Goldbach's Conjecture
Goldbach's conjecture is an idea about prime numbers that is believed to be true but has not been proved: Every even number greater than 4 can be written as the sum of two odd prime numbers. For example: $8 = 3 + 5$. Both 3 and 5 are prime numbers. $20 = 13 + 7 = 17 + 3$. $42 = 23 + 19 = 29 + 13 = 31 + 11 = 37 + 5$. Notice that there can be more than one Goldbach pair. The conjecture says only that there is at least one, and has nothing to say about whether there may be more. Can you find Goldbach pairs for all even integers between 4 and 100? Which have more than one Goldbach pair? Can you find any patterns?

Red Paint
There are 27 small cubes arranged in a 3 by 3 by 3 cube. The top and sides of the large cube are painted red. How many of the 27 small cubes have 0 faces painted? 1 face? 2 faces? 3 faces? 4 faces? 5 faces? 6 faces?

Diagonals
How many diagonals does a polygon with 17 sides have?

Double Your Money!
On the first day, Natasha puts a penny in her piggy bank. On the second day, she puts in another penny, doubling the amount of money in the bank. On the third day, Natasha puts in 2 pennies (the amount already in the bank), again doubling her money. Each day the pattern continues: Natasha puts in the number of pennies needed to double the amount of money in the bank. How long will it take Natasha to save 500,000 pennies? Extension: What if Natasha triples her money each day? How much sooner will she reach 500,000 cents?

Solid Gold
(Well, not really gold!) A solid 3 x 3 x 3 cube weighs 5 pounds and is worth $216. How many dollars is a solid 4 x 4 x 4 cube made of the same material worth?

Thieving Thieves!
When several thieves tried to divide a sum of money by giving $4 to each thief, one thief received nothing. When each thief took $3, they had $1 left over. What is the number of dollars and the number of thieves?

A Piece of Cake
What is the greatest number of pieces you can cut a round cake into by making four straight cuts with a knife? Extensions: 1. What is the maximum number of pieces produced by five cuts? Is there a pattern to these answers? 2. Is this the same problem as dividing a circle into the maximum number of areas by drawing four straight lines?

Square Share
Lines can divide squares into smaller squares. For example, 2 lines divide a square into 4 smaller squares; 4 lines will divide a square into nine smaller squares. How many lines would be required to divide a square into 100 smaller squares? Into 400 smaller squares? Into n smaller squares?
Fast Draw
Form exactly two squares by drawing five lines. By drawing six lines. By drawing seven lines. Extensions: 1. Can you draw two equilateral triangles using four lines? 2. Can you draw two equilateral triangles using five lines?

Four fours
Using exactly four 4s and the operations addition, subtraction, multiplication and division, make equations that equal 0, 1, 2, 3, 4, 6, 7, 8, 9. Remember to use the order of operations correctly! You may use parentheses. Extension: Can you make other numbers? Can you use 3s or another number for a similar activity?

Tile Floor
I want to tile a rectangular floor with congruent square tiles. Blue tiles will form the border and white tiles will cover the interior. The number of blue tiles will equal the number of white tiles. What is the maximum area in square units that can be tiled? Extension: How would this problem be different if there were half as many blue tiles as white tiles? What if there were three times as many blue tiles as white tiles?

Postage Stamp Problem
What is the largest amount of postage that you CANNOT make using only stamps worth 5¢ and 11¢? You may use as many stamps as you wish, in any combination. hello mp

Game Show
Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. (Note: The host will always reveal a goat.) He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors? There is a great deal of information about this problem on the web. If you choose to work on the problem, DO NOT look for resources until after you have submitted your work.

Checkerboard and Dominoes
You are given a checkerboard and 32 dominoes. Each domino covers exactly two adjacent squares on the board. Thus, the 32 dominoes can cover all 64 squares of the checkerboard. Now suppose two squares are cut off at diagonally opposite corners of the board (e.g., upper right, lower left). Is it possible to place 31 dominoes on the board so that all of the 62 remaining squares are covered? If so, show how it can be done. If not, prove that it is impossible.

Counterfeit coin
You have a pile of 24 coins. Twenty-three of the coins have exactly the same weight, but one, which is counterfeit, is heavier than the others (though it looks exactly the same). Your task is to determine which coin is heavier, and therefore counterfeit. You are given a balance scale, which will compare the weights of any two coins or sets of coins. What is the minimum number of weighings that you will need to do to find the counterfeit coin? Extension: a) What if there are 64
coins and one is counterfeit? b) What if you don't know if the counterfeit coin is lighter or heavier? Does it change the solution?

Cheap necklace
You are given four separate pieces of a chain that are each three links in length. It costs 2¢ to open a link and 3¢ to close a link. You are given the pieces of chain with all links closed. Your goal is to obtain a single closed chain, using all links, at a cost of no more than 15¢. Is it possible? Show how, or prove it is impossible.

Multicolored Cube
Arrange 9 red cubes, 9 blue cubes, and 9 white cubes in a large 3 by 3 by 3 cube so that no row or column of cubes contains two cubes of the same color. (Note: Part of the difficulty of this problem lies in finding a way to diagram the solution.)

Water Jar
You are given a jar that will hold exactly 7 quarts of water and a jar that will hold exactly 3 quarts of water. There is an unlimited supply of water, but no other containers. Describe a way to use these jars to measure exactly 5 quarts of water.

Chain Reaction
How can a chain with 63 links be cut in three places so that you could hand a person any number of links from 1 to 63? (A cut link is still counted as a link.)

Locker Problem
There are 500 students and 500 lockers, numbered 1 through 500. Suppose the first student opens each locker. Then the second student closes every second locker. The third student changes the state of every third locker (if it is open, she closes it; if it's closed, she opens it). The fourth student changes the state of every fourth locker. This process continues until the 500th student changes the state of the 500th locker. Which lockers are open after all 500 students have passed through?

Diophantus
Diophantus was a famous Greek mathematician who lived in Alexandria, Egypt, in the third century, A.D. After he died, someone described his life in this puzzle: He was a boy for 1/6 of his life. After 1/12 more, he acquired a beard. After another 1/7, he married. In the 5th year after his marriage, his son was born. The son lived half as many years as his father. Diophantus died 4 years after his son. How old was Diophantus when he died?

Sea Sick
Suppose a boat is located 30 miles from shore and must get a passenger to a hospital that is located 60 miles downshore from the boat's current position. The boat travels at 20 mph, and the ambulance that meets the boat travels at 50 mph. Where should the ambulance meet the boat to minimize the amount of time needed to reach the hospital?
A Round-about way of Painting
You have been hired to create this logo on a storefront window. You want to paint in the middle section of this logo. From the left edge to point 1 is 1 meter across (with the next being 2 meters across, and the last, to point 3, being 3 meters across). How many square meters will you need to paint?

Security Camera (from MAP)
A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop, located at point P. The camera can turn 360 degrees when possible. The plan below shows ten people who are standing in the shop (each letter is a person). Which people can be seen by the camera and where would be the best place to install the camera to see as much of the shop as possible?
Having Kittens (from MAP)
**Boomerangs (from MAP)**

Phil and Cath make and sell boomerangs for a school event. The money they raise will go to charity. They plan to make them in two sizes: small and large. Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving. Cath will decorate them. She only has time to decorate 10 boomerangs of either size. The small boomerang will make $8 for charity. The large boomerang will make $10 for charity. They want to make as much money for charity as they can. How many small and large boomerangs should they make? How much money will they then make?

**Triangle Madness**

Below are two triangles. By moving the pieces from one the other is created, yet in the first the area is greater than the second (one square is not there). How can this be? Can certain shapes have the same area but be reconfigured to have a different area?

**Fly on a Cube**

A fly is sitting on one corner of a sugar cube that has a volume of one cubic inch. Only walking, what paths might the fly take to get to the opposite corner and which is the shortest possible path it can take?