Andrea Kung

Urban Academy Laboratory High School

IG/Twitter: @akismetoo

Teaching Logic & Proofs through Games & Number Theory

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Gabriella Weisberg

Humanities Preparatory Academy

Twitter: @QuestionsNYC

Objectives & Agenda

Our Plan:

- Introduction on the Proofs & Games course
  - Why games?
  - The tiniest bit of game theory
- Norms for Game Play
- Game of 27
  - Base Case (a restricted one-pile version of Nim)
  - Variations
  - Nim
- Debrief
- Further Q&A

Our Goals for You Today:

- Play game(s)!
- Productive Struggle
- Find some math in games
- Find some joy in math

- Have at least one activity to take back to work with you to share with your students, colleagues, families or friends.
- Have some resources

Proofs & Games: Student as Mathematician

- NYC Department of Education Certified College Preparatory Course
- Portfolios and Performance-Based Assessment Tasks (PBATs)
- Teacher-Developed and Inquiry-Based
- Developed with the goals of fostering student perseverance through the mathematical thinking process and writing proofs.
- Student as Mathematician Philosophy

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Why play analog games?

aka, neat brain stuff without citations for later

From the 2017 Games in Education Symposium

Why are games particularly engaging?

- interactive, exploration, discovery, collaboration

Analog vs. Digital

- Studies show in MMORPG (digital), high level of feeling isolation in comparison to D&D and other tabletop games (analog)

Why games? Why inquiry?

- Games prime kids to use these areas of the brains in the future.
- Using an inquiry model follows the engagement of students.

Why is making neuronal connections important?

- Focal point of attention (more firing, strength, speed of processing, elimination of unwanted information)
- Making mistakes strengthens areas of the brain
- Novice players activate multiple areas of the brain, while Expert players use fewer, but more focused areas of the brain (e.g. chess experts use more of the spatial and temporal organizational areas of the brain, not strategy/executive function/frontal lobe!)

Interesting tidbits:

- Frontal lobe (strategy) area does not fully develop until we are in our twenties.
- Younger students struggle with games where they have to deal with lying of peers in the game because they do not have enough social context to draw appropriate conclusions.

What kind of games exist?

aka, the tiniest bit of game theory ideas

		The result of your input/action	
		Random (Surprise!)	Deterministic (Known)
Opponent Strategies	Pre-determined	Roulette, Slot Machines, Blackjack, Risk	Tower of Hanoi, Sudoku, Rubik's Cube
	Intelligent	Poker, Scrabble	Chess, Checkers

Additional Classifications We Consider:

- **Zero-sum** vs. Non-zero-sum
- **Perfect information** vs. Imperfect information
- **Finite** vs. Infinite
- Cooperative vs. **Non-cooperative**
- Simultaneous vs. **Sequential**

Proofs & Games... One More Thing...

Once you've solved:

- Don't give it away! :)
  - Can you support a peer through the process of discovery?
  - Can you think of another way to approach the problem?
  - How would you prove that?
- Extensions
  - We have them!
  - But, can you come up with your own?
- If you have already played this game, please let us know, and we will move you onto Nim.

Game of 27 (Base Case)

Scenario:

There is a bag with 27 tokens.

Two people take turns drawing tokens out of the bag.

Each person may take exactly 1, 2, 3, 4 or 5 tokens on their turn.

The person who takes the last token wins.

Directions:

- Find a strategy to win.
- Keep track of your work!
- Be prepared to justify yourself.

(time to investigate)

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Nim

Scenario:

There are *n* tokens in *p* piles.

Two people take turns taking as many tokens from any given pile. (You cannot take from multiple piles on your move.)

The person who takes the last token(s) wins.

Directions:

- Find a strategy to win.
- Keep track of your work!
- Be prepared to justify yourself.

(time to investigate)

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Reflection: The Student Experience

What was your student as mathematician experience like?
How do you think your students would react to a similar task?

Starting the conversation...

- When you made a poor choice,
  - How did you know?
  - How did you adjust?
- When you made a better choice,
  - How did you know you?
  - How would you prove that you're right?
- What other ideas do you have?
- What was the most difficult part of this exercise for you?
- How did you overcome this?
- Where did the struggles come from?
- **How do you think your students might struggle?**

Workshop Input:

- **Identify sequences and patterns**
- **Writing an equation**
- **Division, Remainders, Arithmetic**
- **Symmetry**
- **(Multiples of 6, "Symmetry")**
- **Modular Arithmetic (27 mod 6 = 3)**
- **Induction, Recursion**
- **Different Entry Levels**
- **Complementary Sets**
- **Visualization**

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Discussion: Teacher Experience

How would you support your students through struggle?

Some guiding questions for our group...

- In this particular activity, what are some prompts that you think would be helpful for students when they are frustrated, stuck, or struggling?
- How does this experience help you reflect on your planning for engaging students in productive struggle?
- How would you differentiate this activity?

Workshop Input:

- When do you know you've won? (Working Backwards)
- Does it matter who goes first? Why?
- With perfect play...
- What if you lose? (Variation, 1 more than the "multiple")
- Multiple players

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Final Thoughts

What makes a game?

- Objects to play with
- Opening arrangement
- Rules
- Goal(s)

Do you think games are a good way way to engage students in mathematical thinking?

Every game can be mathematized!

More things are games than you think!

Solution, you can find on the website! (See sample papers)

Thank You!

New York Performance Standards Consortium

Center for Inquiry in Teaching and Learning

Math for America

Booth #363

Q & A

All of our materials from this workshop (and more!) are published/being published on our website. Please feel free to use them.

http://tinyurl.com/proofsandgames

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