CONCRETE

PICTORIAL

ABSTRACT

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LET’S TALK ABOUT IT

• What do you know about CPA?

• Do you currently use CPA?

• Do you use all three components of CPA?

• Is one component more valuable than another in CPA?
• Research consistently show the most common cause of math struggles is the application of a procedure without understanding its underlying concept and usually points to an undeveloped or missing conceptual model. The brightline distinction for whether students possess conceptual understanding tends to be their use of a model.

SOLVING FOR WHY UNDERSTANDING, ASSESSING AND TEACHING STUDENTS WHO STRUGGLE WITH MATH

– JOHN TAPPER
CONCRETE, PICTORIAL, ABSTRACT

- Concrete
  - Concrete materials – physical models
- Pictorial/Representational
  - 2 dimensional written/pictorial representation – drawn models
- Abstract – Equations
  - Symbols
  - Numbers
  - Algorithms
WHO IS IT FOR?

• CPA is used whole class rather than with students who appear to be struggling. It’s a way to get everyone’s thinking.

• By comparing the way that students use each type of model in CPA teachers can get a good view of students understanding. With this information, teachers can plan more effectively to teach all students in their classes.
CONCRETE TANGIBLE, HANDS-ON TOOLS

- Connecting cubes
- Square tiles
- Two colored circles
- Base 10 blocks
- Measuring tools
- Coins
- Buttons
- Pattern blocks

Some common tools used in elementary mathematics instruction.
Photo Credit: Lisa de Garcia & FLDOE
It is imperative to initially teach a math concept by describing & modeling it using concrete models/objects. This aids in their concrete level of understanding for which to grow from.

- Afford students multiple opportunities to use concrete objects.
- Misuse of manipulatives occurs when teachers tell children to “Do exactly as I do.”

Teaching Student-Centered Mathematics PreK-2, Pearson, Van de Walle, Lovin, Karp, Bay-Williams
PHYSICAL AND VIRTUAL MANIPULATIVES

Physical
- Build the foundation for conceptual understanding

Virtual
- Assist learners in bridging to abstract
NLVM EXAMPLE
• The goal of pictorial representations is that children are able to manipulate ideas and make connections in a meaningful manner.

• Allow children (in most instances) the opportunity to choose their own representations to reason through a problem and to communicate their ideas to others. By doing so, it affords teachers insight into student thinking about the mathematical idea.

Teaching Student-Centered Mathematics PreK-2, Pearson, Van de Walle, Lovin, Karp, Bay-Williams
Figure 2.3

A kindergartner shows her thinking about ways to make 5.

How many ways can five people be on two stories of a house?
PICTORIAL EXAMPLES
Modeling at the abstract level involves using numbers and mathematical symbols.

As a teacher moves through a concrete to pictorial to abstract sequence of instruction, the abstract numbers and/or symbols should be used in conjunction with the concrete materials and representational drawings.

Look for opportunities to use children’s representations during classroom discussions to help make sense of the more abstract mathematical symbols.

*Teaching Student-Centered Mathematics PreK-2, Pearson, Van de Walle, Lovin, Karp, Bay-Williams*
ABSTRACT EXAMPLES
K-2 CPA TASKS
Kindergarten
K.CC.A.3

Standard: Know number names and the count sequence.

- **CCSS.MATH.CONTENT.K.CC.A.1**
  Count to 100 by ones and by tens.

- **CCSS.MATH.CONTENT.K.CC.A.2**
  Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

- **CCSS.MATH.CONTENT.K.CC.A.3**
  Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Task: Students will be able to count the items found in a bag, create a visual representation, and provide a numerical value to the items.
Concrete

Counting Items: Count the number of items next to the picture.

- Pennies: 3
- Paper Clips: 9
- Jelly Beans: 13
- Cubes: 10

Pictorial

Counting Items: Count the number of items next to the picture.

- Pennies: 3
- Paper Clips: 9
- Jelly Beans: 13
- Cubes: 10

Abstract

Counting Items: Count the number of items next to the picture.

- Pennies: 3
- Paper Clips: 9
- Jelly Beans: 13
- Cubes: 10
K.CC.B.4

Standard: Count to tell the number of objects.

- **CCSS.MATH.CONTENT.K.CC.B.4**
  Understand the relationship between numbers and quantities; connect counting to cardinality.

- **CCSS.MATH.CONTENT.K.CC.B.4.A**
  When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

- **CCSS.MATH.CONTENT.K.CC.B.4.B**
  Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

- **CCSS.MATH.CONTENT.K.CC.B.4.C**
  Understand that each successive number name refers to a quantity that is one larger.

- **CCSS.MATH.CONTENT.K.CC.B.5**
  Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

**Task:** Students will be able to count the items found in each section of the counting mat, create a visual representation, and provide a numerical value to the items found in each section.
Concrete

Pictorial

Abstract

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>00</td>
<td>000</td>
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<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
K.CC.B.4

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  Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

**Task:** As the teacher reads the book “Ten Apples on Top” by Dr. Seuss, he/she will stop each time an apple is added. The students will then add a counter to the ten-frame to represent the apples being added. After each page, the teacher will stop to ask the children how many apples there are after each counter is added to the ten-frame.
You can do three, but I can do more.
You have three, but I have four.

2 + 1 = 3

2 + 1 = 3

3 + 1 = 4
K.OA.A.3

Standard: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- **CCSS.MATH.CONTENT.K.OA.A.1**
  Represent addition and subtraction with objects, fingers, mental images, drawings\(^1\), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

- **CCSS.MATH.CONTENT.K.OA.A.2**
  Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

- **CCSS.MATH.CONTENT.K.OA.A.3**
  Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

- **CCSS.MATH.CONTENT.K.OA.A.4**
  For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

- **CCSS.MATH.CONTENT.K.OA.A.5**
  Fluently add and subtract within 5.

Task: Students will be able to count the items given and come up with as many ways as possible to make nine.
Concrete  |  Pictorial  |  Abstract
---|---|---
+ | 0 0 0 | 2 + 7 = 9
| + | 0 0 0 | |
| | 0 0 | |

Concrete  |  Pictorial  |  Abstract
---|---|---
+ | 0 0 0 | 4 + 5 = 9
| + | 0 0 0 | |
| | 0 0 | |
K.NBT.A.1

Standard: Work with numbers 11-19 to gain foundations for place value.

- [CCSS.MATH.CONTENT.K.NBT.A.1](#)
  Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Task: Students will be able to decompose teen numbers using a ten frame and an equation.
First Grade
Standard: Represent and solve problems involving addition and subtraction.

- **CCSS.MATH.CONTENT.1.OA.A.1**
  Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.¹

- **CCSS.MATH.CONTENT.1.OA.A.2**
  Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Task: Students will be able to add and subtract within 20 using the following word problem:

- In Ms. Wellington’s class you can earn tickets and use them to buy things that you like. Sasha wants to earn enough tickets to buy everything on the list and also be able to buy extra play time with a friend. How many tickets will Sasha need to save in order to earn all of the activities that she wants to do?
<table>
<thead>
<tr>
<th>Item</th>
<th>Number of earned tickets needed</th>
<th>Tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toy from the treasure box</td>
<td>1</td>
<td><img src="image1" alt="Admit One Tickets" /></td>
</tr>
<tr>
<td>Snacks</td>
<td>2</td>
<td><img src="image2" alt="Admit One Tickets" /></td>
</tr>
<tr>
<td>Computer time</td>
<td>3</td>
<td><img src="image3" alt="Admit One Tickets" /></td>
</tr>
<tr>
<td>Extra play time with a friend</td>
<td>4</td>
<td><img src="image4" alt="Admit One Tickets" /></td>
</tr>
</tbody>
</table>

**Pictorial**

```
0 + 0 = 0
0 + 0 = 0
0 + 0 = 0
```

**Abstract**

1 + 2 = 3  
3 + 3 = 6  
6 + 4 = 10  
10 + 4 = 14
Standard: Understand and apply properties of operations and the relationship between addition and subtraction

- **CCSS.MATH.CONTENT.1.OA.B.3**
  Apply properties of operations as strategies to add and subtract. Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)

- **CCSS.MATH.CONTENT.1.OA.B.4**
  Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.

Task: Students will be able to apply strategies to add and subtract numbers using fact family triangles. Students will come up with as many equations possible that represent what is on the triangle.
Standard: Work with addition and subtraction equations

- **CCSS.MATH.CONTENT.1.OA.D.7**
  Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.

- **CCSS.MATH.CONTENT.1.OA.D.8**
  Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = _ - 3$, $6 + 6 = _$.

Task: Students will be able to compare the number of items in each box. Students will write an equation that reflects what is in the box and compare them to see if they are equal.
Concrete

Pictorial

Abstract

4 + 3 = 2 + 2 + 2 + 1

7 = 7 + 1
1.NBT.B.2

Standard: Understand place value

- **CCSS.MATH.CONTENT.1.NBT.B.2**
  Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

- **CCSS.MATH.CONTENT.1.NBT.B.2.A**
  10 can be thought of as a bundle of ten ones — called a "ten."

- **CCSS.MATH.CONTENT.1.NBT.B.2.B**
  The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

- **CCSS.MATH.CONTENT.1.NBT.B.2.C**
  The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

- **CCSS.MATH.CONTENT.1.NBT.B.3**
  Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Task: As the teacher reads the book “The Very Hungry Caterpillar” by Eric Carle, he/she will stop each time a food item is eaten. The students will then add a counter to the ten-frame to represent the food items being eaten. After each page, the teacher will stop to ask the children how many items there are after each counter is added to the ten-frame.
On Thursday he ate through four strawberries, but he was still hungry.

On Friday he ate through five oranges, but he was still hungry.

On Saturday he ate through.....

That night he had stomach-ache!
1.NBT.C.4

Standard: Use place value understanding and properties of operations to add and subtract

- **CCSS.MATH.CONTENT.1.NBT.C.4**
  Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

- **CCSS.MATH.CONTENT.1.NBT.C.5**
  Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

- **CCSS.MATH.CONTENT.1.NBT.C.6**
  Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**Task:** Students will be able to use base ten blocks to show and explain their thinking by applying addition strategies within 100.
Concrete

Pictorial

Abstract

23 + 17
23 + 10 + 7
23 + 7 = 30
30 + 10 = 40
2.OA.A

Standard: Represent and solve problems involving addition and subtraction

- **CCSS.MATH.CONTENT.2.OA.A.1**
  Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹

Task: Students will be able to use addition and subtraction strategies by showing and explaining their thinking by using the following word problem:

- Jacob loves to eat jolly ranchers. He brought 34 jolly ranchers to school to share with his best friend Rhonda. If Jacob gives Rhonda 15 jolly ranchers, how many jolly ranchers will he have left over after sharing them with Rhonda?
Concrete

Pictorial

Abstract

34 - 15 (14 + 1)
34 - 14 = 20
20 - 1 = 19
2.NBT.A.1

Standard: Understand place value

- CCSS.MATH.CONTENT.2.NBT.A.1
  Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
  Understand the following as special cases:

- CCSS.MATH.CONTENT.2.NBT.A.1.A
  100 can be thought of as a bundle of ten tens — called a "hundred."

- CCSS.MATH.CONTENT.2.NBT.A.1.B
  The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

- CCSS.MATH.CONTENT.2.NBT.A.2
  Count within 1000; skip-count by 5s, 10s, and 100s.

- CCSS.MATH.CONTENT.2.NBT.A.3
  Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

- CCSS.MATH.CONTENT.2.NBT.A.4
  Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Task: Students will be able to use base ten blocks to show and explain their thinking by representing the value of each digit found in a three digit number.
Standard: Measure and estimate lengths in standard units.

- **CCSS.MATH.CONTENT.2.MD.A.1**
  Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

- **CCSS.MATH.CONTENT.2.MD.A.2**
  Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

- **CCSS.MATH.CONTENT.2.MD.A.3**
  Estimate lengths using units of inches, feet, centimeters, and meters.

- **CCSS.MATH.CONTENT.2.MD.A.4**
  Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Task: As the teacher reads the book “How big is a foot?” by Rolf Myller, he/she will give the students a chance to explore some of the things that happened in the story with their classmates. The teacher will ask the students to explain why the bed created by the apprentice is not what the king expected. The teacher will then ask the students to go around the room and find items that measure about a foot long.
The king told the carpenter to make a bed for the queen.

The king took off his shoes and walked carefully around the queen.

The carpenter took off his shoes and measured the size of the bed with his little feet.

Concrete

12 1 inch pieces

Pictorial

Abstract

1 foot = 12 inches

Math

1 foot = 12 inches
WEBSITES FOR C.P.A.TASKS

- Florida- CPalms- [http://www.cpalms.org/Public/](http://www.cpalms.org/Public/)
- Georgia- [https://www.georgiastandards.org/Georgia-Standards/Pages/Math-K-5.aspx](https://www.georgiastandards.org/Georgia-Standards/Pages/Math-K-5.aspx)
- Texas- [https://www.texasgateway.org/resource/k%E2%80%932-diagnostic-assessments](https://www.texasgateway.org/resource/k%E2%80%932-diagnostic-assessments)
- [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/)

Howard County Public Schools- [https://hcpss.instructure.com/courses/124](https://hcpss.instructure.com/courses/124) (Kindergarten)
PLEASE TWEET US WITH YOUR EXPERIENCES!

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