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Harnessing the Power of Mathematical Models to Re-Envision Early Childhood Routines

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WELCOME to Our Session!!
We are ...

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The Power of Models to Re-Envision Routines

How can we use mathematical models and routines to support young learners with developing the ability to reason with structures inherent in our number system?
Gravemeijer’s Trajectory for Mathematical Models

Model of the situation → Model of student thinking → Model as a tool for thinking
The Power of Models to Re-Envision Routines

The Bead String
As you watch this video, think about:

- What mathematical big ideas Muhab has constructed?
- How does the bead string model supports his thinking?
Talk to a neighbor about what you noticed in this video:

- What mathematical big ideas Muhab has constructed?
- How does the bead string model supports his thinking?
The Power of Models to Re-Envision Routines

Muhab

- mathematical big ideas (unitizing);
- structuring the number system;
- use of logical reasoning (if ... then);
- ability to communicate his ideas clearly;
- uses the model as a tool to think with.
How do kindergarteners develop these sophisticated big ideas over time?
How can we support the development of these big ideas for our students?

Routines!
Seeger's Quote

There is a ten and a seven in the number seventeen.
Routine: How many days have we been in school?
We began to wonder:

*How can we provide more opportunities for students to practice with and develop ownership of the model?*
Build My Number:
A Differentiated Practice Routine
Students work in partnerships or trios.

One student turns over a number card for everyone to see.

Each student builds the number on the bead string and checks what they’ve built with their partner.

Different groups work with different quantities (within 10, 20, or 100) depending on their goals.
Mini-lessons Using the Bead String: 
"Reading" the bead string
How many?

“Eight!”

How do you know?

Child 1: “1, 2, 3, 4, 5, 6, 7, 8!”
Child 2: “5... 6, 7, 8!”
Child 3: “Because 5 and 3 make 8!”
Describe what you see.

Child 1: “I see red and white beads.”

Child 2: “I see 5 red beads and 3 white beads.”
5 and ...
How does this lesson evolve?

5 and ...
How does this lesson evolve?

5 and ...

<table>
<thead>
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<th>5 and ...</th>
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<td>5 and 3</td>
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<td>5 and 2</td>
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<tr>
<td>5 and 4</td>
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<td>5 and 1</td>
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</table>
How does this lesson evolve?

5 and ...

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<th>5 and ...</th>
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<td>5 and 3</td>
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<td>5 and 4</td>
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<td>5 and 1</td>
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</table>
How does this lesson continue to evolve?

10 and ...

20 and ...

10

20
Who are my students?
- What do they know?
- What strategies do they need to develop?

What is the goal of my lesson?
- How can I design a string (choose and sequence problems) that address this goal?

What might students do and say in the lesson, and how can I respond?
- What will students strategies be? How will I model these strategies?
- What struggles or misconceptions will students’ have? How will I address these?
# Mini-lesson Planning Sheet

<table>
<thead>
<tr>
<th>Group size:</th>
<th>small</th>
<th>whole class</th>
<th>Type of lesson:</th>
<th>game</th>
<th>mental math minilesson</th>
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<tbody>
<tr>
<td>Mathematical goal:</td>
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<td>Social Goal:</td>
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<td>Model:</td>
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<td>Big Ideas:</td>
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<td>Strategies:</td>
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<td>Student Struggles:</td>
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<tr>
<td>Possible Student Errors:</td>
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</table>

<table>
<thead>
<tr>
<th>Teacher DOES</th>
<th>Teacher SAYS</th>
<th>Kids Say</th>
<th>Teacher WRITES</th>
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<tbody>
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Mathematical Goal: Develop the ability to use conceptual and perceptual subitizing with quantities less than 10, and to be able to describe visual images as “5 and.”

Social Goal: Develop students’ ability to listen and attend to the lesson and answer the questions being posed or paraphrase or question the reasoning of their peers.

Model: Bead string grouped in 5s (alternating red and white beads).

Big Ideas: Early work with the big ideas of cardinality (e.g., the quantity names the set) and part-whole relations (e.g., addition as the combination of two distinct sets [or two parts]).
# 5 and Mini-lesson

<table>
<thead>
<tr>
<th>Teacher DOES</th>
<th>Teacher SAYS</th>
<th>KIDS SAY</th>
<th>Teacher WRITES/SAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear the bead string. Then pull over 5 beads again.</td>
<td>How many beads is this?</td>
<td>5!</td>
<td>Say, So these are our 5 beads.</td>
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<tr>
<td><strong>Again, no discussion here.</strong></td>
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<tr>
<td>We’re going to do a lesson today called, “5 and.” In this lesson, we are going to read the bead string using the language “5 and.”</td>
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<td></td>
<td>Write “5 and” in big letters at the top of the board.</td>
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</table>
How does this work support the development of computational fluency?

10 and ...
As you watch a portion of this small group mini-lesson, take note of how the model is being used to support students’:

• understanding of mathematical big ideas; and,
• reasoning and communication.
$10 + 2 = 9 + 3$
In the next clip...

- How does the model support Marlon’s thinking?
- What big ideas are coming to the surface?

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9 + 7 = 9 + (1 + 6)

9 + 7 = (9 + 1) + 6

9 + 7 = 10 + 6
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The Ten Frame
Describe What You See
How might students describe this image?

Child 1: “I see red dots and yellow dots.”

Child 2: “I see three red on the top, three red on the bottom. Two yellow on the top and two yellow on the bottom.”

Child 3: “I see six red dots on the left and four yellow dots on the right.”

Child 4: “I see six red dots and four yellow dots.”
10s Frame: If, Then
10s Frame: If, Then

Structure of the activity:

- There are 10 dots in the 10-frame.
- I am going to tell you how many are red.
- If you know how many are red, then how many are yellow?
- Does the image I show you match your thinking?
10 dots.
If I have 5 red dots on my frame, then ...
10 dots.
If I have 8 red dots on my frame, then ...
10 dots.
If I have 10 red dots on my frame, then ...
10 dots.
If I have 6 red dots on my frame, then ...
10 dots.

If I have 10 yellow dots on my frame, then ...
10 dots.
If I have 9 red dots on my frame, then ...
10 dots.
If I have 9 yellow dots on my frame, then ...
As you watch a portion of this small group mini-lesson, take note of how the ten frame is being used to support students’:

- understanding of mathematical big ideas; and,
- reasoning and communication.
10s Frame: If, Then

• *What are the mathematical ideas coming up in this activity?*

• *How is the model supporting students’ ability to reason and communicate their thinking?*
Quick image of 10 dots; some are red; some are yellow.

• Develop part-whole relations to 10.

• The structure of the model can allow for visualization using the 5 structure inherent in the model.

• Because the whole (10) and one part is known, one can reason around the missing part.
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The 100s Frame

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True or Not True?
Structure of the activity:

• First I am going to show you an addition expression.

• Then I am going to show you a 100-frame. All the squares will be filled in with red and blue.

• Some squares will be red and some squares will be blue.

• Does the addition expression match the image I show you?

• Thumbs up if the expression truly matches the image.

• Thumbs down if the expression does not truly match the image.
80 + 20
80 + 20
80 + 20
40 + 60
40 + 60
40 + 60
$40 + 60$
Quick image of 100 squares; some are blue; some are red.

- Develop part-whole relations to 100.
- Because of the speed of the image, the eye is focused on deconstructing the image into tens (visualizing units of 10).
- The eye is drawn to the parts and the juxtaposition of these leads to reasoning with a landmark problem, 50 + 50.
- If 50 + 50 is 100, and I only see two blue tens, I know, the images cannot be 50 + 50 because one of the addends is < 50.
- Half does not have to be represented by congruent shapes.
In Summary:

- Mathematical models that are rooted in the structure of our number system (e.g. the bead string and the 10/100 frame) can serve as powerful tools for early childhood students to reason abstractly and quantitatively.

- The consistent use of these models within routines can help students develop big ideas in early number, addition, subtraction, and place value.

- The strategic use of and questioning around these models can directly impact students’ ability to move beyond computation and reason within the structures of the number system.
Thanks for coming to our session!

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