Productive Struggle with the Seven Billion People Problem

National Council of Teachers of Mathematics
Annual Meeting
April 27, 2018 ▪ Washington, DC

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Agenda

How can a cognitively demanding real-world task such as the Seven Billion People Problem promote productive struggle?...and help shape students' mathematical dispositions?

- Let's do the problem! (and share strategies)
- Productive struggle: task choice, implementation, and classroom norms
- Shaping students' mathematical dispositions
- Extensions to other disciplines
- Connections to SMP and time for questions

The Seven Billion People Problem

In November 2011, the 7 billionth person was born.

a. Is it true that if you laid out all the people on earth end to end, they would encircle the earth 266 times?

b. Is it true that if all 7 billion people stood shoulder to shoulder, we would all fit into Los Angeles?

Missing any information?...

a. Circumference of the Earth?

≈ 24,901 miles

How many feet in a mile?

5280 feet = 1 mile

Height of the average person?

b. What's the area of LA?

≈ 500 square miles

How much area does one person take up?

Let’s solve!

Discuss solution strategies…

Anticipated Trouble Areas

- Converting square feet to square miles
- Interpreting scientific notation on calculators
- Estimating the height of an average person

What *is* Productive Struggle?

For students:

- Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle
- Asking questions
- Persevering in solving problems

For teachers:

- Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them

(NCTM, *Principles to Actions: Ensuring Mathematical Success for All,* 2014, p. 52)

Choosing a Task for Productive Struggle

Levels of cognitive demand:

Lower-Level Demands	Higher-Level Demands
Memorization	Procedures with connections
Procedures without connections	Doing mathematics

A task with high cognitive demand:

- is often non-routine
- calls for solution strategies that are not obvious
- invites debate
- will "embody the complexities of real-life situations"

(Stein, Smith, Henningsen, & Silver, 2009)

Implementing a Task for Productive Struggle

Taking time to build classroom norms where…

- mistakes become an accepted part of the problem solving
- sufficient time is available to work in groups and discuss possible strategies
- students realize that the teacher is *not* the sole authority on mathematical justification

Implementing a Task for Productive Struggle

Support productive struggle in learning mathematics Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

(NCTM, *Principles to Actions: Ensuring Mathematical Success for All,* 2014, p. 52)

Dispositions: How did *you* feel while solving this problem?

- Sixteen preservice teachers enrolled in an elementary/middle level mathematics methods course solved the Seven Billion People Problem
- The preservice teachers completed a four-item reflection after solving the problem
- Here are their thoughts…

Murawska, J. M. (Jan./Feb, 2018). The seven billion people problem: Fostering productive struggle. *Mathematics Teaching in the Middle School. 23*(4), 209-214

Describe your initial reaction when starting to solve this problem.

Sample responses:

I was really concerned of my ability to complete this problem. I was overwhelmed and did not know where to start.

It was very frustrating. There were too many unknowns.

Annoyed by the guessing of exact averages and exact measurements.

Scary!

Statistics:

- 56% expressed feelings of apprehension or discomfort
- 38% cited concerns regarding missing information and unknowns

When you were finished solving the problem with your group, how confident were you of your conclusion?

Sample responses:

50/50 because our answer changed multiple times. We had 1 answer if we took average child, infant, and adult and another if it was just average adult.

↑ *confidence when we discussed in whole group as a class.*

Statistics:

- 50% were in the middle (*semi-confident, fairly confident*)
- 31% stated stronger levels of confidence (*very confident, confident, pretty confident*)

To what extent did your collaborative experience on this task impact your disposition toward the learning of mathematics?

Sample responses:

I really enjoyed it. If I were doing it by myself I wouldn't have enjoyed it as much because I would have gotten too overwhelmed.

It helped tremendously. I don't think I would have been able to do this alone; it would have been too stressful.

Statistics:

- 94% had a positive response toward group work

Assuming that you will teach sixth grade [or higher], do you think you will use this problem in your mathematics classroom one day? Why or why not?

Sample responses:

Yes, I love the aspect of having students become comfortable with reasonable assumptions rather than explicit "answers."

Yes! Absolutely because it involved so much math and math concepts, builds team work, and gives students real life context of the problem.

Yes because it asks them to think critically.

Statistics:

- 94% wrote an enthusiastic yes or a maybe

So what do these responses mean?

- Initial reactions: uncomfortable!
- Opportunity for group discussions: increased confidence levels and *enjoyment*!
- We should do more of these types of tasks.

Additional Resources for the Seven Billion People Problem

Quick 7 Billion Quiz:
http://news.nationalgeographic.com/news/2011/10/111031-population-7-billion-earth-world-un-seven/

Cool photographs:
http://ngm.nationalgeographic.com/2011/01/seven-billion/olson-photography?source=news_7billion

Additional Resource for Similar Problems

Fermi Problems:
http://mathforum.org/workshops/sum96/interdisc/sheila1.html

Alignment to Common Core Geometry and Measurement Content Standards

Topics include estimating, converting within a measurement system, circumference, area, and modeling with geometry

- 4.MD.1
- 5.MD.1
- 7.G.4
- 7.G.6
- G-MG.2

Standards for Mathematical Practice (SMP)

- Eight standards "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students."

- Mathematically proficient students:
  1. *Make sense of problems and persevere in solving them*
  2. *Reason abstractly and quantitatively*
  3. *Construct viable arguments and critique the reasoning of others*
  4. *Model with mathematics*
  5. *Use appropriate tools strategically*
  6. *Attend to precision*
  7. *Look for and make sure of structure*
  8. *Look for express regularity in repeated reasoning*

(Common Core State Standards for Mathematics, 2011, p. 6)

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If **we want students to…**

1. Make sense of problems and persevere in solving them

Then the task should require…

true problem solving, where the solution is not immediately obvious.

If **we want students to…**

3. Construct viable arguments and critique the reasoning of others

Then the task should…

lend itself to debate and discussion.

If we want students to…

4. Model with mathematics

- Mathematically proficient students apply mathematics to solve problems arising in everyday life, society, and the workplace
- Students are comfortable making assumptions and approximations to simplify a complicated situation

Then the task should…

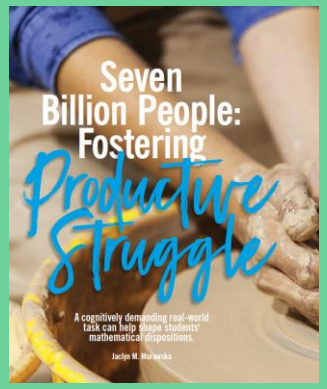
require students to make assumptions, because real-life data is messy!

References

Common Core State Standards (CCSS). (2011). *Common Core State Standards for Mathematics.* Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Murawska, J. M. (Jan./Feb, 2018). The seven billion people problem: Fostering productive struggle. *Mathematics Teaching in the Middle School. 23*(4), 209-214.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston: NCTM.



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The End