PD and Instructional Tools for Advancing ELLs’ Mathematics and Language Through an Integrated Approach

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Washington, DC
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Rodrigo J. Gutiérrez, Ph.D., University of Arizona
Warm-up (Handout): How are you integrating mathematics and language learning for English learners?
Welcome!

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Table: ESOL Enrollment Data
- Total ESOL Population: 24,445

Table: ESOL Student Data
- Languages: ~130
- Countries: ~125
- Country of Birth:
  - United States (61%)
  - El Salvador (19%)
  - Guatemala (5%)
  - Honduras (4%)
  - Afghanistan (1%)
An excellent mathematics program requires that all students (including English learners) have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential. 

(from Principles to Action, NCTM)

Teaching children mathematics necessitates attending to the language of the mathematics classroom, including the mathematical content and the multi-faceted discourse. This is even more critical for English learners. The key to equity and access for English learners - who are acquiring a new language and a new content at the same time - lies in intentional planning and facilitating mathematics learning together with language learning by strategically engaging them in meaningful activities that allow for authentic use, development, and advancement of English language skills in the context of mathematics.

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Session Agenda

• Introductions and Audience Check-in
• ESOL – UMD Focus Group: A Unique Partnership
• Exploration with Instructional Tools
• Take-aways and Why this Matters
• Closure and Questions

Session Outcome:
Participants will leave with actionable take-aways for addressing equity and access for ELLs in mathematics through integration of language and mathematics learning
What’s on the Table?

• Handouts
• Folders (2 per table)
• Posters
The Changing Landscape for Teaching Mathematics

- Growing numbers of English learners in public schools
- CCSS: Mathematical Content AND Practices
- WIDA Language Development Standards
  - ESOL teachers support language development (L, S, R, W) in ALL content areas, including mathematics
- Teachers of Mathematics = Teachers of Language
  
  Focus on promoting student DISCOURSE for both mathematics AND language development!

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OVERLAPPING KNOWLEDGE FOR TEACHING MATHEMATICS TO ENGLISH LANGUAGE LEARNERS

Discourse in Mathematics & the Language of Mathematics

Teaching and Learning of Mathematics - Content & Pedagogy

English Language Acquisition & English Learners

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PGCPS ESOL & UMD CfME Partnership

2014-15
FOCUS GROUP
Engaging ELLs in Mathematical Discourse*

2015-16
FOCUS GROUP
Supporting ELLs in Mathematics

2016-17
FOCUS GROUP
Mathematical Literacy for ELLs*

2017-18
FOCUS GROUP
Rigorous Math Instruction for ELLs (with County Mathematics Department)

GRAD COURSES
(ITQ Grant): ELL Math (ES)

GRAD COURSES
(ITQ Grant): ELL Math South

PLC
ESOL Office-Based (Teaching Teams)

PLCs
School-Based (Teacher-Facilitated)

ESOL Coaching Support

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Standards That Guide Our Collaborative Work

- Discourse in Mathematics & the Language of Mathematics
- Teaching and Learning of Mathematics - Content & Pedagogy
- English Language Acquisition & English Learners
- WIDA ELD Standards
- Mathematics Education/CCSS

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Standard 3: The Language of Mathematics

English language learners communicate information, ideas, and concepts necessary for academic success in the content area of mathematics.
The Language of Mathematics: Defining Features
Planning - Teaching - Monitoring - Assessing

Vocabulary:
everyday words
academic words
mathematical terms

Language Forms & Conventions:
language structures
singular & plural nouns
comparatives & superlatives
types of sentences
subject-verb agreement

Linguistic Complexity:
extended & logically connected utterances of speech
(oral & written)

Word/Phrase Level
Sentence Level
Discourse Level

Language Domains: Listening, Speaking, Reading, Writing

Adapted by Galina (Halla) Jmourko from The Defining Features of the Academic Language in WIDA’s Standards, WIDA Consortium, Draft, 2011

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#ELLsCan Framework:

- **Focusing on students’ abilities**
- **Identifying next steps for planning and teaching**

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### CAN DO Descriptors (PreK-12)

#### Figure 5M: CAN DO Descriptors for the Levels of English Language Proficiency, PreK-12

For the given level of English language proficiency, English language learners can:

<table>
<thead>
<tr>
<th>Level 1: Entering</th>
<th>Level 2: Beginning</th>
<th>Level 3: Developing</th>
<th>Level 4: Expanding</th>
<th>Level 5: Bridging</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LISTENING</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Listen to stories, pictures, words, phrases</td>
<td>Listen to stories, pictures, words, phrases</td>
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<td>Listen to stories, pictures, words, phrases</td>
<td>Listen to stories, pictures, words, phrases</td>
</tr>
<tr>
<td><strong>SPEAKING</strong></td>
<td></td>
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</tr>
<tr>
<td>Name objects, people, pictures</td>
<td>Name objects, people, pictures</td>
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<td>Name objects, people, pictures</td>
</tr>
<tr>
<td>Answer WH-questions (who, what, when, where, why)</td>
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</tr>
<tr>
<td><strong>READING</strong></td>
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<td></td>
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<tr>
<td>Match nouns and symbols to words, phrases or environmental print</td>
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</tr>
<tr>
<td>Identify concepts about print and test features</td>
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</tr>
<tr>
<td><strong>WRITING</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Label objects, pictures, diagrams</td>
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www.wida.us
Guiding Principles of Language Development (LD)

First languages, Cultures, Life Experiences

Context

Language - Content Connection

Supports

Meaningful Interaction

Listening, Speaking, Reading, & Writing

Social, Instructional, & Academic Language

Guiding Principles of Language Development

1. Students’ languages and cultures are valuable resources to be tapped and incorporated into schooling.

2. Students’ home, school, and community experiences influence their language development.

3. Students draw on their metacognitive, metalinguistic, and metacultural awareness to develop proficiency in additional languages.

4. Students’ academic language development in their native language facilitates their academic language development in English. Conversely, students’ academic language development in English informs their academic language development in their native language.

5. Students learn language and culture through meaningful use and interaction.

6. Students use language in functional and communicative ways that vary according to context.

7. Students develop language proficiency in listening, speaking, reading, and writing interdependently, but at different rates and in different ways.

8. Students’ development of academic language and academic content knowledge are inter-related processes.

9. Students’ development of social, instructional, and academic language, a complex and long-term process, is the foundation for their success in school.

10. Students’ access to instructional tasks requiring complex thinking is enhanced when linguistic complexity and instructional support match their levels of language proficiency.

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Previous *Isolated* PD Efforts

**Prince George’s County Public Schools**

- Book Study–Based PD: 4-6 paid evening sessions
- Focus Group 2013-14: 4 half-day sessions on *Mathematical Discourse for ELs* (video analysis)
- **Need:** More research-based mathematical pedagogy

**Center for Mathematics Education, UMD**

- Outreach Courses: Graduate evening courses for practicing teachers with a focus on both content and pedagogy
- **Need:** More authentic connections to classrooms and local contexts for working with English Learners
OVERLAPPING KNOWLEDGE FOR TEACHING MATHEMATICS TO ENGLISH LANGUAGE LEARNERS

Mathematics Education/CCSS

Teaching and Learning of Mathematics - Content & Pedagogy

Discourse in Mathematics & the Language of Mathematics

English Language Acquisition & English Learners

WIDA ELD Standards

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Engaging ELLs in Mathematics Discourse

Focus Group

Professional Development Structure, Components, and Process:

❖ Participants: 25 mainstream and ESOL teachers (Gr. 2-6)
❖ 7 full-day sessions (substitutes provided)
❖ Language of Mathematics component (with math content as the basis of LD)
❖ Mathematical Pedagogy component (with attention to the role of language)
❖ Show-and-Tell small group AND whole group sharing:
  ➢ Implementation of new learning/strategy in the classroom
  ➢ Evidence through classroom artifacts, student work, videos
  ➢ Feedback from colleagues
❖ LOTS of research-based resources
❖ Planning time
❖ Personalized on-site support (planning, coaching, videotaping, debriefing)

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Mathematical Pedagogy Component
Emphasizing Teaching Practices

- Problem Solving-Based Mathematics
  - Teaching Mathematics for Understanding

- Formative Assessment Protocols (CGI)

- The Language of Area and Perimeter

- Number Sense Games

- Math Talk and Discourse Moves
  - Patterns of Questioning
  - Talk Moves
  - Writing

- Using Children’s Literature for Teaching Mathematics
Mathematical Discourse: Productive Classroom Discussions

• Anticipating Student Responses
  • Plan teacher reactions and questions

• Patterns of Questioning
  • IRE, Funneling, and Focusing

• Talk Moves (Chapin, O’Connor, & Anderson, 2009)
  1. Revoicing
  2. Repeating
  3. Reasoning
  4. Adding on
  5. Waiting
Integrated Approach to Mathematics - Language Teaching and Learning

**Isolated Approach**
- Addressing ELs’ areas of needs in isolation
- Limited opportunities to practice language & math skills
- Limited professional collaboration

**Integrated Approach: Student Level**
- Connecting to ELs’ backgrounds & experiences
- Math sense-making through language & non-linguistic representations
- Advancing language skills through interaction with math content & other students/adults

**Integrated Approach: Educator Level**
- Exploring & applying interrelated nature of math & language learning
- Building on teachers’ professional expertise
- Increasing opportunities for collaboration

Shared responsibility for ELs’ learning experiences in math

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## Language Development Supports for ELLs to Increase Comprehension and Communication

### Environment
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

### Sensory Supports*
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

### Graphic Supports*
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

### Interactive Supports*
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

### Verbal and Textual Supports
- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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Exploration of Instructional Tools

**Cubing Game**

**Three-Way Tie**

**2x2 Sentence Builders**

**Problem-Solution Space**

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Discussion Prompts
(Back of Notes)

• Make Sense of the Tool:
  - Explore and engage
  - Review student work

• Benefits for Mathematical Knowledge and Skills

• Benefits for Developing Language

• Other Benefits/Considerations

• Applications

Make sure to consider CCSS Practices and WIDA Principles!
Be prepared to share with the whole group!

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(12 minutes)
(12 minutes)
Share out

• Concise description of Instructional Tool
• Discussion of benefits for developing both mathematics AND language

(min/tool)

*PLEASE DO NOT TAKE STUDENT WORK*
Cubing Game: Looking at a Concept from Different Perspectives

Describe

Area = 20 cm²
Perimeter = 18 cm

Compare/Contrast

What is the perimeter of the red shape?
What is the perimeter of the orange shape?
What is the perimeter of the green shape?
What is the perimeter of the blue shape?

Define

Perimeter
Area

Apply

Connect/Associate

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Let's Investigate and Prove the Connections!

Name: ______________________
I can describe how ____________, ____________, and ____________ relate to each other!
2x2 or 3x3 Sentence Builders

Fraction → Numerator
Denominator → Decimal Fraction
Convert → Number

Equally → Parts

The Whole
My Problem-Solution Space

I am learning how to make sense of a math problem and how to make a convincing argument about my solution.

- Paraphrase or retell the problem in your own words.
- Create and label a visual model to represent the problem and the solution.
- Use numbers to solve the problem.
- Write your answer in complete sentences.
- Use specific information or examples from the problem to support your answer.
- Apply what you know mathematically to make a convincing argument about your solution.

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Manuel saw some birds this week. He saw 2 blue jays on Monday, 5 cardinals on Tuesday, then again 4 blue jays on Wednesday, and again 7 cardinals on Thursday. On Friday, Manuel saw 6 blue jays.

If the pattern continues, what is the number and type of bird Manuel will see on Saturday?
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Instructional Tools: Take-Aways

• Have **explicit** language focus (vocabulary, sentences) AND support development of mathematical ideas

• Integrate **multiple** ways to support language and mathematics learning (visual, graphic, interactive)

• Offer ways to **differentiate** support for ELs of varying language skills (word level, sentence level, discourse level)

• Offer multiple ways for formative **assessment** (language AND mathematics)

• Create opportunities to **practice** CC Mathematical Practices AND L/S/R/W skills

*NOTE:* Because these are “non-traditional” ways of engaging students in mathematics, it is important to strategically coordinate the use of **Tools** with appropriate mathematical **Tasks**.
ABC Taxonomy: Tracking New Learning

Session 7
Why This Matters: Teacher Shifts

• Advanced Knowledge about Content, Pedagogy, and Tools
• Elevated Status of ESOL Teachers = Shared Expertise
• Growth in Teacher Collaboration and Leadership
• More Productive Classroom Norms and Instructional Strategies
• Multiple Lenses: Language, Mathematics, Environment, Participation
• New Noticings About Practices and Students’ Abilities
• Shifts in Teacher Beliefs of and Knowledge for Teaching Math with ELs
Teaching English Learners
Includes Improving Reflective Practices

- Teacher Journaling -

<table>
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<th>Language Strength/Challenges:</th>
<th>Participation/Attitudes/Motivation:</th>
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**Rationale**

Reflecting on the Use of Instructional Tools/Practices/Supports

**Teaching Practices**

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<th>I noticed that my student(s) _______</th>
<th>Other</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Students’ Math and Language Skills</td>
<td>Students’ Participation, Attitudes, Feelings, Motivation, etc.</td>
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Moving from “My English learners can’t do this...” to “My students can’t do this YET. Therefore, I need to _____ , so that my #ELLsCan.”
Resources

- TODO-math.org
- NCTM:
  - Principles to Actions
  - Access and Equity: Promoting High-Quality Mathematics
  - Beyond Good Teaching
- TESOL: CCSS in Math for ELLs
- Intentional Talk
- ELs in the Mathematics Classroom
- Math Tools: Grades 3-12

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TODOS Conference 2018
It’s ALL about ALL Students Learning Quality Mathematics: Advocating for Equity and Social Justice
June 21-23, 2018
Scottsdale Plaza Resort, Scottsdale, AZ

Mathematic Teaching and Learning Conference Topics
• Equity and Social Justice
• Language, Literacy and Culture
• Building on Community Strengths
• Tasks, Technology, Social Media, and Assessment
• Advocacy and Activism

Pre-Conference
June 21st 8am-3pm
Leading for Social Justice
in Mathematics

NCTM Booth #159

A Mathematics Equity Organization
Equitable, rigorous, and coherent mathematics!
http://www.todos-math.org
Insights and New Ideas?
How are you integrating mathematics and language learning for English learners?
THANK YOU!

Feel free to contact us for more information, resources, etc.

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