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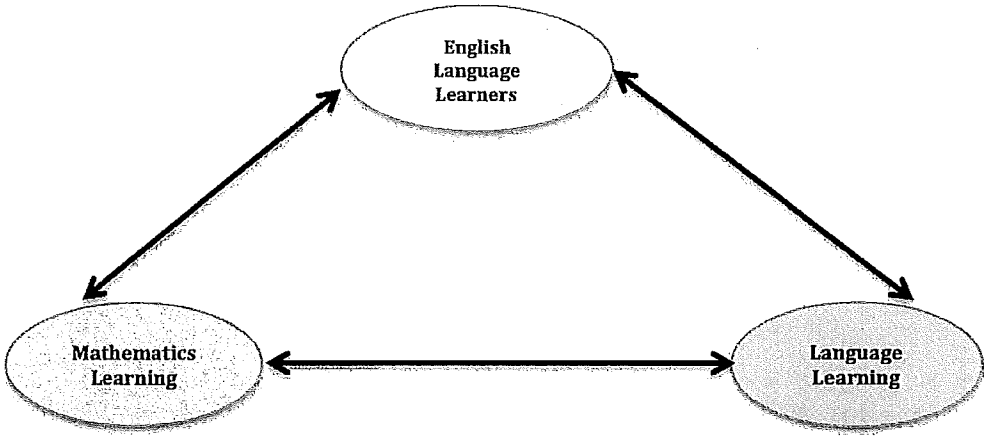
PD and Instructional Tools for Advancing ELLs' Mathematics and Language Through an Integrated Approach

WARMUP: The *Three Way Tie** graphic support promotes writing and discussion about the relationships between three topics. With that in mind...

How are you integrating mathematics and language learning for English Learners?



Key Ideas and Notes	Interpretations, Connections, Applications

Take-Aways (Next steps, to read, to share, etc.)

**Modified from "Math Tools (Grades 3-12)" by H.F. Silver, J.R. Brunsting, & T.Walsh; 2008.*

Instructional Tool: _____

1. **Make Sense of the Tool:**
   a. What is it? How does it work?

   b. How/When have you used something similar?

2. **Based on your exploration of the Tool AND review of student work, reflect on:**
   a. Benefits for Developing Mathematical Knowledge and Skills (e.g., CCSSM Practices)

   b. Benefits for Developing Language (e.g., WIDA: Word, Sentence, Discourse; L/S/R/W)

   c. Other Benefits/Considerations

3. **Applications: How/When might you implement this Tool?**

CCSS Mathematical Practice (What STUDENTS Do)	NCTM Mathematics Teaching Practices (What TEACHERS Do)
1) Make sense of problems and persevere in solving them*	❑ Establish mathematics goals to focus learning
2) Reason abstractly and quantitatively	❑ Implement tasks that promote reasoning and problem solving
3) Construct viable arguments and critique the reasoning of others*	❑ Use and connect mathematical representations*
4) Model with mathematics*	❑ Facilitate meaningful mathematical discourse*
5) Use appropriate tools strategically	❑ Pose purposeful questions*
6) Attend to precision*	❑ Build procedural fluency from conceptual understanding
7) Look for and make use of structure	❑ Support productive struggle in learning mathematics
8) Look for and express regularity in repeated reasoning	❑ Elicit and use evidence of student thinking

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*	Graphic Supports*	Interactive Supports*	Verbal and Textual Supports
• Real-life objects (realia) or concrete objects • Physical models • Manipulatives • Pictures & photographs • Visual representations or models such as diagrams or drawings • Videos & films • Newspapers or magazines • Gestures • Physical movements • Music & songs	• Graphs • Charts • Timelines • Number lines • Graphic organizers • Graphing paper	• In a whole group • In a small group • With a partner such as *Turn-and-Talk* • In pairs as a group (first, two pairs work independently, then they form a group of four) • In triads • Cooperative learning structures such as *Think-Pair-Share* • Interactive websites or software • With a mentor or coach	• Labeling • Students' native language • Modeling • Repetitions • Paraphrasing • Summarizing • Guiding questions • Clarifying questions • Probing questions • Leveled questions such as *What? When? Where? How? Why?* • Questioning prompts & cues • Word Banks • Sentence starters • Sentence frames • Discussion frames • Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

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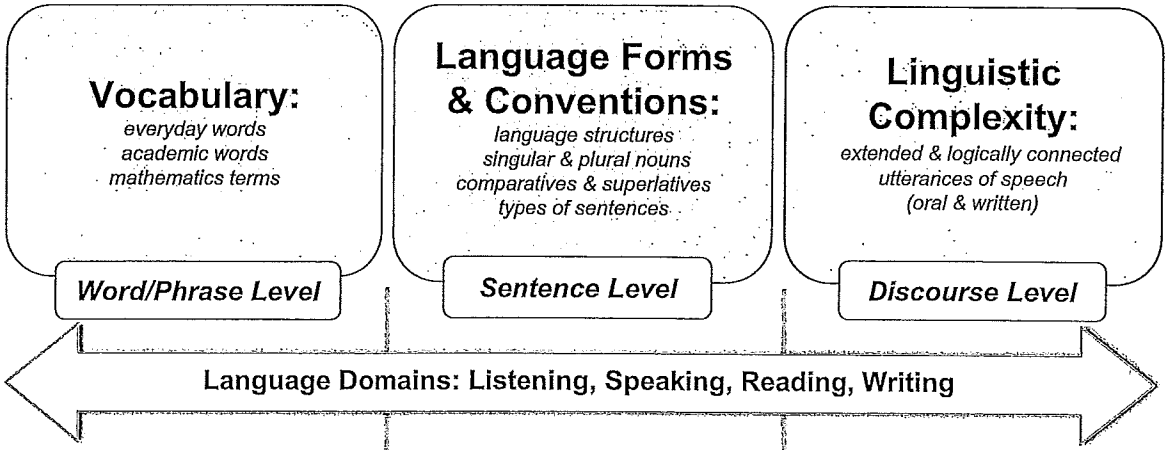
WIDA™ English Language Development Standards

Standard 1: Social and Instructional Language
Standard 2: The Language of Language Arts
Standard 3: The Language of Mathematics
Standard 4: The Language of Social Studies
Standard 5: The Language of Science

English language learners communicate information, ideas, and concepts necessary for academic success in the content area of mathematics.

The Language of Mathematics: Defining Features

Planning – Teaching – Monitoring – Assessing



Adapted by Galina (Halla) Jmourko, ESOL Coach, PGCPS from *The Defining Features of the Academic Language in WIDA's Standards*, WIDA Consortium, Draft, 2011

WIDA™

The Cornerstone of WIDA's Standards:
Guiding Principles of Language Development

1. Students' languages and cultures are valuable resources to be tapped and incorporated into schooling.
   Escamilla & Hopewell (2010); Goldenberg & Coleman (2010); Garcia (2005); Freeman, Freeman, & Mercuri (2002); González, Moll, & Amanti (2005); Scarcella (1990)

2. Students' home, school, and community experiences influence their language development.
   Nieto (2008); Payne (2003); Collier (1995); California State Department of Education (1986)

3. Students draw on their metacognitive, metalinguistic, and metacultural awareness to develop proficiency in additional languages.
   Cloud, Genesee, & Hamayan (2009); Bialystok (2007); Chamot & O'Malley (1994); Bialystok (1991); Cummins (1978)

4. Students' academic language development in their native language facilitates their academic language development in English. Conversely, students' academic language development in English informs their academic language development in their native language.
   Escamilla & Hopewell (2010); Gottlieb, Katz, & Ernst-Slavit (2009); Tabors (2008); Espinosa (2009); August & Shanahan (2006); Genesee, Lindholm-Leary, Saunders, & Christian (2006); Snow (2005); Genesee, Paradis, & Crago (2004); August & Shanahan (2006); Riches & Genesee (2006); Gottlieb (2003); Schleppegrell & Colombi (2002); Lindholm & Molina (2000); Pardo & Tinajero (1993)

5. Students learn language and culture through meaningful use and interaction.
   Brown (2007); Garcia & Hamayan, (2006); Garcia (2005); Kramsch (2003); Díaz-Rico & Weed (1995); Halliday & Hasan (1989); Damen (1987)

6. Students use language in functional and communicative ways that vary according to context.
   Schleppegrell (2004); Halliday (1976); Finocchiaro & Brumfit (1983)

7. Students develop language proficiency in listening, speaking, reading, and writing interdependently, but at different rates and in different ways.
   Gottlieb & Hamayan (2007); Spolsky (1989); Vygotsky (1962)

8. Students' development of academic language and academic content knowledge are inter-related processes.
   Gibbons (2009); Collier & Thomas (2009); Gottlieb, Katz, & Ernst-Slavit (2009); Echevarria, Vogt, & Short (2008); Zwiers (2008); Gee (2007); Bailey (2007); Mohan (1986)

9. Students' development of social, instructional, and academic language, a complex and long-term process, is the foundation for their success in school.
   Anstrom, et.al. (2010); Francis, Lesaux, Kieffer, & Rivera (2006); Bailey & Butler (2002); Cummins (1979)

10. Students' access to instructional tasks requiring complex thinking is enhanced when linguistic complexity and instructional support match their levels of language proficiency.
    Gottlieb, Katz, & Ernst-Slavit (2009); Gibbons (2009, 2002); Vygotsky (1962)

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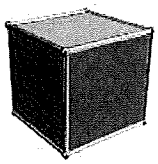

Cubing Game

Purpose: To look at a concept from different perspectives.

- **Describe** it.
- **Apply** it.
- **Compare** it with.../**Contrast it** to...
- **Connect** it to/**Associate** it with...
- **Create** a visual representation of it/a story problem.
- **Define** it.

Now, Let's Play a Cubing Game!

Concept: AREA

1. **With a partner, read the ways/perspectives below.**

• Describe	• Compare/Contrast	• Create
• Apply	• Connect/Associate	• Define

2. **Now discuss how you might use these perspectives when talking about area. See some examples below but you can't use these examples when you play.**

Examples:

1. Describe: *Area is a space that the rectangle covers.*
2. Apply: *My dad needed to figure out the area of the kitchen floor when he was buying some new tiles.*
3. Compare/Contrast: *The area of this rectangle is larger than that one because it covers more space.*
4. Connect/Associate: *I associate area with the bulletin board in our classroom.*
5. Create: *When I created a flowerbed, I created an area where I'll plant some flowers. This is what it looks like.*
6. Define: *Area is the amount of space inside a plane figure.*

3. **Now, take turns tossing a cube. When the cube lands, use the perspective that faces the sky to discuss AREA.**

4. **Have FUN!!!**

Adapted by Galina (Halla) Jmourko, ESOL Coach, PGCPS from *Literacy + Math = Creative Connections in the Elementary Mathematics* by Jennifer L. Altieri; © 2010 International Reading Association.

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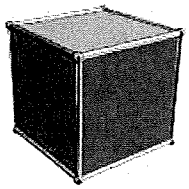
We are looking at _____from different perspectives:



- ☐ **Describe** it.
- ☐ **Create** its visual representation.
- ☐ **Apply** it.
- ☐ **Connect** it to/**Associate** it with...
- ☐ **Compare** it /**Contrast it** to...
- ☐ **Define** it.

1. How can you **describe** _____?

_____

_____

_____.

2. How can you **create** a visual representation of _____?

3. How can you **apply** _____?

_____

_____

_____.

4. How can you **connect** _____ to _____?

_____

_____

_____.

5. How can you **compare/contrast** _____?

_____

_____

_____.

6. How can you **define** _____?

_____

_____

_____.

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2x2 or 3x3 Sentence Builders

(also known as *Structural Indexing*)

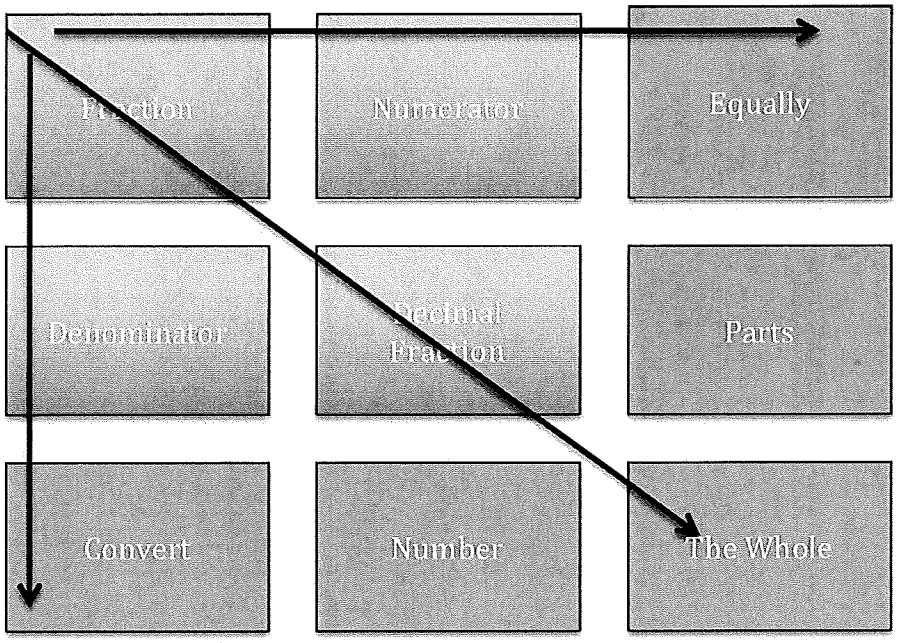
Purpose:

- To use new math vocabulary in sentences in order to reinforce students' understanding of math concepts;
- To help students link related math words in order to construct complete sentences that are mathematically correct;
- To practice math vocabulary in order to build students' ability to use vocabulary fluently and create logical arguments independently

Steps:

1. Create 4* key index cards. Each card has a math word.
2. Place the cards in a 2 x 2**array.
3. Ask students to work in pairs to create a sentence using the two*** words in each column, row, and diagonal. A total of 3-6 sentences can be created. Students can be asked to create sentences orally or in writing.
4. Emphasize that sentences MUST be mathematical, complete, and correct.

*9 cards can be used to challenge students.
** Or 3 x 3 array if 9 cards are used.
*** Or using the three words in each column, row, and diagonal.
When a challenge arises, you might allow students to rearrange the cards to create make sentences that make sense mathematically.



Benefits for ELLs

My Classroom Take-Aways:

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Name _____

2x 2 Sentence Builders: Use Words to Create Sentences.

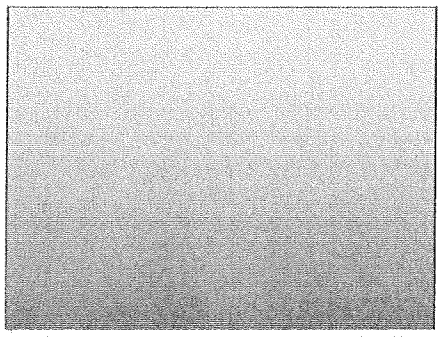
Make sure that your sentences are complete and make sense mathematically.

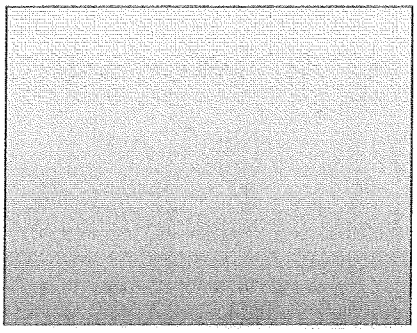
Use arrows to indicate which words you used.

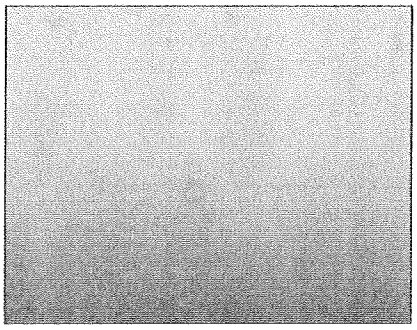
I am learning about _____.

I can use these important words (see below) to talk about _____.

I can create at least 3 sentences using two words *horizontally, vertically, or diagonally.*







1. _____

_____

2. _____

_____

3. _____

_____

4. _____

_____

Adapted by Galina (Halla), ESOL Coach, PGCPS from "Supporting Writing Skills in English Language Learners" by R. M. Santa Cruz & I. Sanchez-Gutierrez; NCTM, March 2009.

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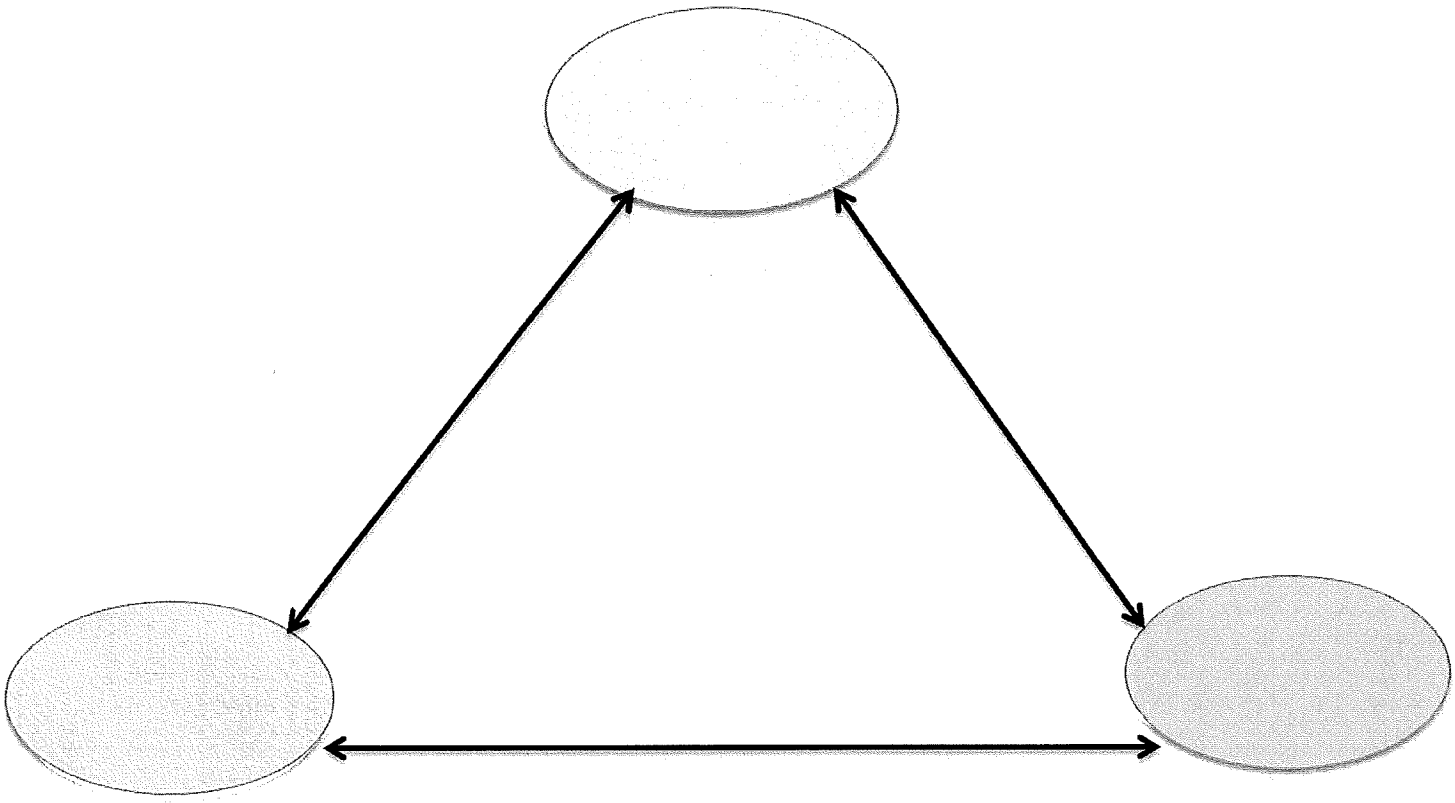
Three-Way-Tie Graphic Support

Let's Investigate and Prove the Connections/Relationships

Purpose: To provide an opportunity to think and discuss relationships between mathematical concepts or terms.

Steps:

1. Identify an important mathematical concept/term.
2. Graphically triangulate the concept/term with two other related concepts/terms.
3. Along each arrow/side, write a sentence that shows a relationship between the two concepts/terms.
Make sure that your sentences are complete and mathematically reasonable.
4. After you complete three sentences, share your sentences with a partner.
5. Discuss what you noticed about the sentences. Reflect how the sentences are similar or different. Why?



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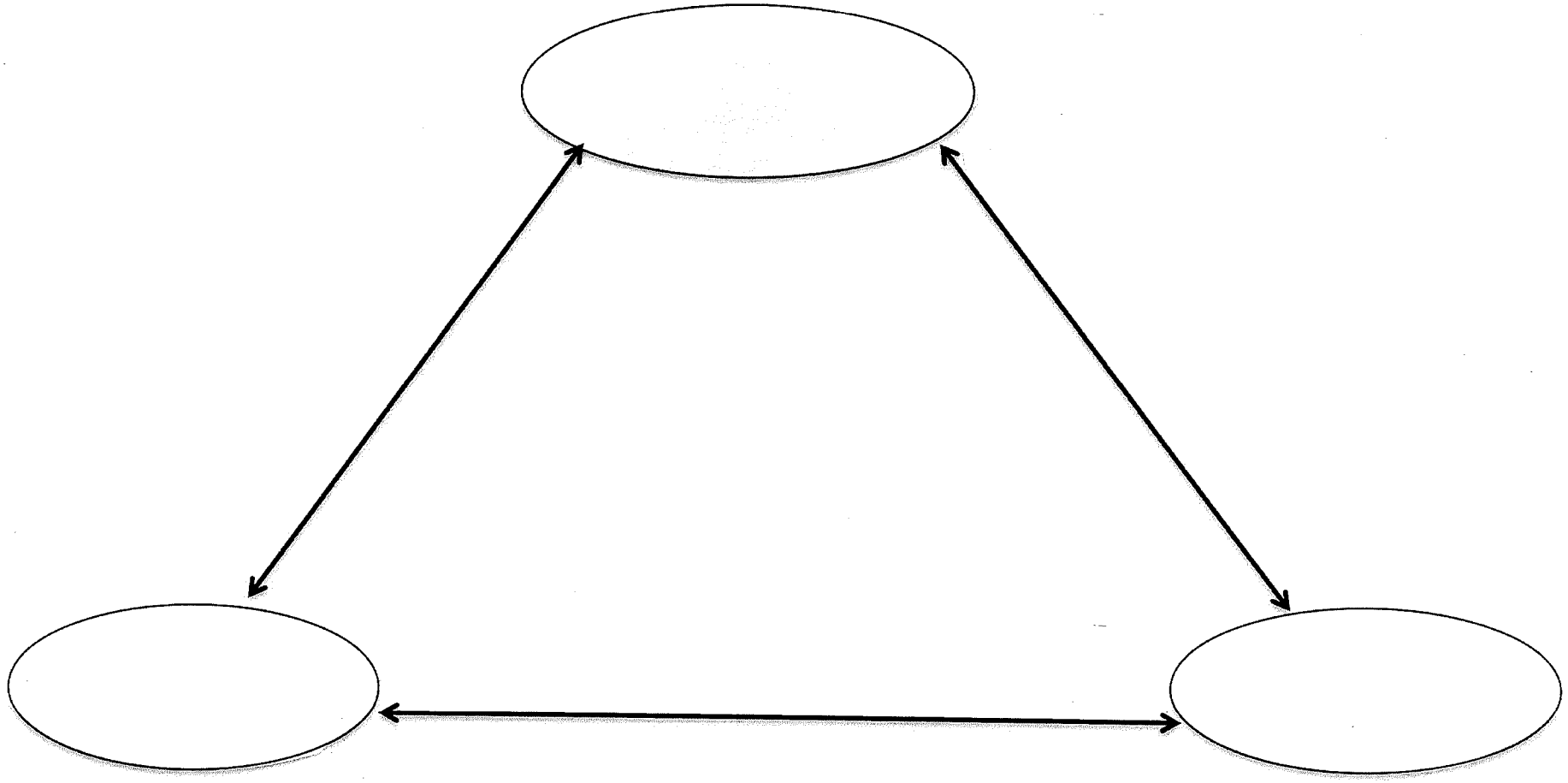
Three -Way -Tie Concept Map by _____

Student Name

Let's Investigate and Prove the Connections!

I can describe how _____, _____, and _____ relate to each other!



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Name: _____

My *Problem-Solution* Space

Date: _____

I am learning how to **make sense of a math problem** and how to **make a convincing argument** about my solution.

- ◊ **Paraphrase** or retell the problem in your own words.
- ◊ Create and label a **visual model to represent** the problem and the solution.

- ◊ Use **numbers to solve** the problem.

- ◊ Write your **answer** in complete sentences.
- ◊ Use specific **information** from the problem to support your thinking.
- ◊ Apply what you know mathematically to make a **convincing argument about your solution.**

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I am a Math Detective. I know how to make sense of the problem and how to solve it!

Name: _____ **Date:** _____

- ◊ I can **visualize** the problem.
- ◊ I can **retell** the problem.
- ◊ I can **create a visual model** to show important math facts.
- ◊ I can **label** the model.

- ◊ I can use **numbers or a number sentence to solve** the problem.

- ◊ I can **explain** how I solved the problem.

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