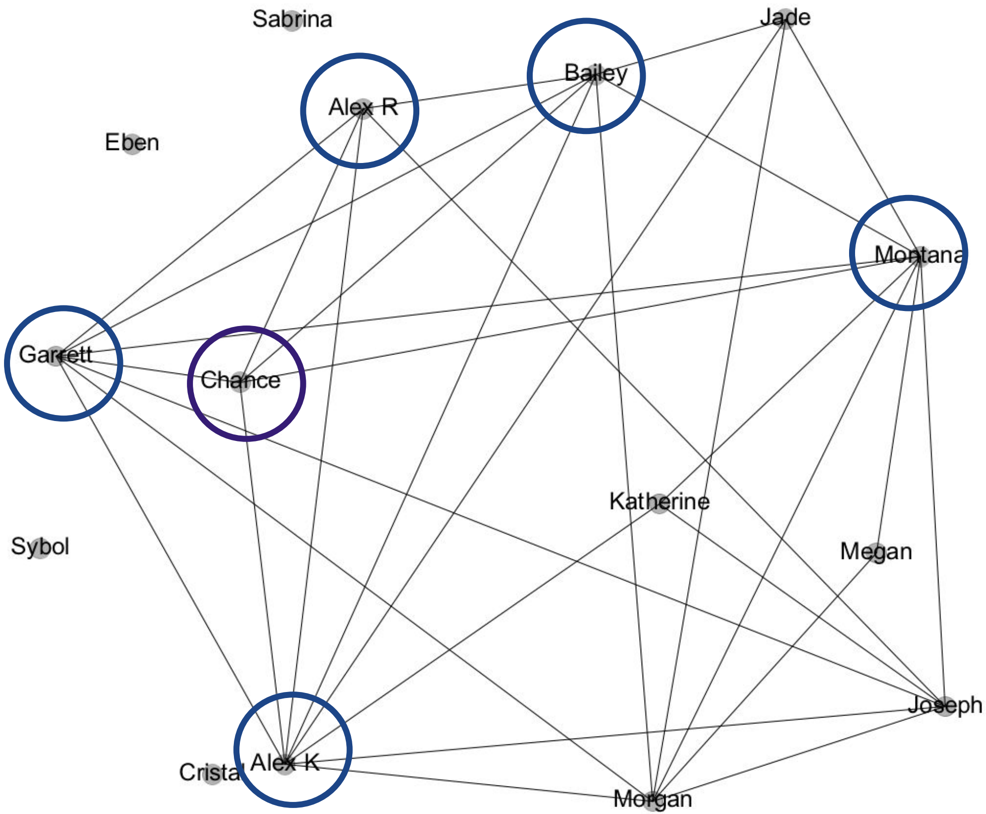
$C(v) = 8/10 = 0.8$




$$C(v) = \frac{\text{number of edges in subgraph made up of } A}{\text{number of edged in a complete subgraph with } n(A) \text{ nodes}}$$

# For example:

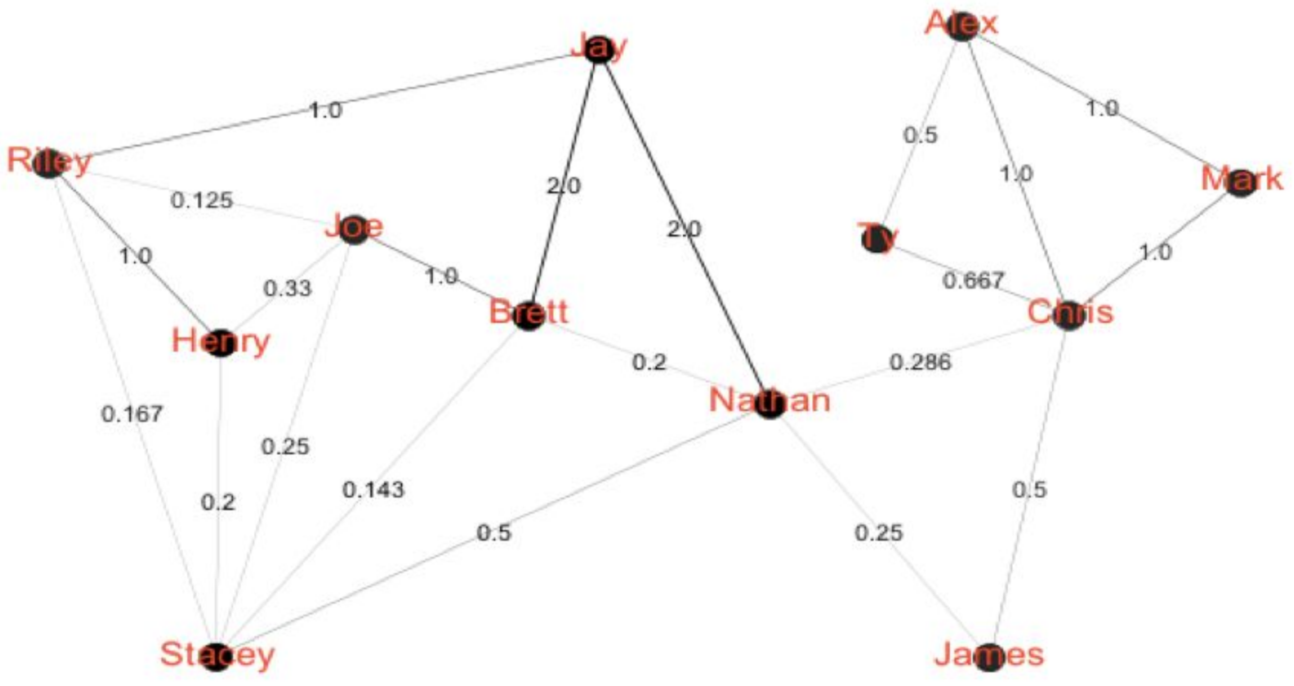
A large clustering coefficient would that reposting by a small number of friends would quickly give the illusion of a majority




$$C(v) = \frac{\text{number of edges in subgraph made up of } A}{\text{number of edged in a complete subgraph with } n(A) \text{ nodes}}$$

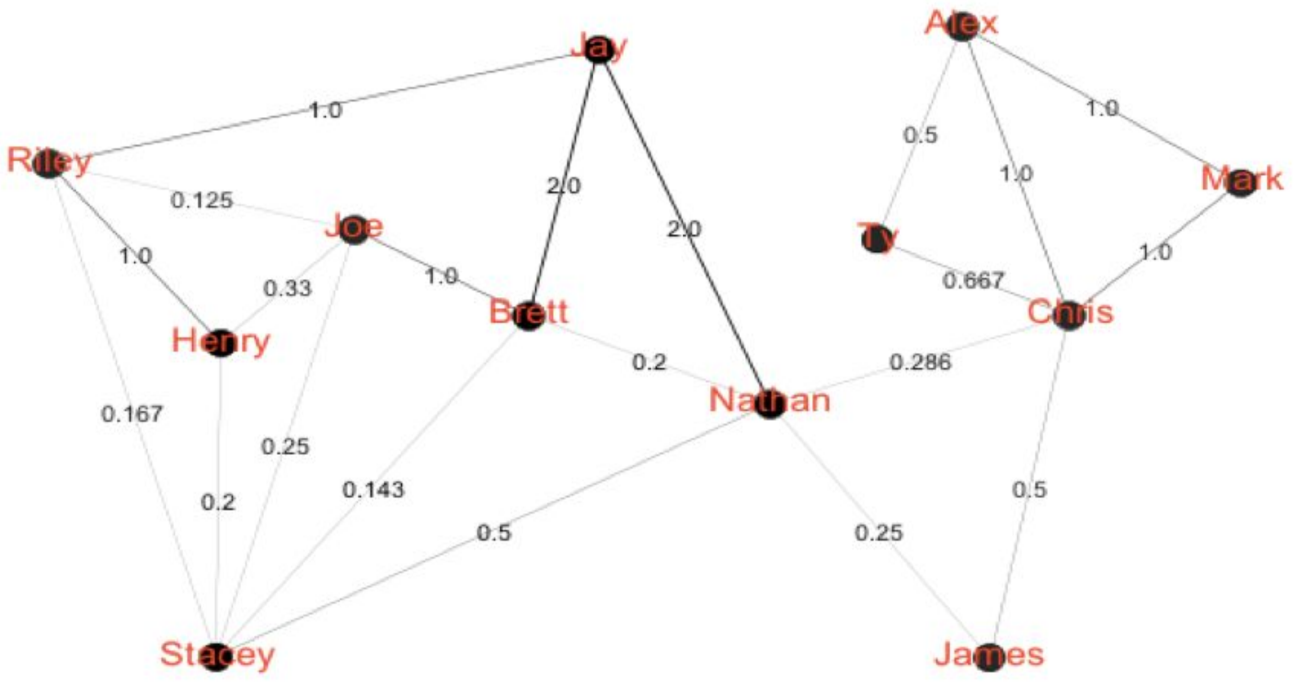
# Measures of Centrality

The *farness* of a vertex is the sum of all the shortest paths to other vertices. The *closeness centrality* is the reciprocal of the farness.



# Measures of Centrality

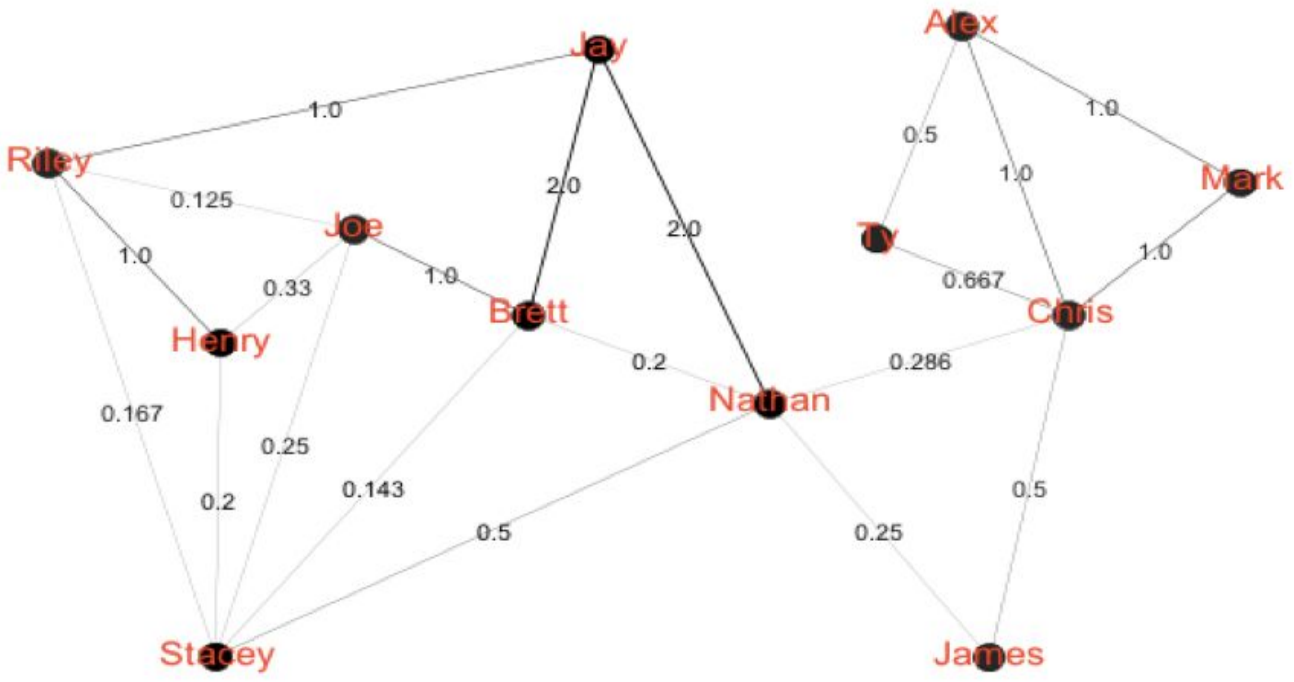
| | closeness |
|---|---|
| Nathan | 1.545 |
| Stacey | 1.818 |
| Chris | 1.818 |
| Brett | 1.909 |
| Jay | 2 |



# Measures of Centrality

The *betweenness centrality* is a measure of how many shortest paths pass through the vertex

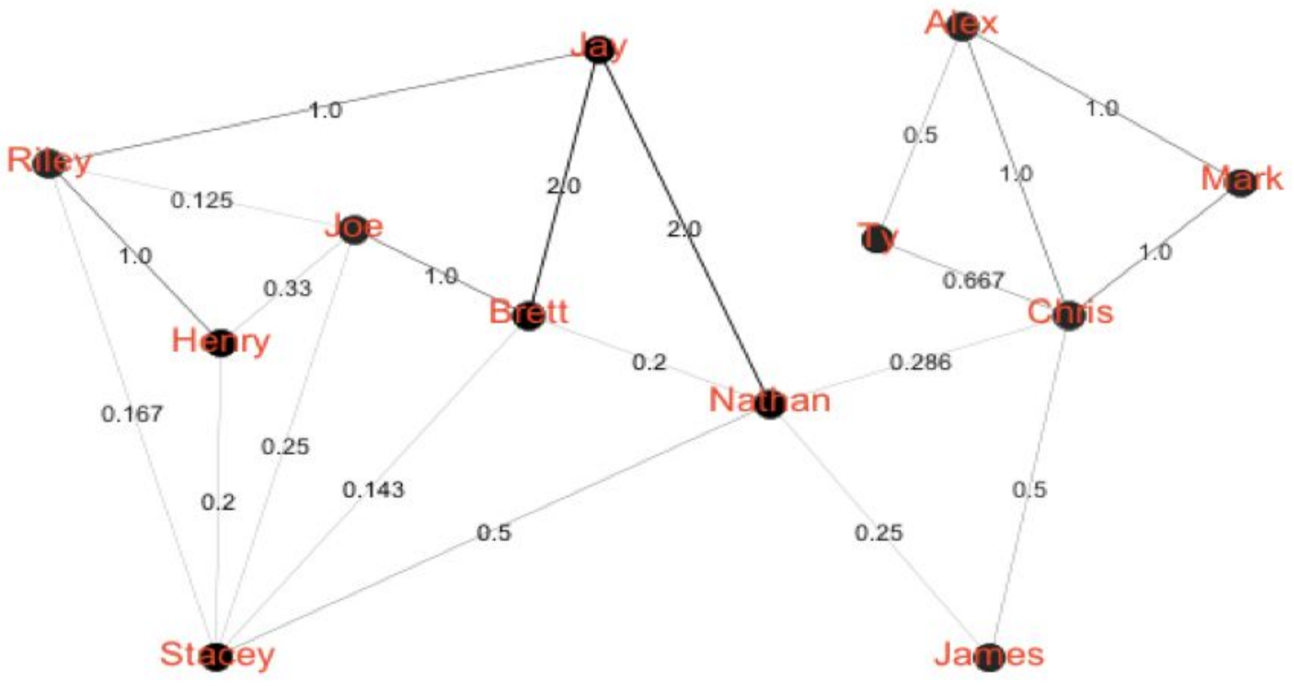
For each pair of vertices, find the shortest paths, and then compute the fraction that pass through the vertex in question.



# Measures of Centrality

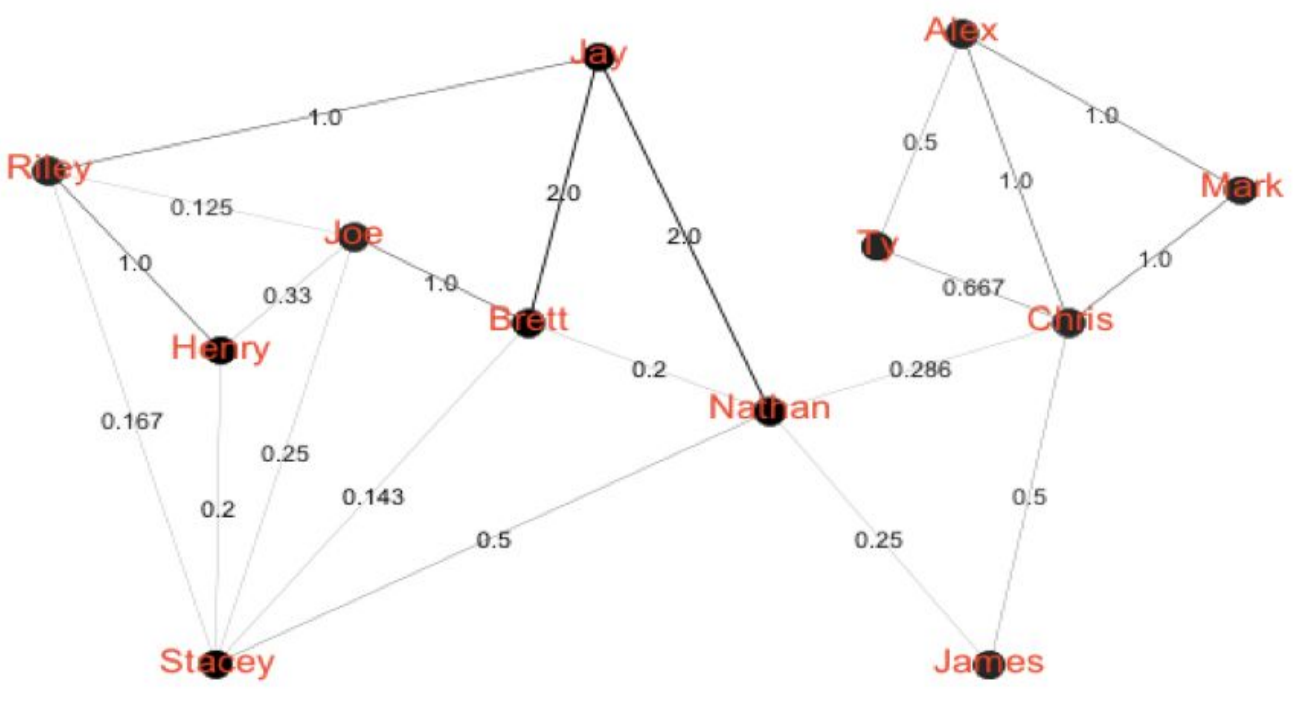
The *betweenness centrality* is a measure of how many shortest paths pass through the vertex

For a given vertex, add those fractions up for every pair.



# Measures of Centrality

| | betweenness |
|---|---|
| Nathan | 30.333 |
| Chris | 24.5 |
| Stacey | 12.833 |



# Other Cool Stuff To Do ..

## Degrees of Separation

- On Twitter, almost everyone is within 5 degrees of separation
- On Facebook, the average a couple years ago was 4.74. 99.6% of users are separated by 5 degrees
- Wikipedia links are another good network

## Weight vertices instead of edges

- Eigenvector Centrality
- Page Rank

# Other Cool Stuff To Do ..

On average, your friends will have more friends than you - why?

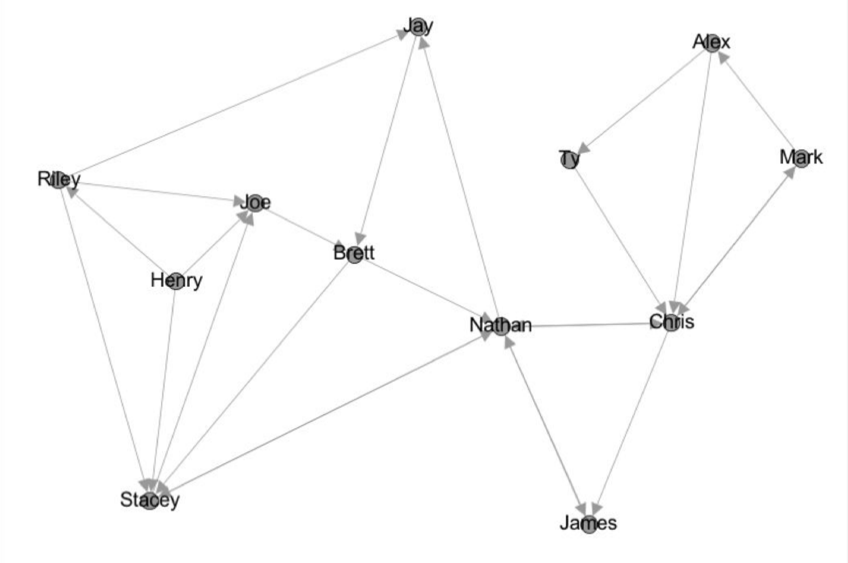
Disease spread

- Who should be vaccinated in order to limit the spread most effectively (degree distribution
- What networks are vulnerable (mean shortest path
- Check out Angela & George Shiflet, “Getting the ‘edge’ on the next flu pandemic: We should’a ‘node’ better”. Published on shodor.org (http://shodor.org/petascale/materials/UPModules/socialNetworks/

# Other Cool Stuff To Do ..

## Directed Edges

- Twitter provides an excellent model for directed edges
- What will change when edges are directed? Will our algorithms still work?
- Who is most influential now?

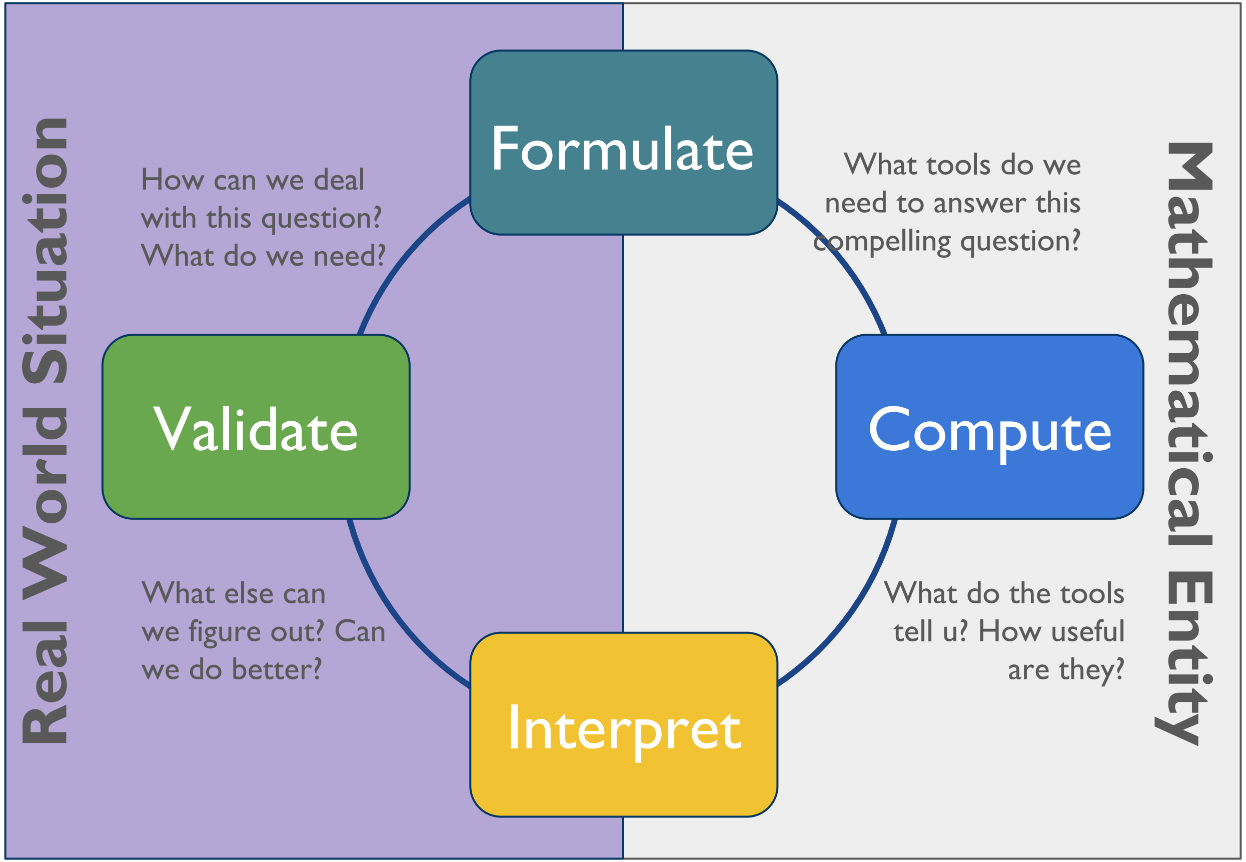


# Social Networks and Graph Theory

Rich mathematical modeling in action ..

Resources for analyzing graphs:

- Gephi (free)
- NodelXL (Excel plugin, Windows only)



# Thank you!

Todd Abel
University of Central Arkansas
tabel1@uca.edu

Christina Pennington
Ashe County (NC) High School
christina.pennington@ashe.k12.nc.us