Problem Solving Tasks and Games the Develop Meaning by Connecting Multiple Strategies

NCTM Annual Conference 2018 Washington D.C.

K-2 Workshop

Session 462

You need a packet and a paper clip.

Solve Nan and Bert Problems on pages 3-5.

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UNIVERSITY OF ILLINOIS AT CHICAGO

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Problem Solving Tasks and Games the Develop Meaning by Connecting Multiple Strategies

NCTM Annual Conference 2018 Washington D.C.
K-2 Workshop
Session 1310

Jennifer Leimberer

Teaching Integrated Mathematics and Science (TIMS) Project

Learning Sciences Research Institute, University of Illinois at Chicago

mathtrailblazers@uic.edu

University of Illinois at Chicago
Learning Sciences Research Institute

METRO CHICAGO MATH INITIATIVE

Math Trailblazers

Math Trailblazers
Research and Revision Study

2003–2006	Research on implementation of 2nd edition in classrooms
2006–2009	Revision and field test of new materials in grades 1–5
2008–2009	Student Achievement Study
2010–2014	Final revision of materials for publication

National Science Foundation
WHERE DISCOVERIES BEGIN

Math Trailblazers

Research Studies

- Whole Number Study–UIC & KSU 2003–2008
- Implementation Study– –2006
- Fractions and Ratios– –2006
- Video Study–UIC 2003–2006
- Field Test Study–UIC 2006–2010
- Student Achievement Study–UIC 2009–2011
- Embedded Assessment Study–UIC 2010 - 2014

How can instruction support that learning?

How do students learn?

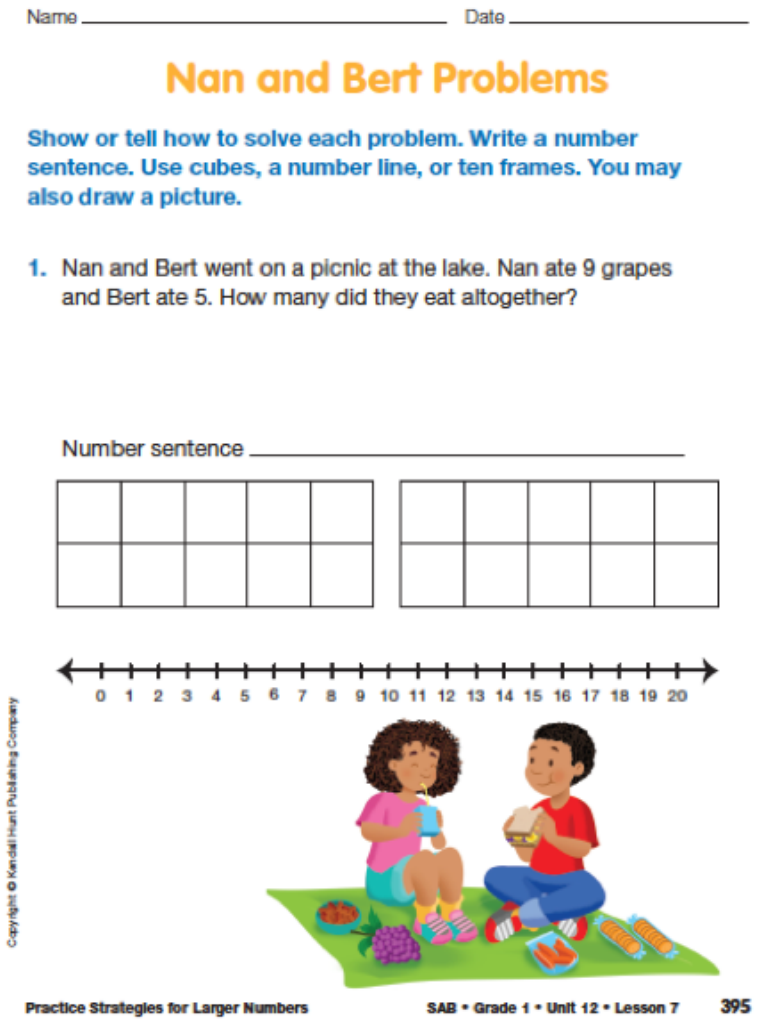
Research Studies

- Whole Number Study–UIC & KSU 2003–2008
- Implementation Study–UIC 2003–2006
- Fractions and Ratios–UMN 2004–2006
- Video Study–UIC 2003–2006
- Field Test Study–UIC 2006–2010
- Student Achievement Study–UIC 2009–2011
- Embedded Assessment Study–UIC 2010 - 2014

Nan and Bert Problems (pg. 2-3)

Solve and then discuss with a partner:

- Did you use the same strategy for each problem? Why?
- What strategies would you expect from your students?
- What strategies would you hope for?
- How might the tools support reasoning?



Name _____ Date _____

Nan and Bert Problems

Show or tell how to solve each problem. Write a number sentence. Use cubes, a number line, or ten frames. You may also draw a picture.

1. Nan and Bert went on a picnic at the lake. Nan ate 9 grapes and Bert ate 5. How many did they eat altogether?

Number sentence _____

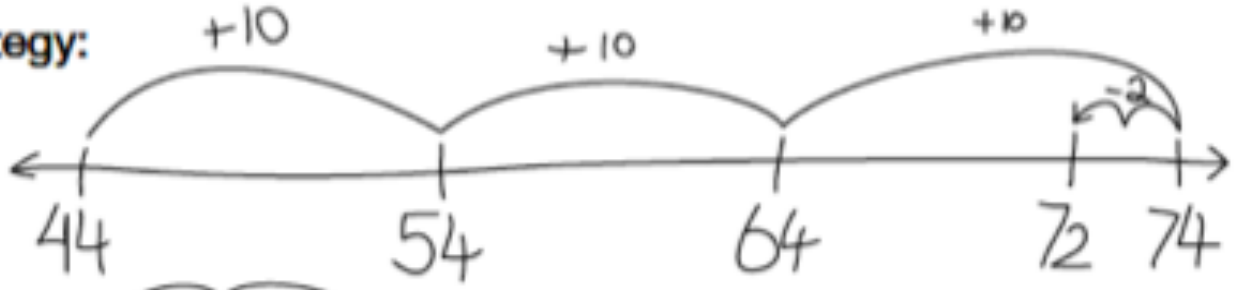
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Chris’s group made 28 hats. Julia’s group made 44 hats. How many hats did both groups make altogether?

Chris

I can think about it better if I make a number line in my head. I think about starting at 44, moving forward 30 and then back 2, since 28 is 2 less than 30. I can write it like this.

Chris's Strategy:



I start at 44 and then add on 30, going by tens: 54, 64, 74. Subtract 2 and it is 72. 72 hats.

Julia's Strategy:

Altogether we made 72 hats. I broke the numbers into tens and ones: 20 + 40 is 60, 8 and 4 is 12, 60 + 12 is 72. We made 72 hats.

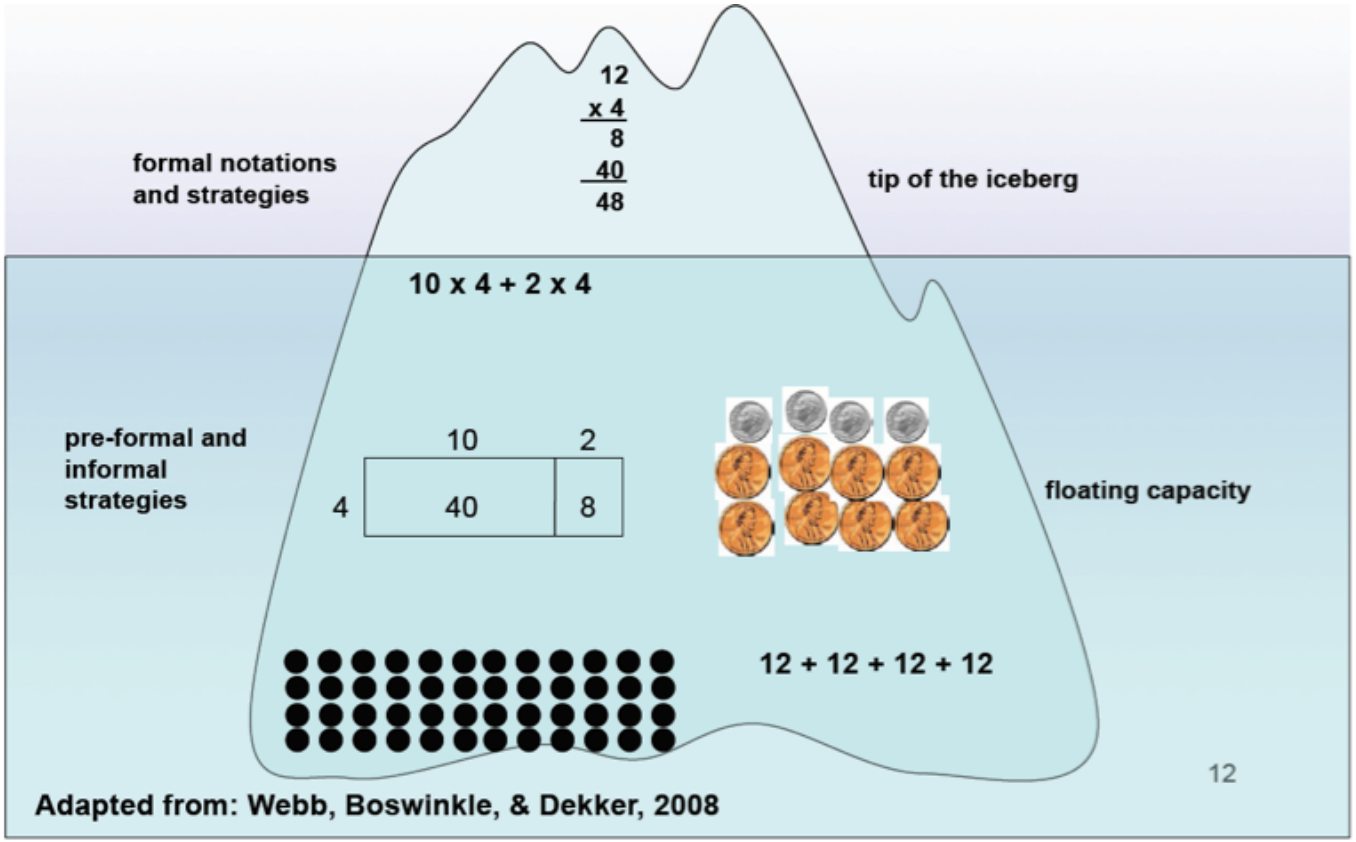
Julia

$$\begin{aligned} 28 &= 20 + 8 \\ +\,44 &= 40 + 4 \\ \hline & 60 + 12 = 72 \text{ hats} \end{aligned}$$

Why Multiple Strategies?

Why Multiple Strategies?

Rationale #1 A range of strategies allows for *sense-making* and development of *conceptual understanding*.



Why Multiple Strategies?

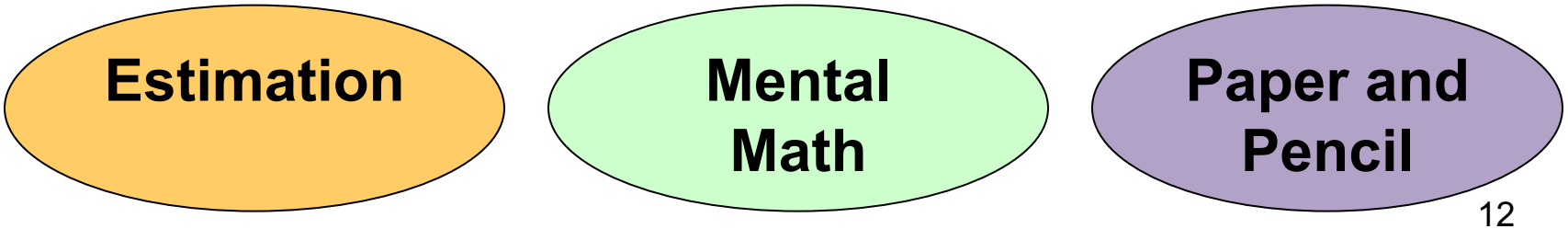
Rationale #2 A range of strategies promotes computational fluency.

-National Research Council

Why Multiple Strategies?

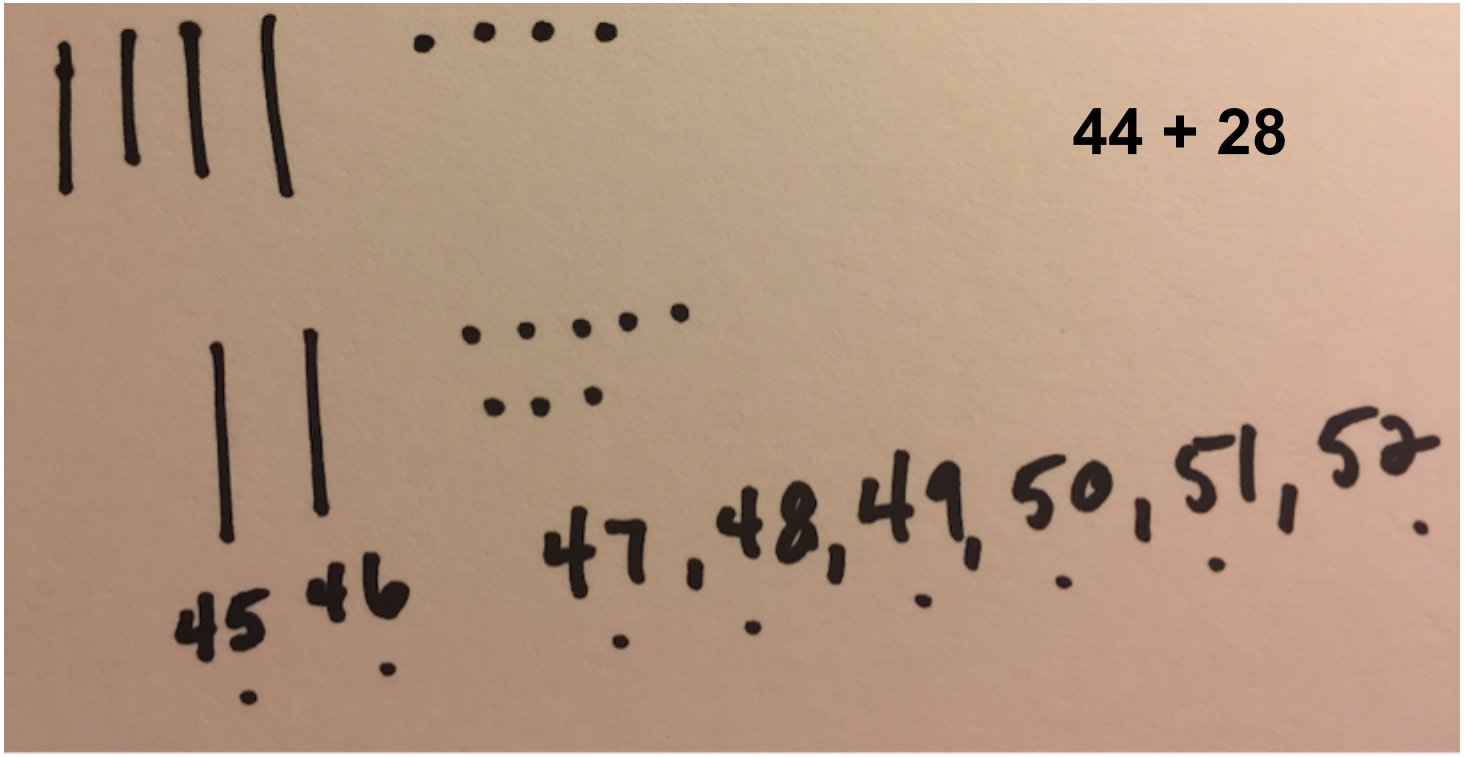
Rationale #3 A range of strategies helps students *access and respond to* mathematical contexts.

There are 5204 Chocos. A customer came in and bought 565. Another customer came in and wanted to buy 4859 pieces of candy. Was there enough candy in the store so that he could buy that much?



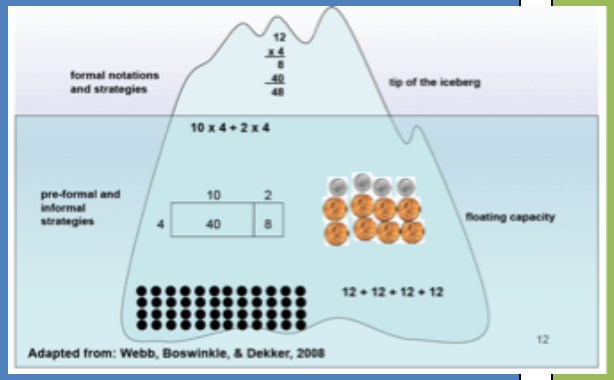
Why Multiple Strategies?

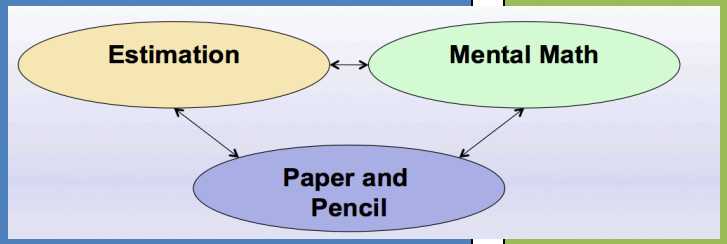
Rationale #4 A range of strategies supports a range of student *identities* and *needs*.



Why Multiple Strategies?

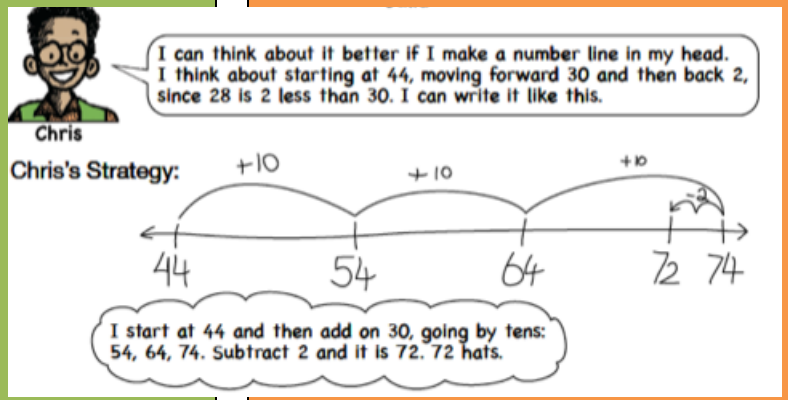
Direct Modeling





Counting Strategies

Reasoning from Known Facts



Carpenter, 1999; National Research Council, 2001

Why Multiple Strategies?

Why Multiple Strategies?

- Too many.
- Keep using the same inefficient strategy.
- Found a favorite.
- Connections between strategies and representations not present.
- Not sure which to use when.
- Strategy is a procedure rather than a way to support reasoning or understanding.

"*To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. The degree of students' conceptual understanding is related to the richness and extent of the connections they have made.*"

- National Research Council, 2001

How multiple strategies???

Connect

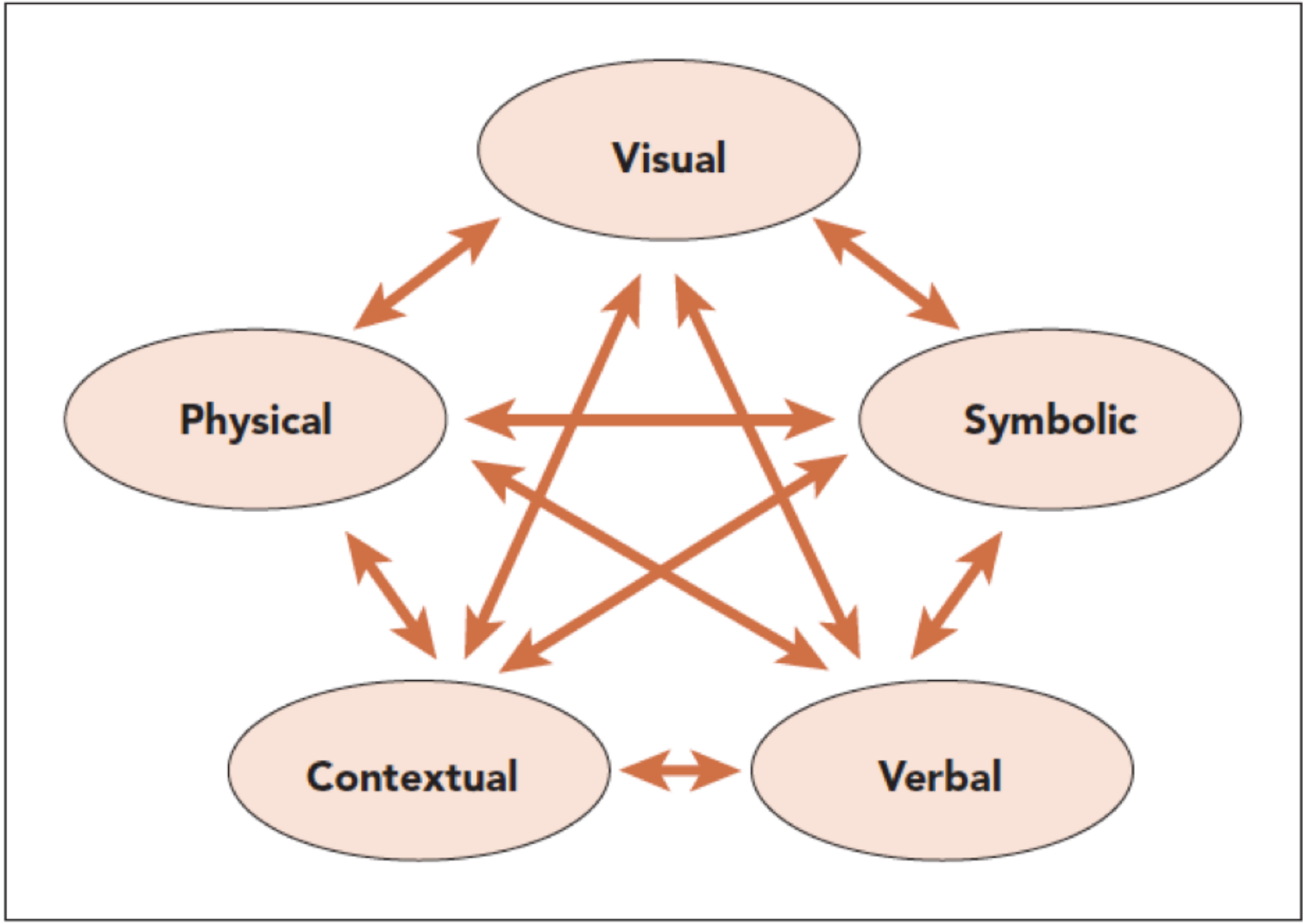
Representations of Reasoning

Connect to Representations

Five frogs on the log.
Three jump off.

There are 2 frogs on the log.

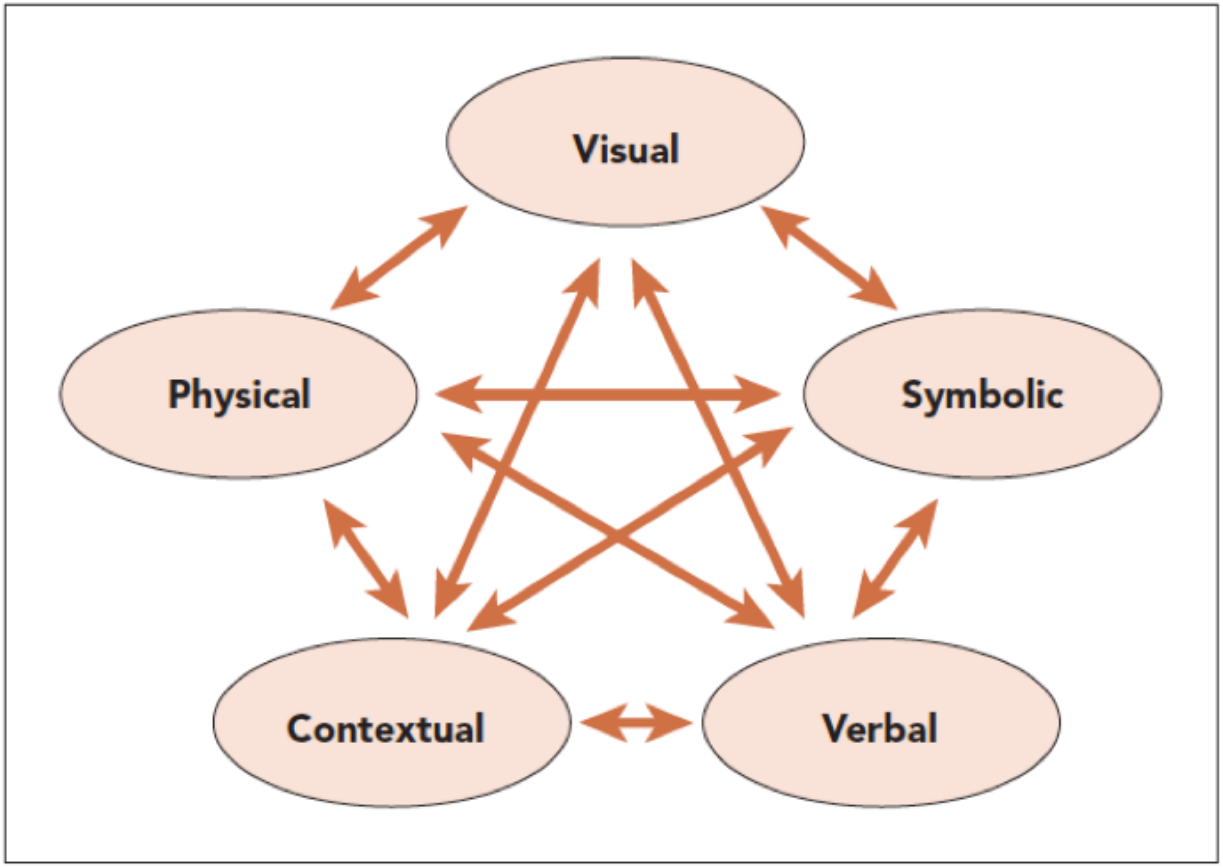
3 minus 5 is 2



From *Taking Action: Implementing Effective Mathematics Teaching Practices, K-5* by Smith, Huinker, Bill, NCTM 2017

Three specific instructional strategies to support students' development of representational competence are suggested by Marshall, Superfine, and Canty (2010, p. 40). They include—

- engaging in dialogue about the explicit connections between representations;
- alternating directionality of the connections made among representations; and
- encouraging purposeful selection of representations.

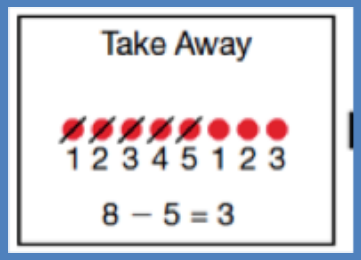


Stages of Conceptual Development

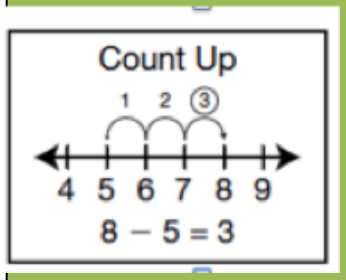
Carpenter, 1999; National Research Council, 2001

Phases to Develop Fluency

Direct Modeling

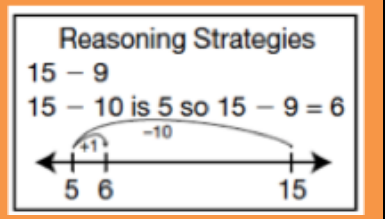


Counting Strategies



Reasoning From Known Facts

I know 3+5 = 8
so, 8 – 5 = 3

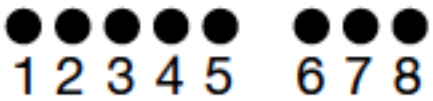


Fluency

- Flexibly
- Accurately
- Efficiently
- Appropriately

Carpenter, 1999; National Research Council, 2001

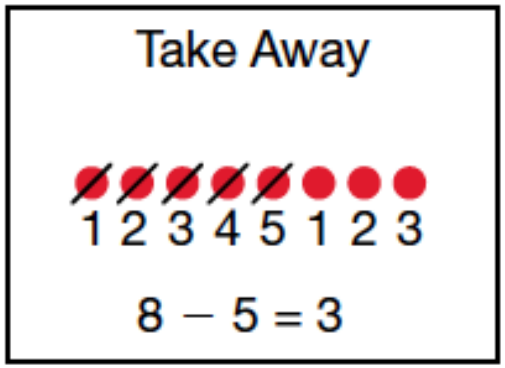
Stages of Conceptual Development

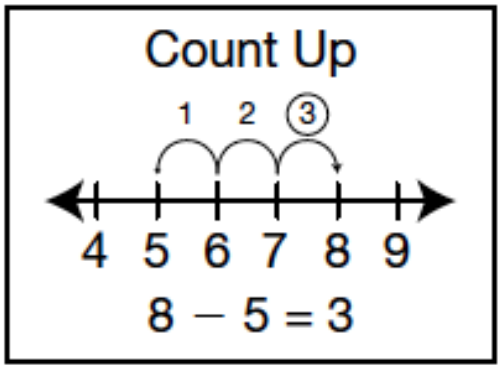
Direct Modeling	Counting Strategies	Reasoning from Known Facts
Counting All  5 + 3 = 8	Counting On  5 + 3 = 8	Reasoning from Known Facts 9 + 6 = 10 + 5 = 15

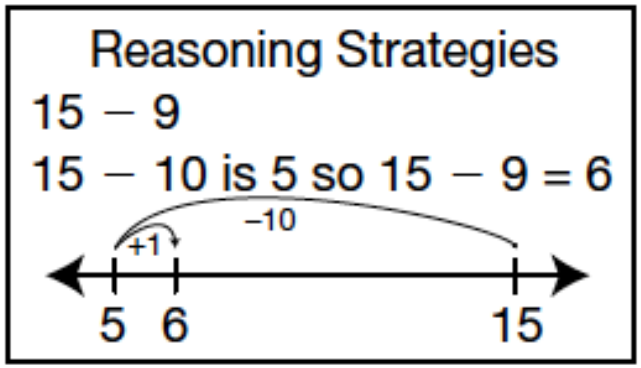
Carpenter, 1999; National Research Council, 2001

Stages of Conceptual Development

Direct Modeling	Counting Strategies	Reasoning from Known Facts

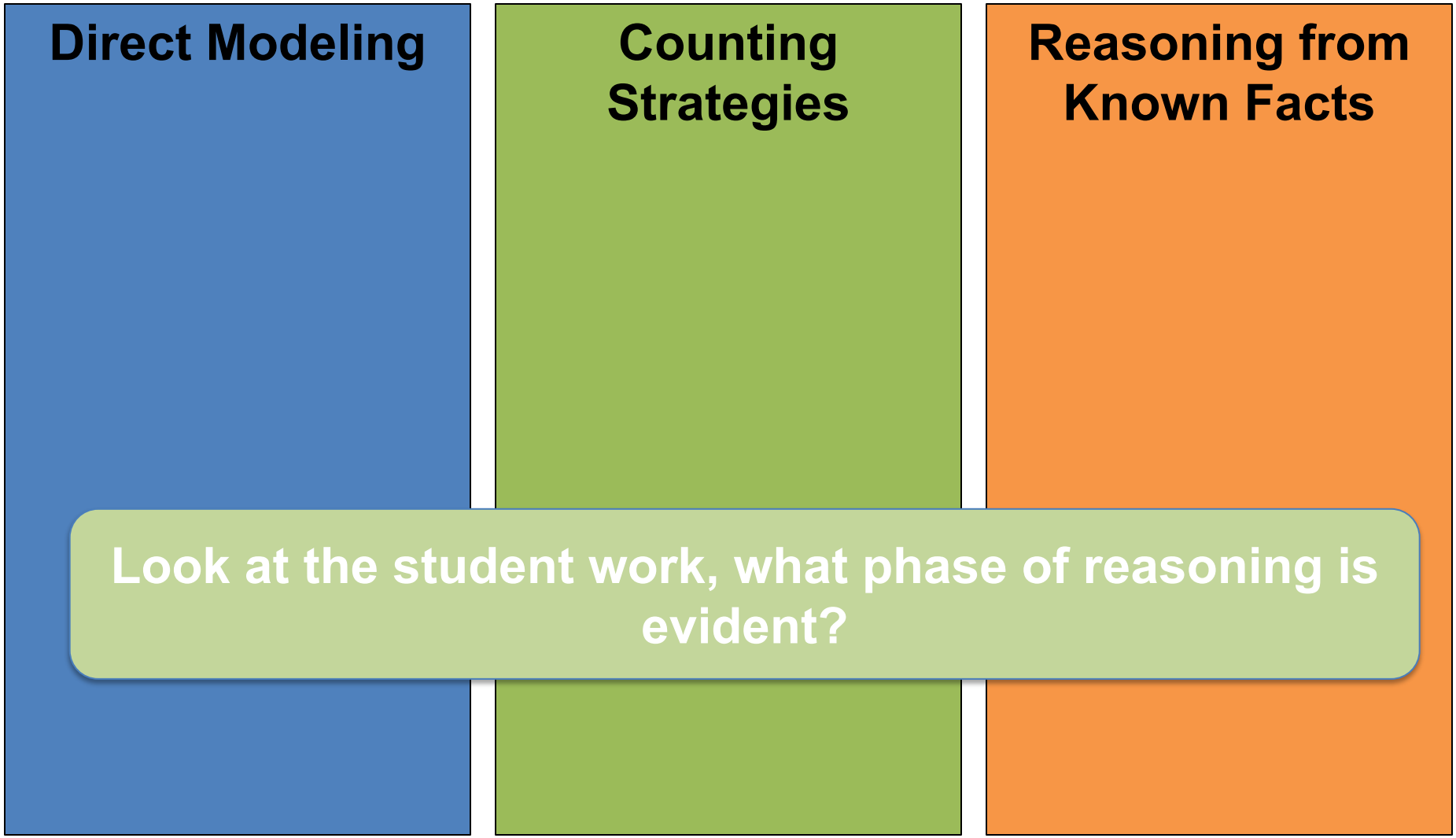






Carpenter, 1999; National Research Council, 2001

Stages of Conceptual Development



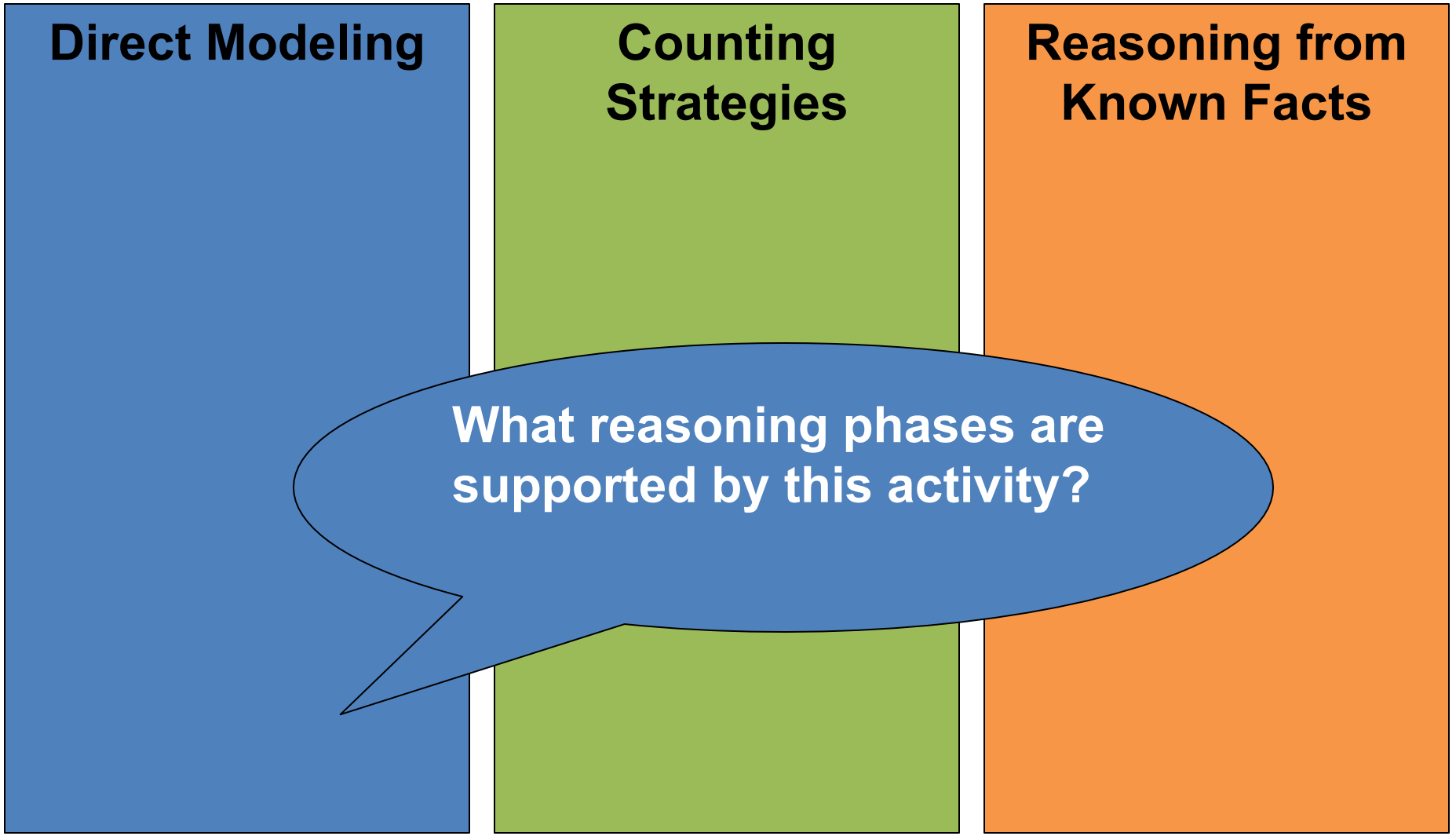
Carpenter, 1999; National Research Council, 2001

"*Discussions that focus on cognitively challenging mathematical tasks, namely those that promote thinking, reasoning, and problems solving, are a primary mechanism for promoting conceptual understanding of mathematics*."

Smith, Hughes, Engle & Stein, 2009, p. 549

What connections are being made?

Stages of Conceptual Development



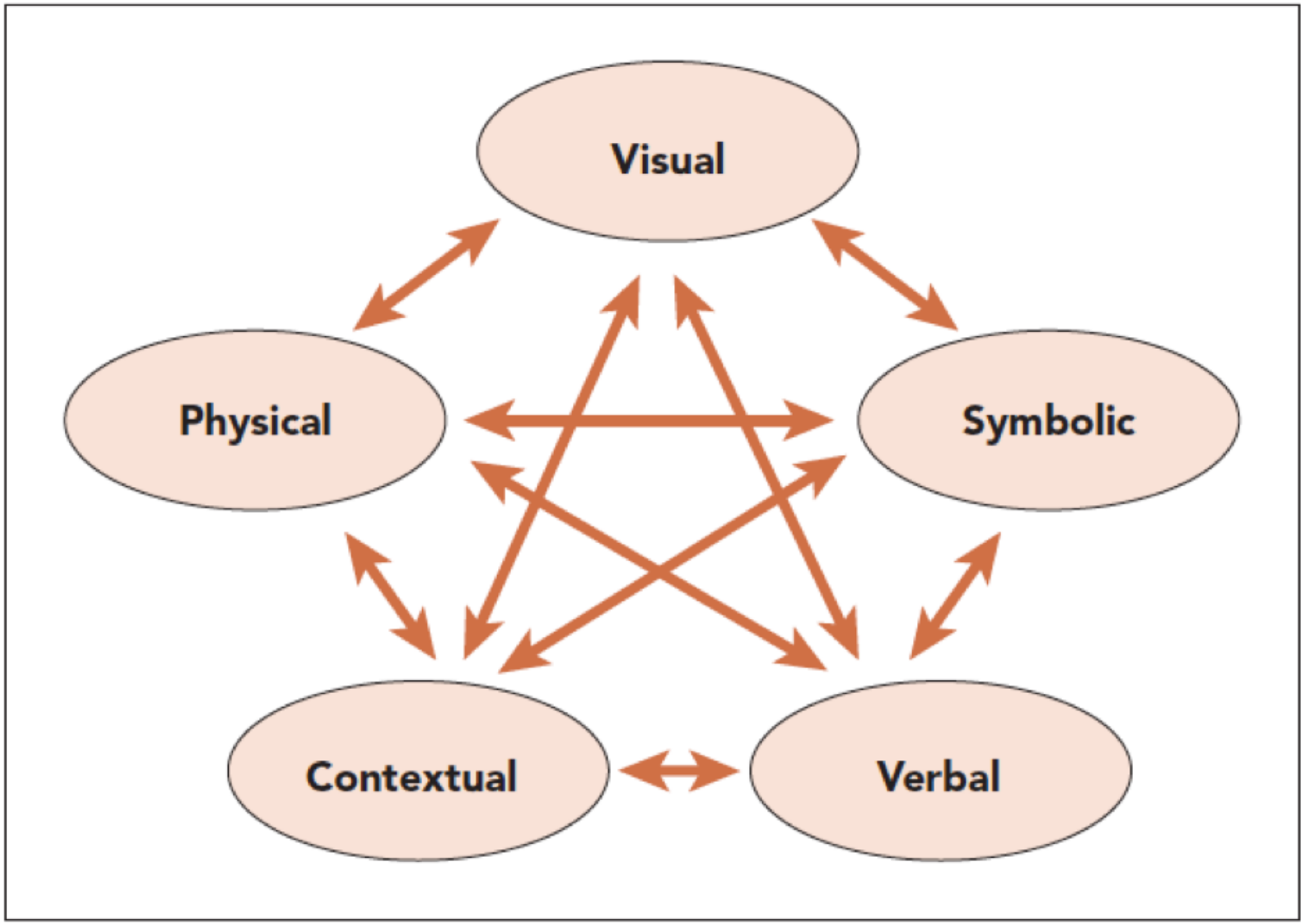
Carpenter, 1999; National Research Council, 2001

What connections are being made?

Partition Ten

Stand up and find a partner.

1. Face your partner.
2. Player 1 puts both arms straight up in the air.
3. Once in the air, Player 1 uses his or her fingers to show a number between 0 and 10.
4. Player 2 puts his or her arms down and displays the number needed to make ten. For example, Player 1 shows 4 fingers and Player 2 shows 6 fingers because $4 + 6 = 10$.
5. On the next turn, Player 2 leads.



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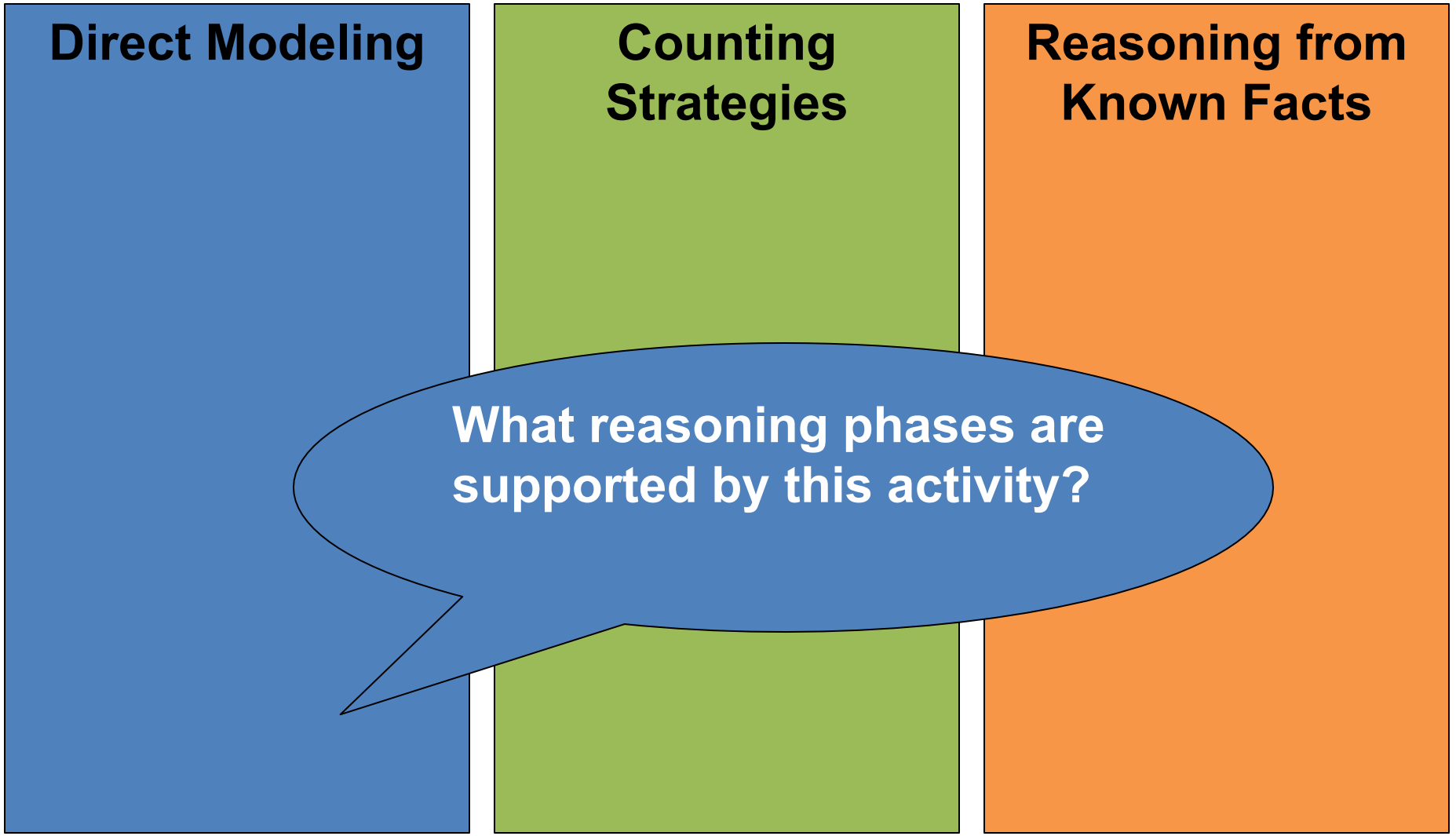
Stages of Conceptual Development



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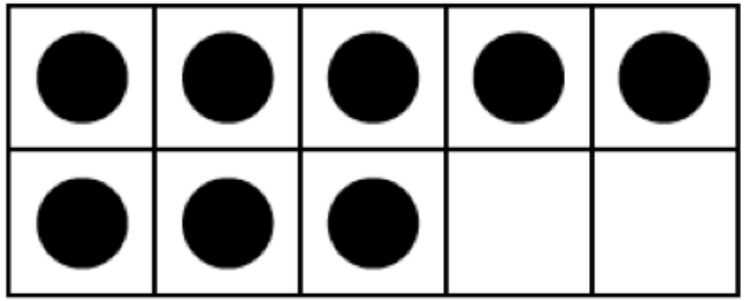
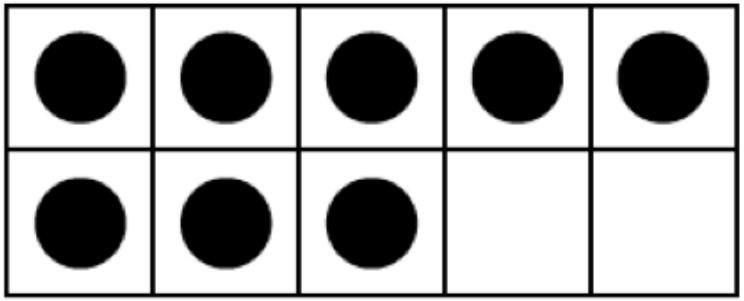
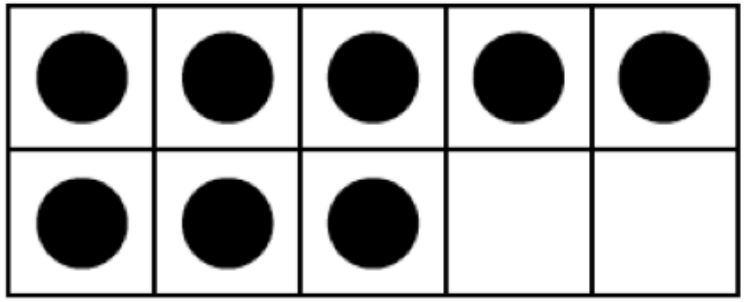
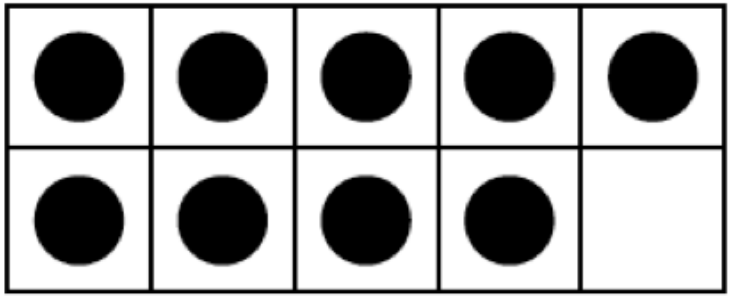




8 + 8 = 8 + 2 + 6

8 + 8 = 5 + 5 + 3 + 3



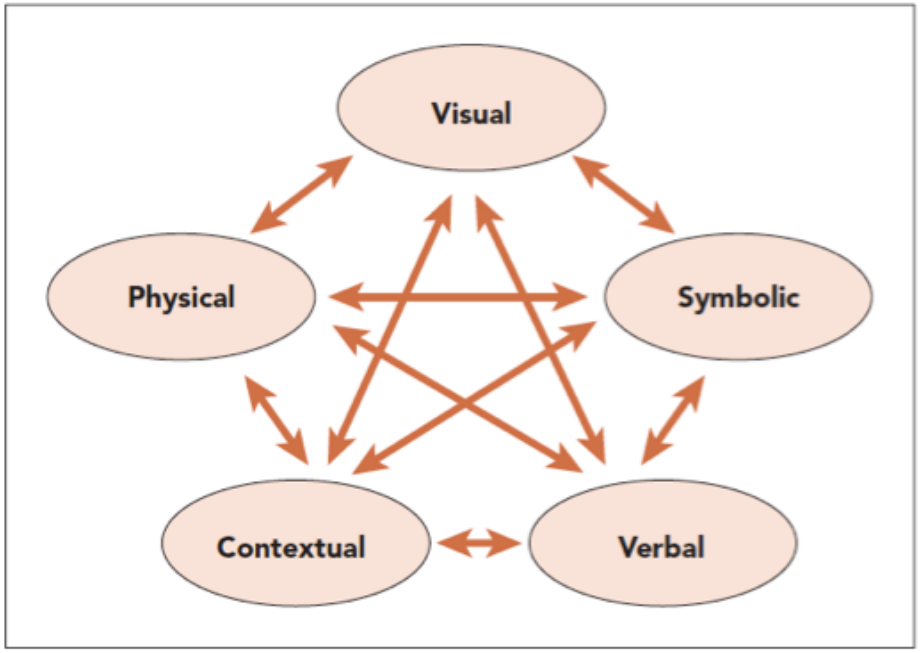


8 + 9 = 9 + 1 + 7

8 + 9 = 8 + 2 + 7

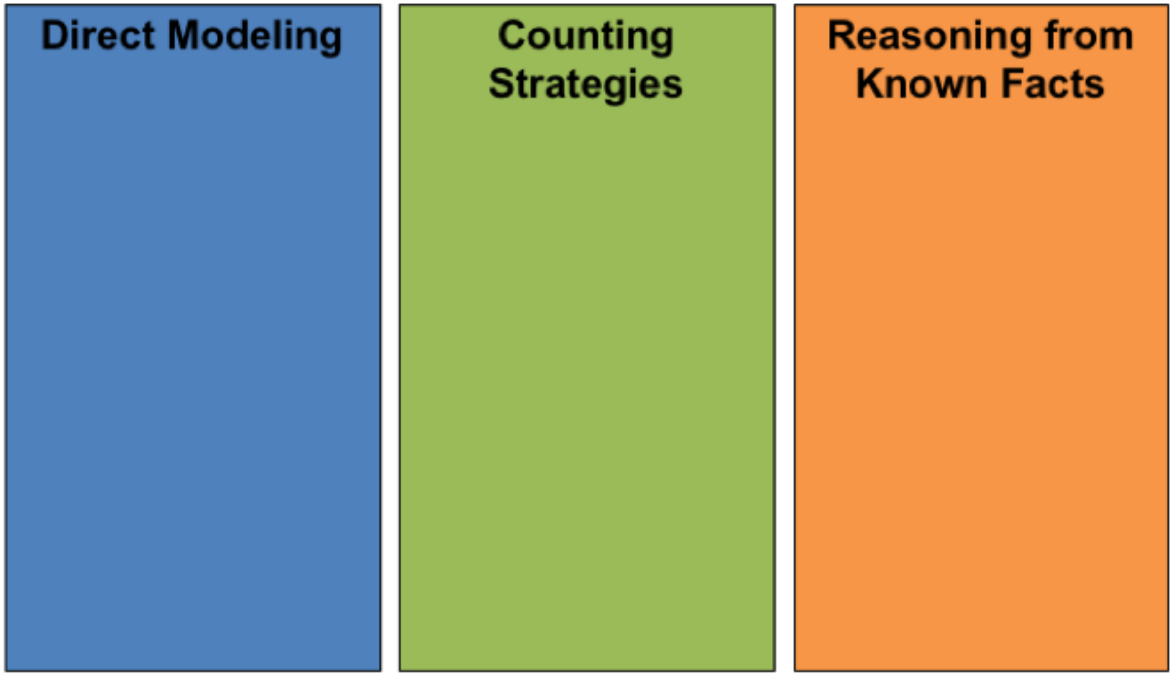
8 + 9 = 5 + 5 + 3 + 4

8 + 9 = 8 + 8 + 1



Connect Representations

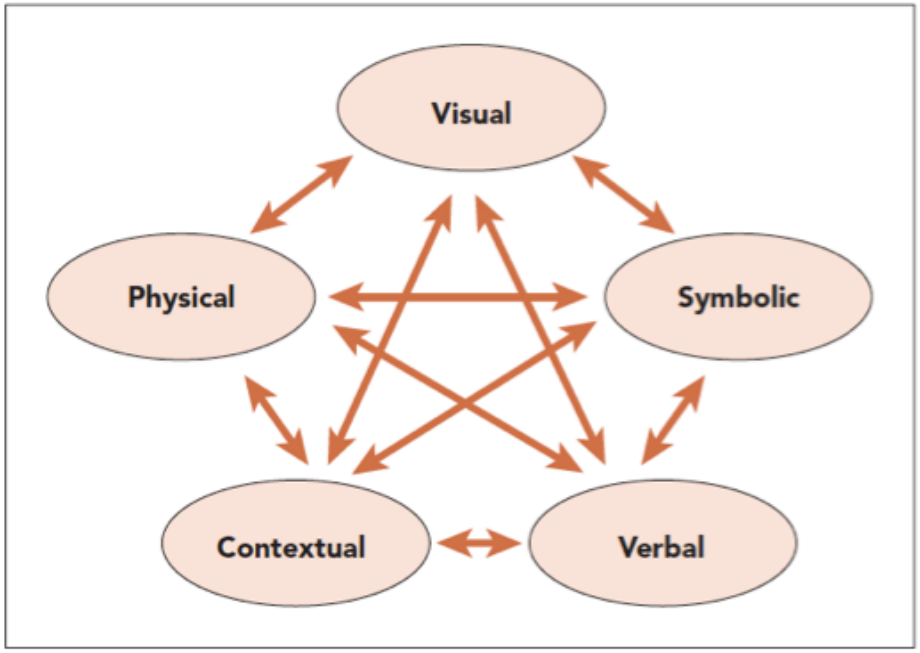
Connect to Reasoning Development



Analyze Kinds of Connections

Find someone from your grade level to play a game with:

- ***Towers of Ten Game*** Grade K
- ***Doubles, Doubles +1, Doubles -1*** Grade 1
- ***Not More Than 100*** Grade 2



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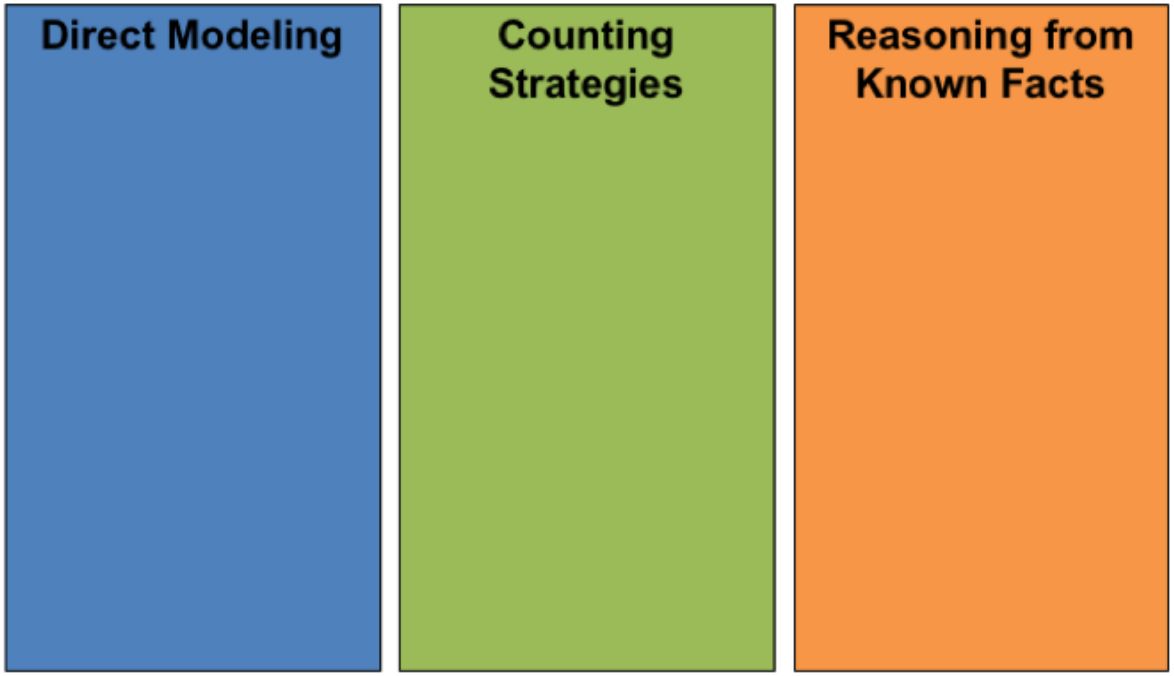
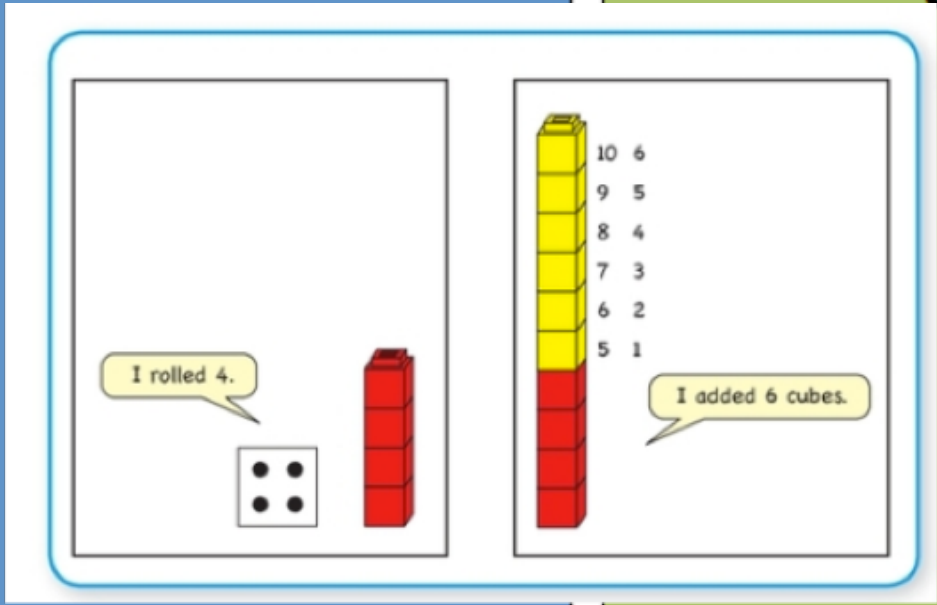


Stages of Conceptual Development

Direct Modeling



Counting Strategies

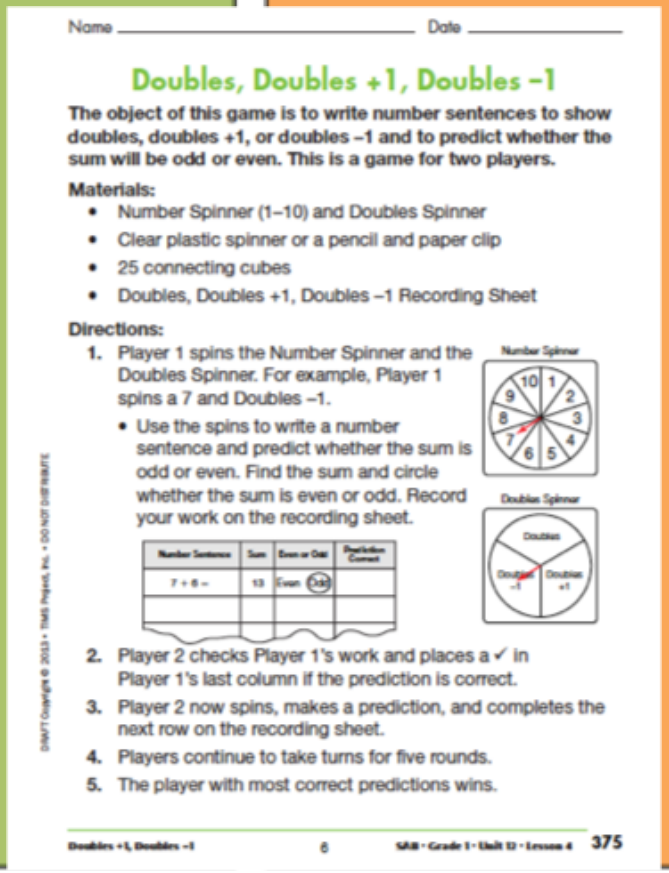
Reasoning from Known Facts

Stages of Conceptual Development

Direct Modeling

Counting Strategies

Reasoning from Known Facts



Name _____ Date _____

Doubles, Doubles +1, Doubles –1

The object of this game is to write number sentences to show doubles, doubles +1, or doubles –1 and to predict whether the sum will be odd or even. This is a game for two players.

Materials:

- Number Spinner (1–10) and Doubles Spinner
- Clear plastic spinner or a pencil and paper clip
- 25 connecting cubes
- Doubles, Doubles +1, Doubles –1 Recording Sheet

Directions:

1. Player 1 spins the Number Spinner and the Doubles Spinner. For example, Player 1 spins a 7 and Doubles –1.
   - Use the spins to write a number sentence and predict whether the sum is odd or even. Find the sum and circle whether the sum is even or odd. Record your work on the recording sheet.

Number Spinner

Doubles Spinner

Number Sentence	Sum	Even or Odd	Prediction Correct
7 + 6 =	13	Even (Odd)	

2. Player 2 checks Player 1's work and places a ✓ in Player 1's last column if the prediction is correct.
3. Player 2 now spins, makes a prediction, and completes the next row on the recording sheet.
4. Players continue to take turns for five rounds.
5. The player with most correct predictions wins.

Stages of Conceptual Development

Direct Modeling

Counting Strategies

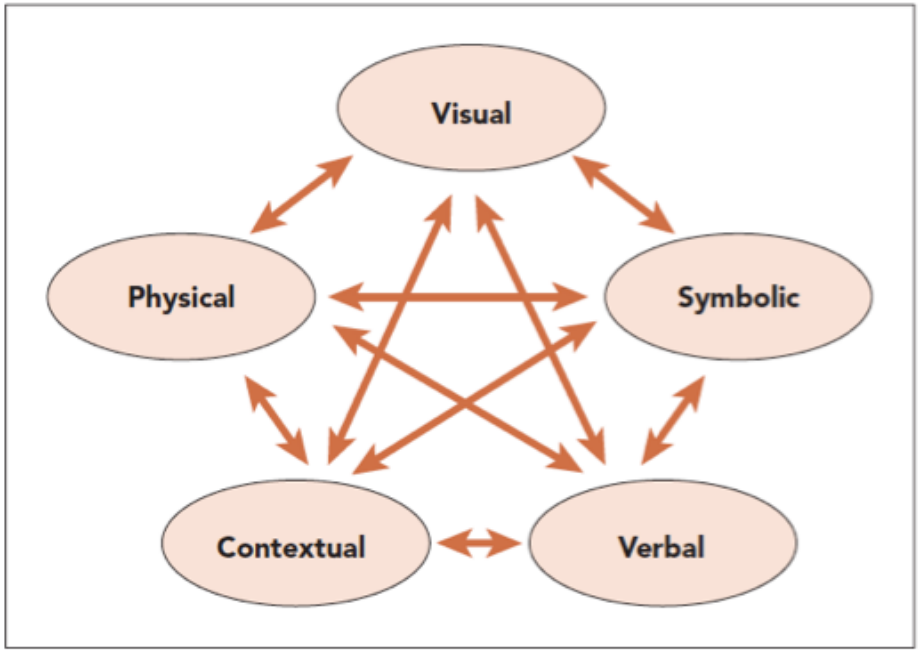
Reasoning from Known Facts

Name _____ Date _____

Sample Recording Sheet

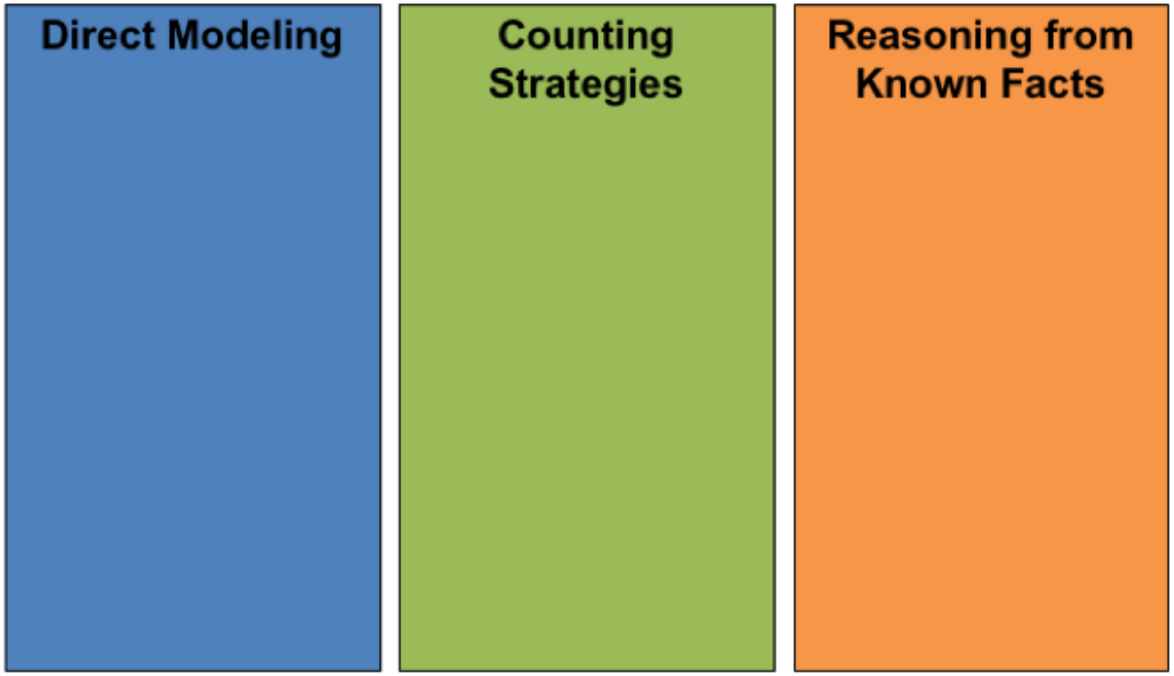
Spin	Tens	Ones	Number Sentence
1	5	0	50 + 0 = 50
2		6	50 + 6 = 56
3	2	0	56 + 20 = 76
4		9	76 + 9 = 85
5	1	0	85 + 10 = 95

Not More Than 100 Spinner



Connect Representations

Connect to Reasoning Development



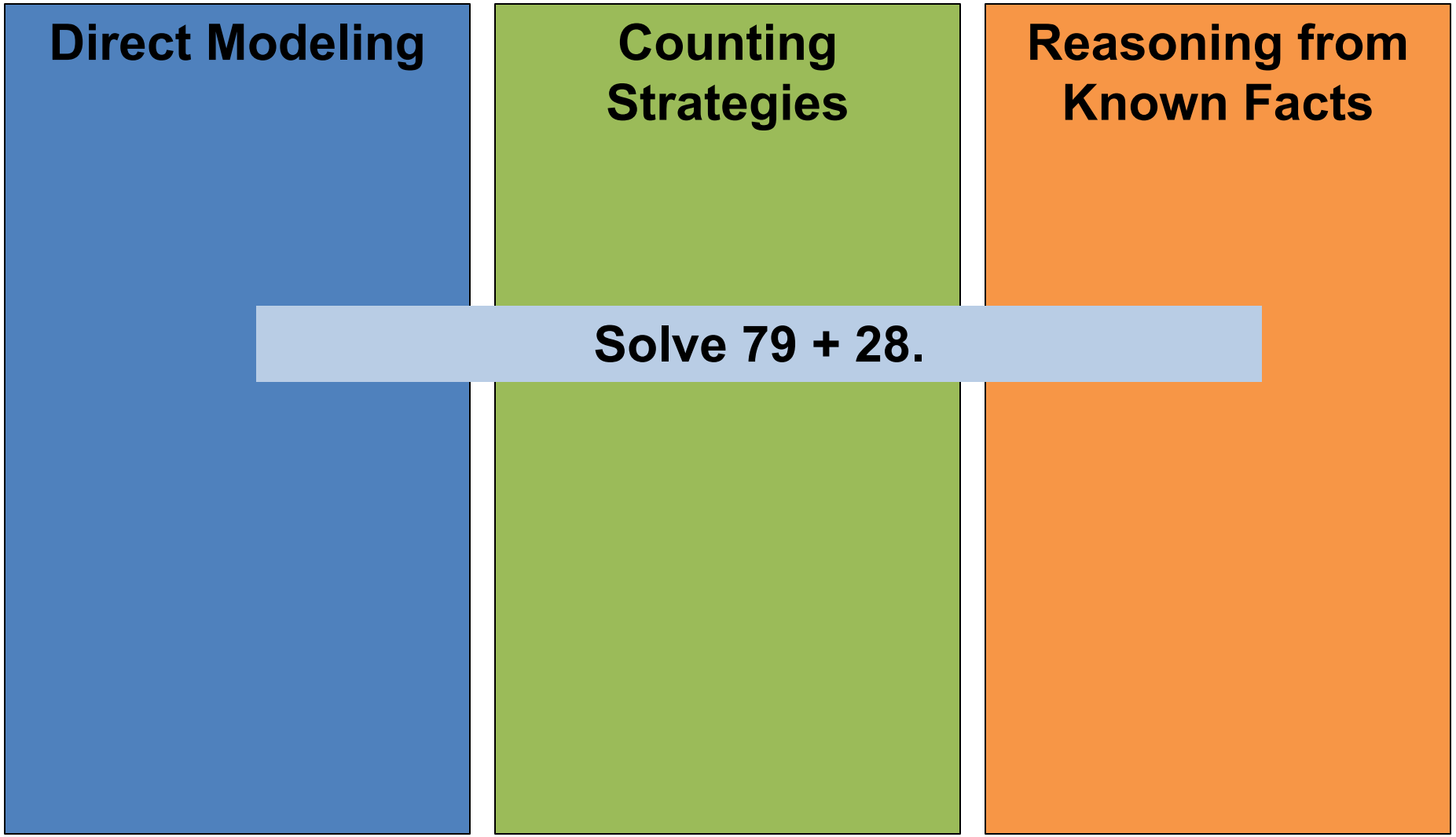
Connect to Student's Thinking

I thought about coins . . .

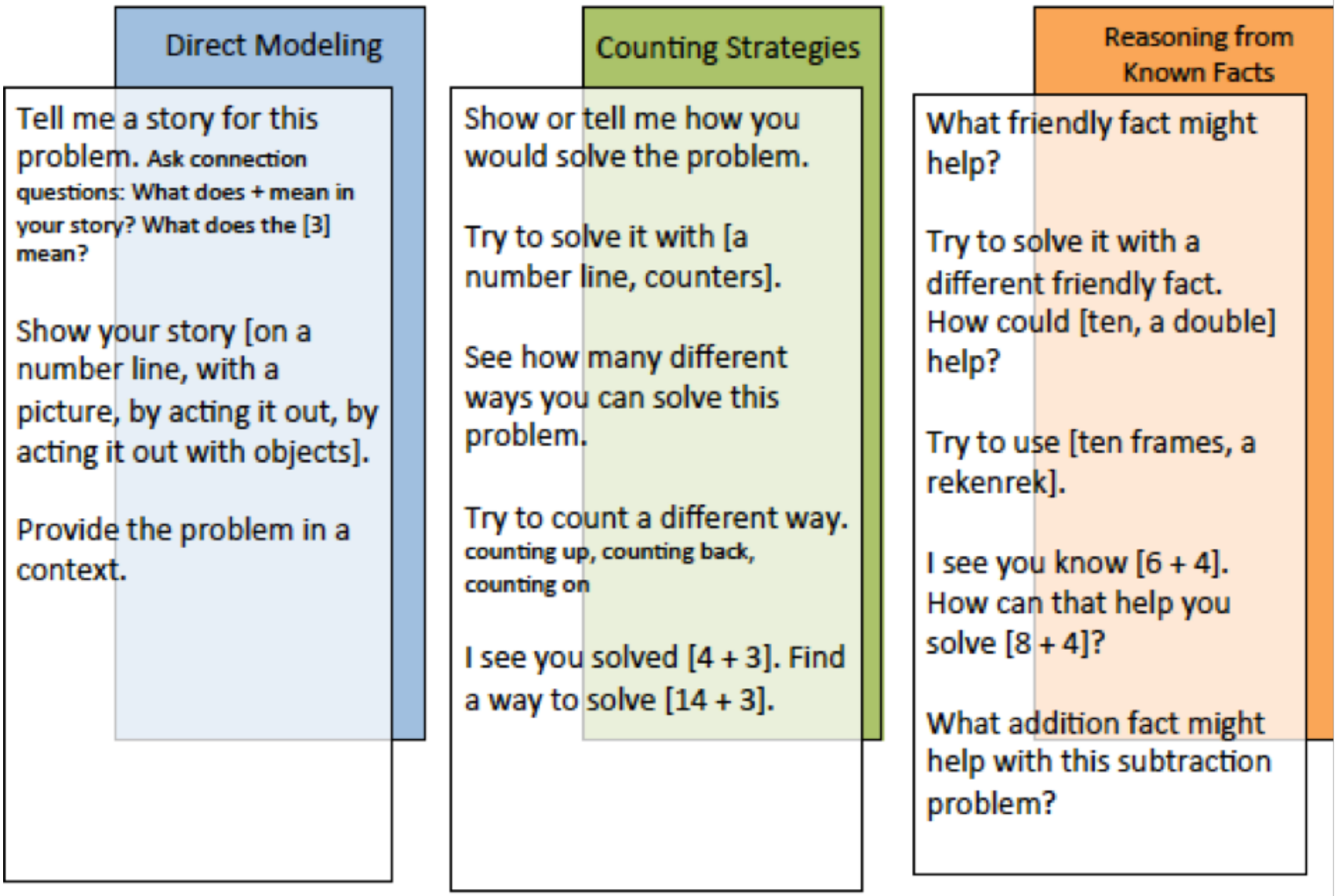
80 + 28 = 108
1 less than 108 is 107.

Are all students in the same place?

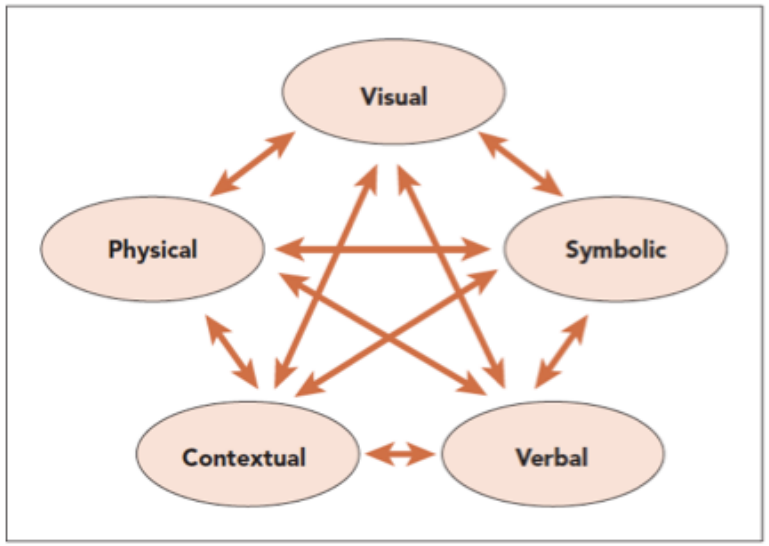
Stages of Conceptual Development



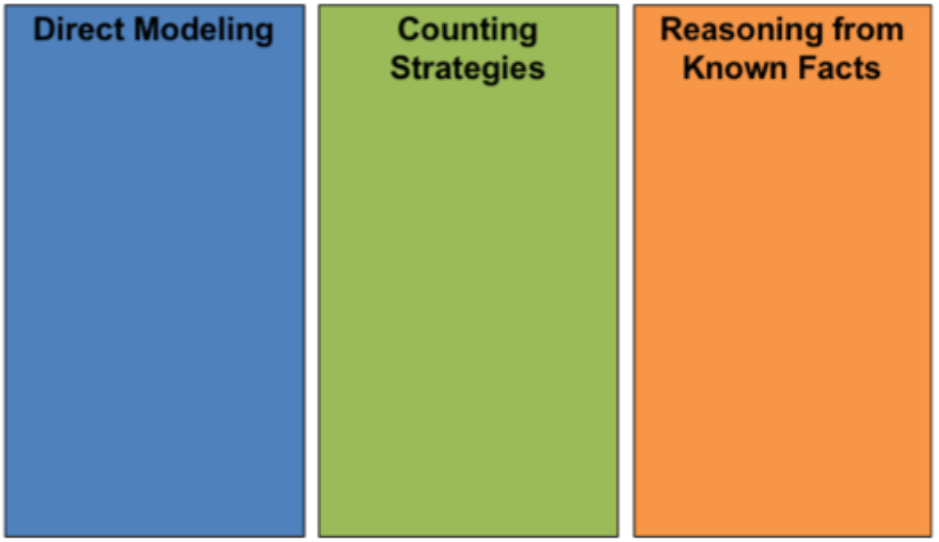
Carpenter, 1999; National Research Council, 2001



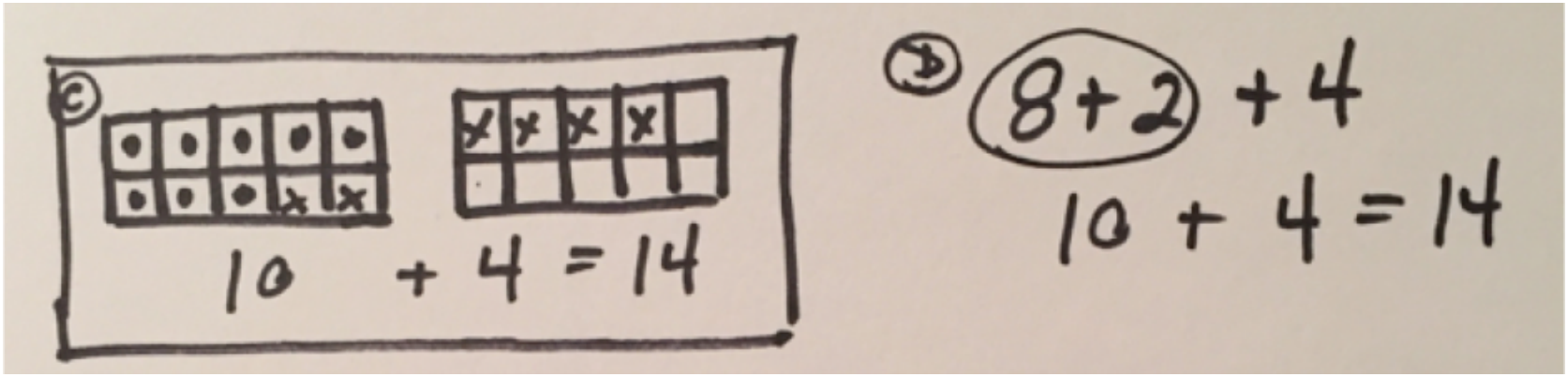
Addition and Subtraction



Connect Representations

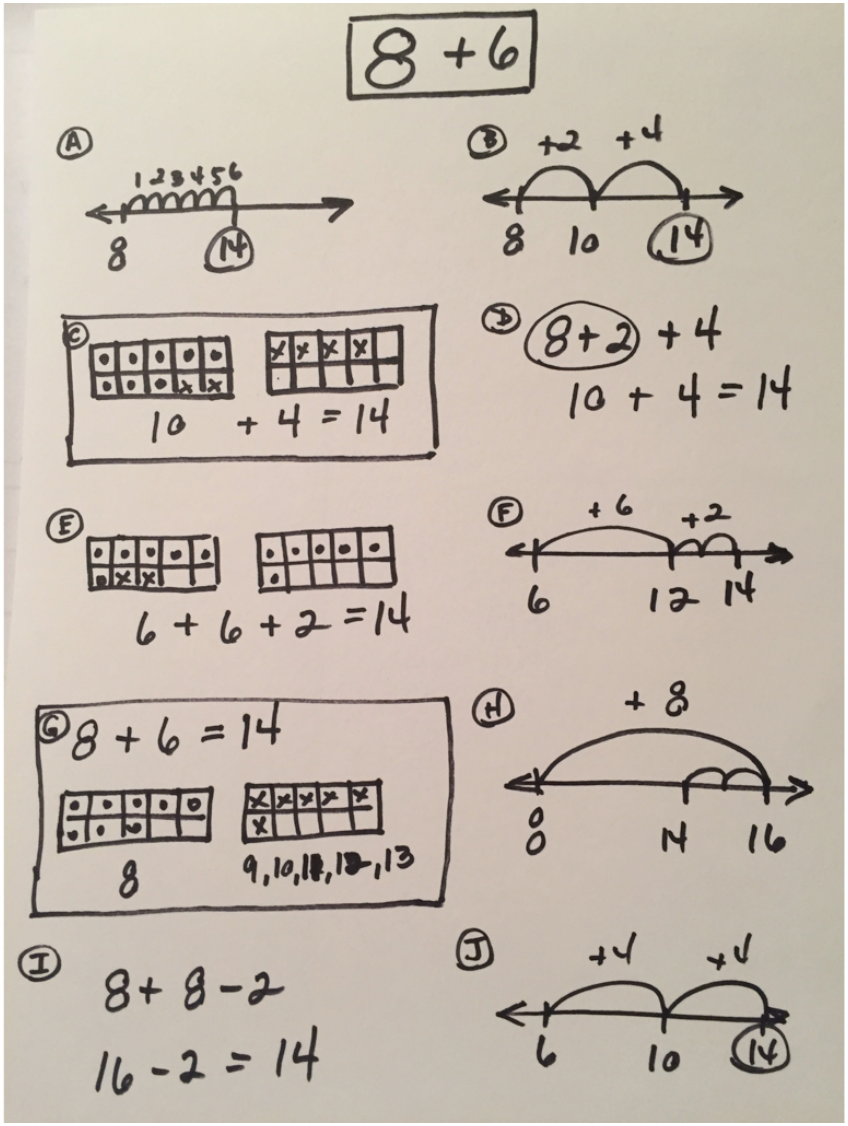


Connect to Reasoning Development



Connect Strategies

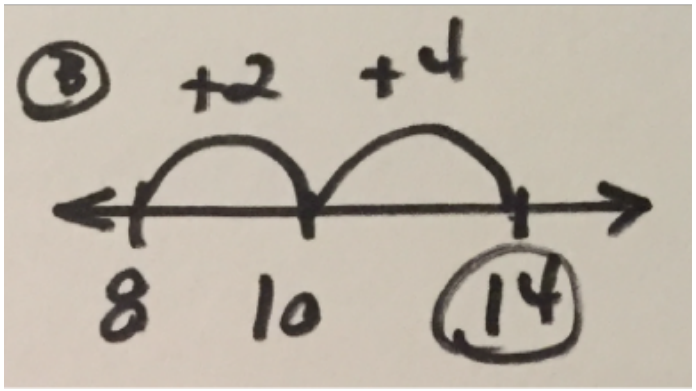
Connect to Strategies (pg. 13)

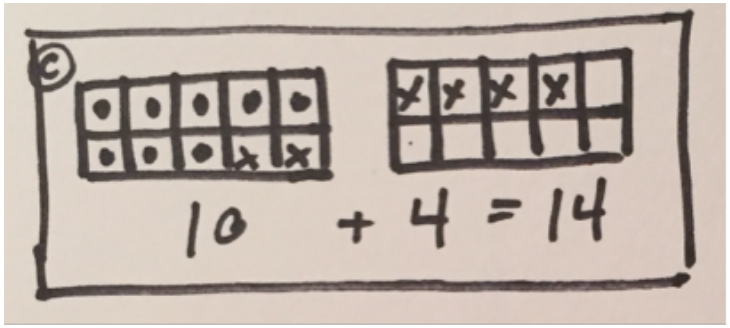


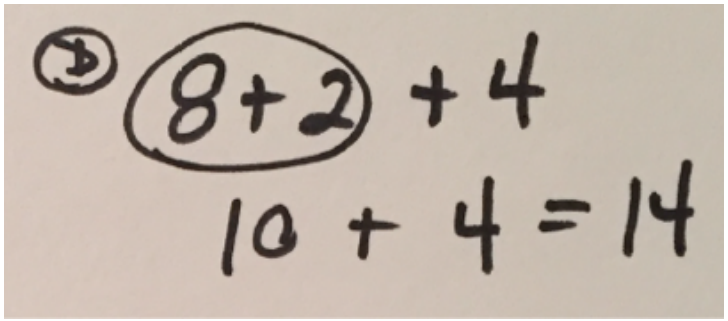
Look at the solutions to 8 + 6

- **Which used the same representation?**
- **Which used the same strategy?**

Connect to Strategies(pg. 6)

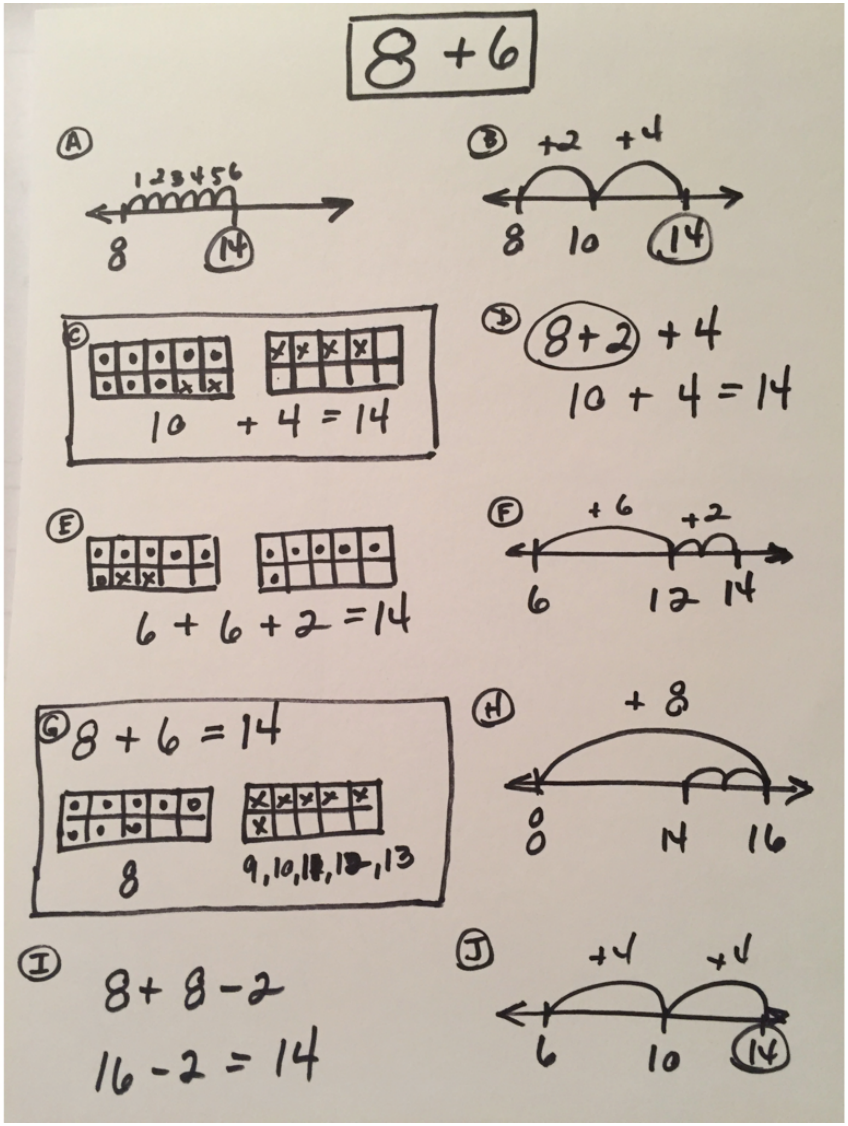






- **Did these three students use the same strategy?**
- **I notice the +2 in Student B solution. Does student C add 2 to 8? How is that shown?**
- **How is the +2 shown in Student D's solution?**

Connect to Strategies (pg. 6)

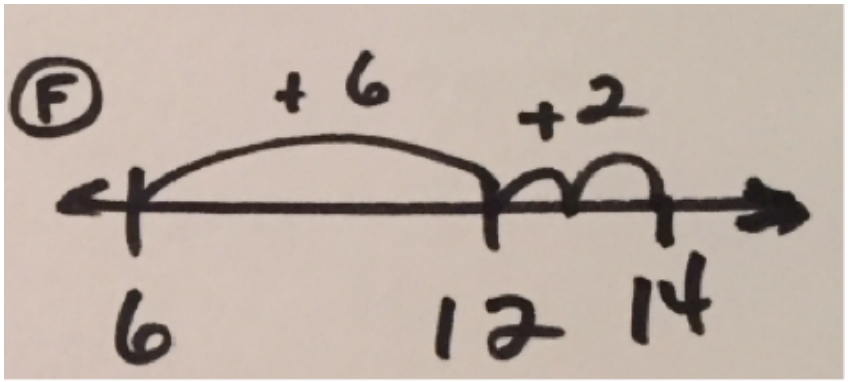


Your turn. Work with a partner.

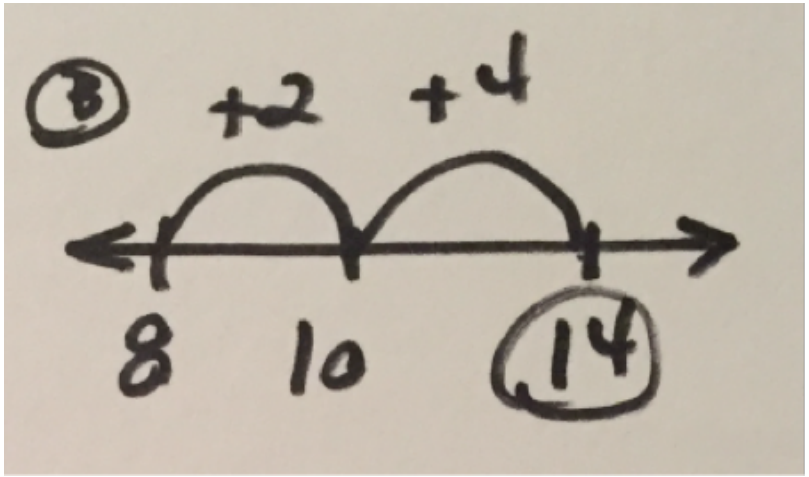
Write questions that help students connect the strategies and representations.

Use Questions on pg. 7 as a guide.

Connect to Strategies



I notice that both of these students counted on. Student F started at 6 and Student B started at 8.



Can they do that? Why does that work?

Ⓐ $74 + 38 = 112$

$70 \quad 4 \quad 8 \quad 30$

$4 + 8 = 12$

$70 + 30 = 100$

$100 + 12 = 112$

Ⓑ

74

$+38$

$12 + 100 = 112$

74

$+38$

12

100

112

Ⓒ $74 + 38 = 22$

$70 + 30 = 100$

$4 + 8 = 12$

112

Ⓓ $74 + 38 = 108$

$70 + 30 = 100$

$100 + 8 = 108$

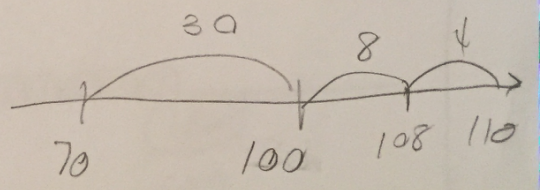
$108 + 4 = 112$

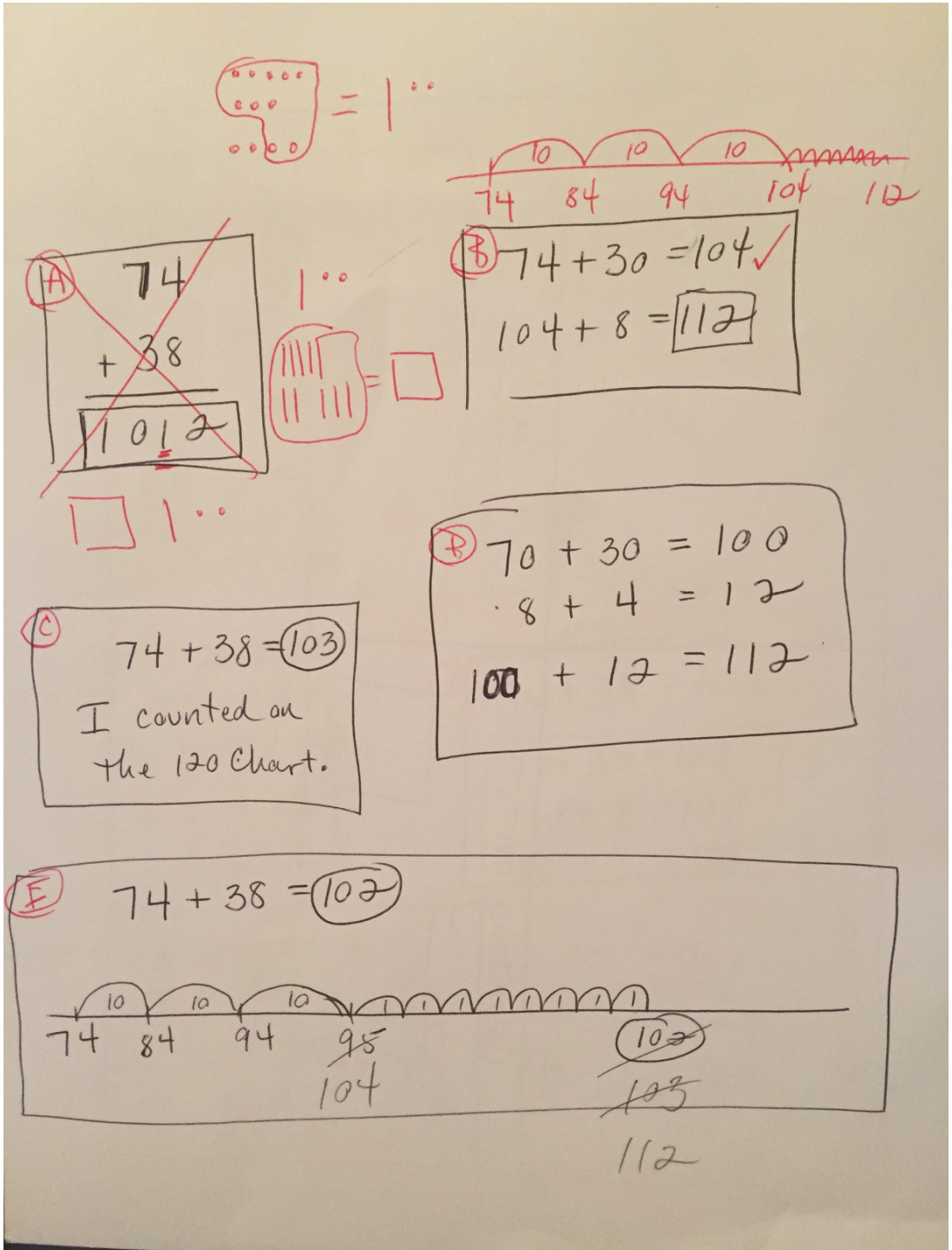
Ⓕ $74 + 38 = 112$

100

12

112





Design your own based on something you are seeing in your classroom.

Connect to Strategies

1. Chris's group made 28 hats. Julia's group made 44 hats. How many hats did both groups make altogether?

Julia's Strategy:

Julia

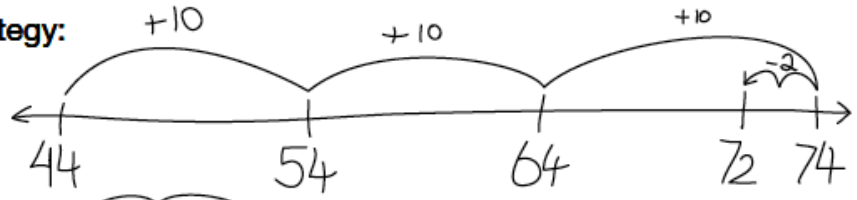
Altogether we made 72 hats. I broke the numbers into tens and ones: 20 + 40 is 60, 8 and 4 is 12, 60 + 12 is 72. We made 72 hats.

$$\begin{aligned} 28 &= 20 + 8 \\ +\ 44 &= 40 + 4 \\ \hline & 60 + 12 = 72 \text{ hats} \end{aligned}$$

Chris

I can think about it better if I make a number line in my head. I think about starting at 44, moving forward 30 and then back 2, since 28 is 2 less than 30. I can write it like this.

Chris's Strategy:



I start at 44 and then add on 30, going by tens: 54, 64, 74. Subtract 2 and it is 72. 72 hats.

2. **A.** How did Julia use tens and ones to add?
   B. How did Chris use tens and ones?

Connect to Student's Strategies

1. Jason solved the following problem. Does his answer make sense? Why or why not?

$$\begin{array}{r} 97 \\ +\ 86 \\ \hline 1713 \end{array}$$

Connect to Context Demands

Sort the problems by strategy.

2 – 1

2 – 2

6 – 5

8 – 7

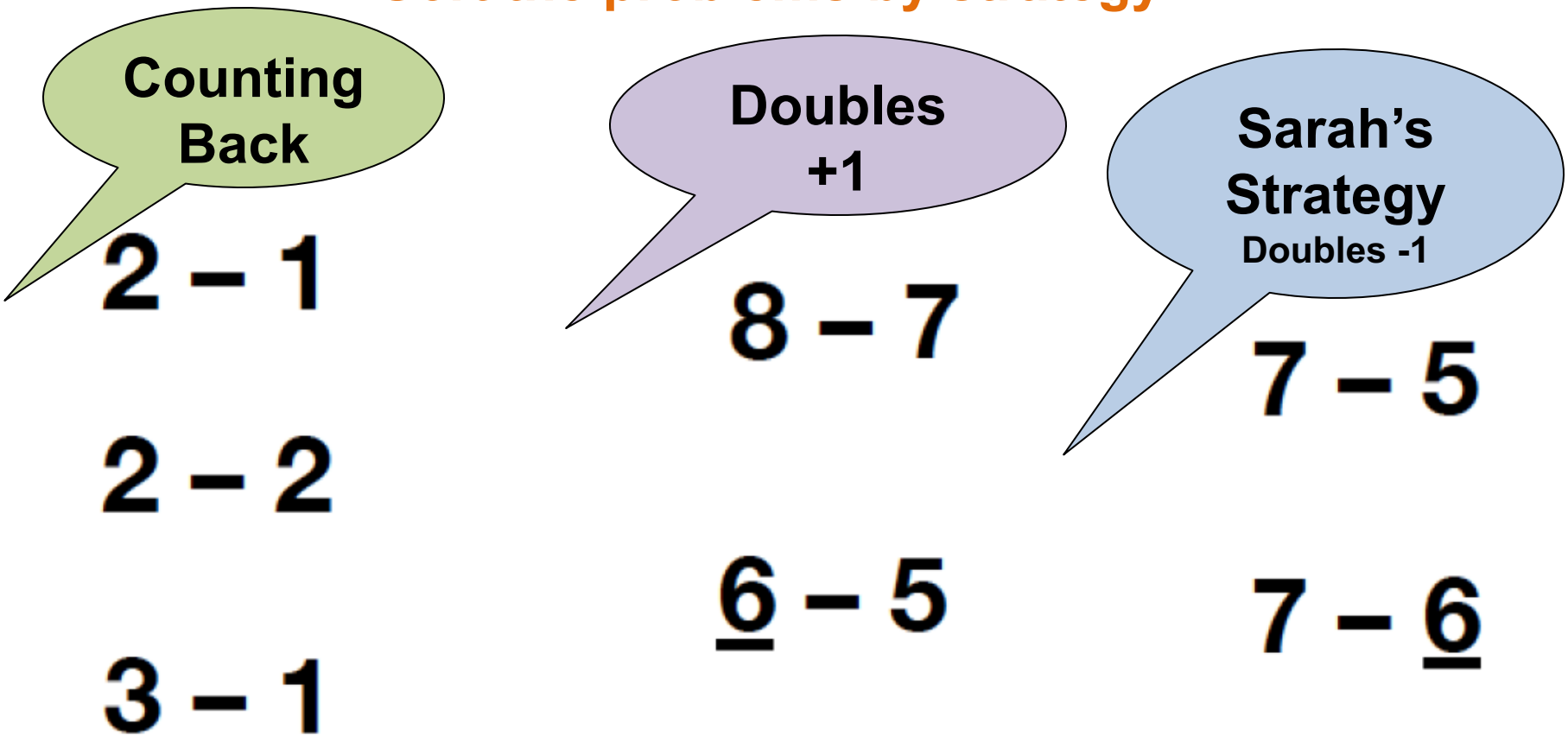
7 – 5

7 – 6

3 – 1

Connect to Context Demands

Sort the problems by strategy.



Connect to Context Demands

9 + 5	8 + 7	8 + 6	8 + 8

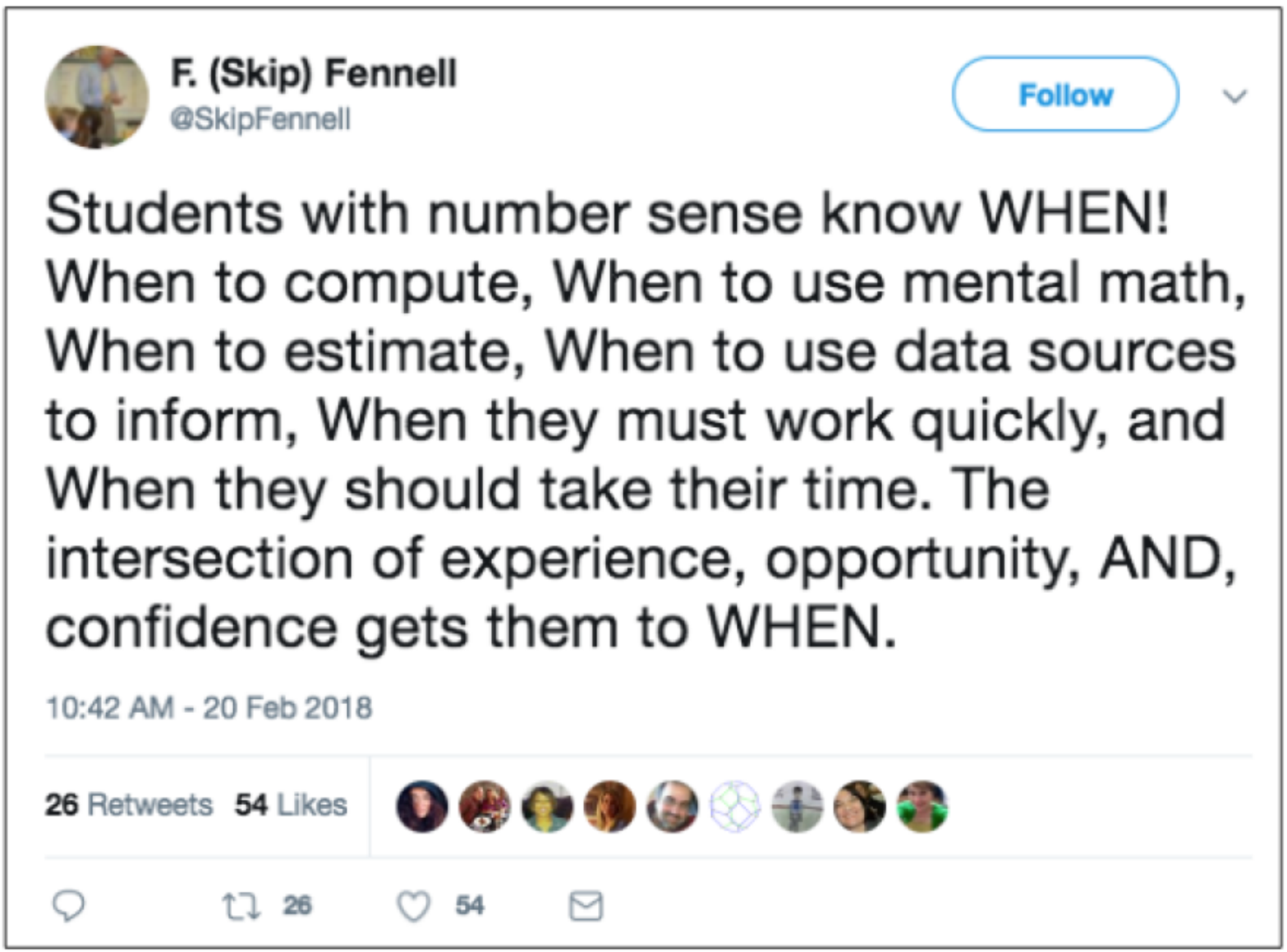
Would you use the same strategy?

9 + 2

Connect to Student's Thinking

How can you use one fact to solve another?

4 – 4	4 – 3	10 – 7	9 – 7


F. (Skip) Fennell
@SkipFennell
Follow
Students with number sense know WHEN! When to compute, When to use mental math, When to estimate, When to use data sources to inform, When they must work quickly, and When they should take their time. The intersection of experience, opportunity, AND, confidence gets them to WHEN.
10:42 AM - 20 Feb 2018
26 Retweets 54 Likes
26
54

Connect to Context Demands

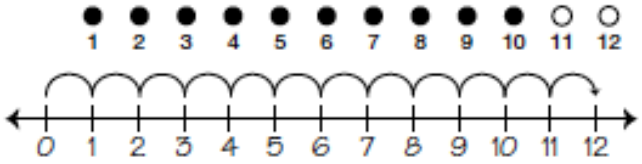
My Addition Strategies Menu for Larger Numbers

Counting All	Making Ten
Counting On	Using Ten
Another Strategy _____	Using Doubles

Addition Strategies Menu for the Facts

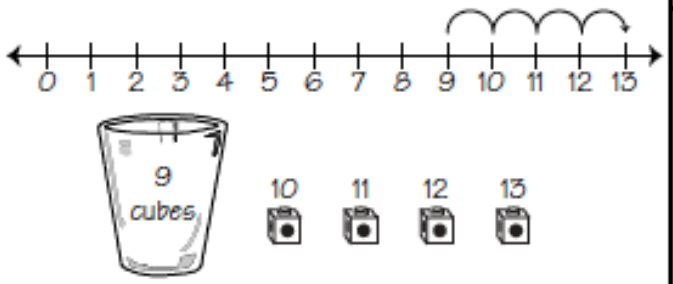
Counting All

$10 + 2 = 12$



Counting On

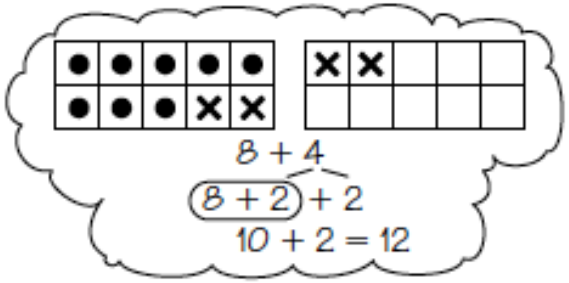
$9 + 4 = 13$



Another Strategy _____

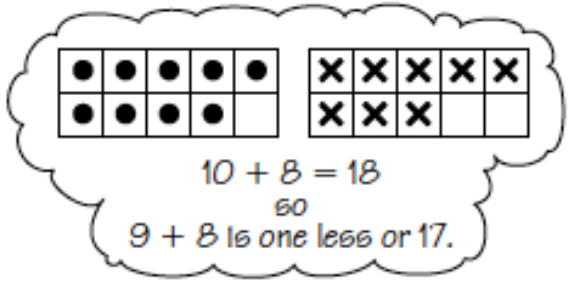
Making Ten

$8 + 4 = 12$



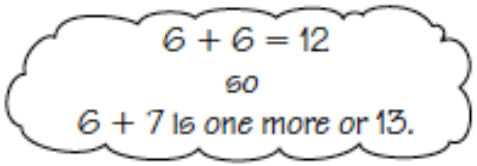
Using Ten

$9 + 8 = 17$



Using Doubles

$6 + 7 = 13$



Addition Strategies Menu

Finding Friendly Numbers

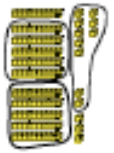
138 + 29

140 + 30 = 170

170 is a reasonable estimate.

Using Base-Ten Pieces

$$\begin{array}{r} 68 \\ +\,55 \\ \hline 123 \end{array}$$

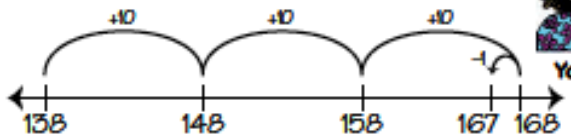


Trade 11 skinnies and 13 bits for
1 flat, 2 skinnies, and 3 bits

Counting On

138 + 29

138 + 30 − 1 = 167



137	138	139	140
147	148	149	150
157	158	159	160
167	168	169	170

138 → 148 → 158 → 167 ← 168

Using Expanded Form

$$\begin{array}{r} 68 = 60 + 8 \\ +\,55 = 50 + 5 \\ \hline 110 + 13 = 123 \end{array}$$

Using All-Partials

$$\begin{array}{r} 68 \\ +\,55 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$$

Using the Compact Method

$$\begin{array}{r} {\scriptstyle 1}\,68 \\ +\,55 \\ \hline 123 \end{array}$$

Use Menus to prompt. . .

Try a method you hardly ever choose.

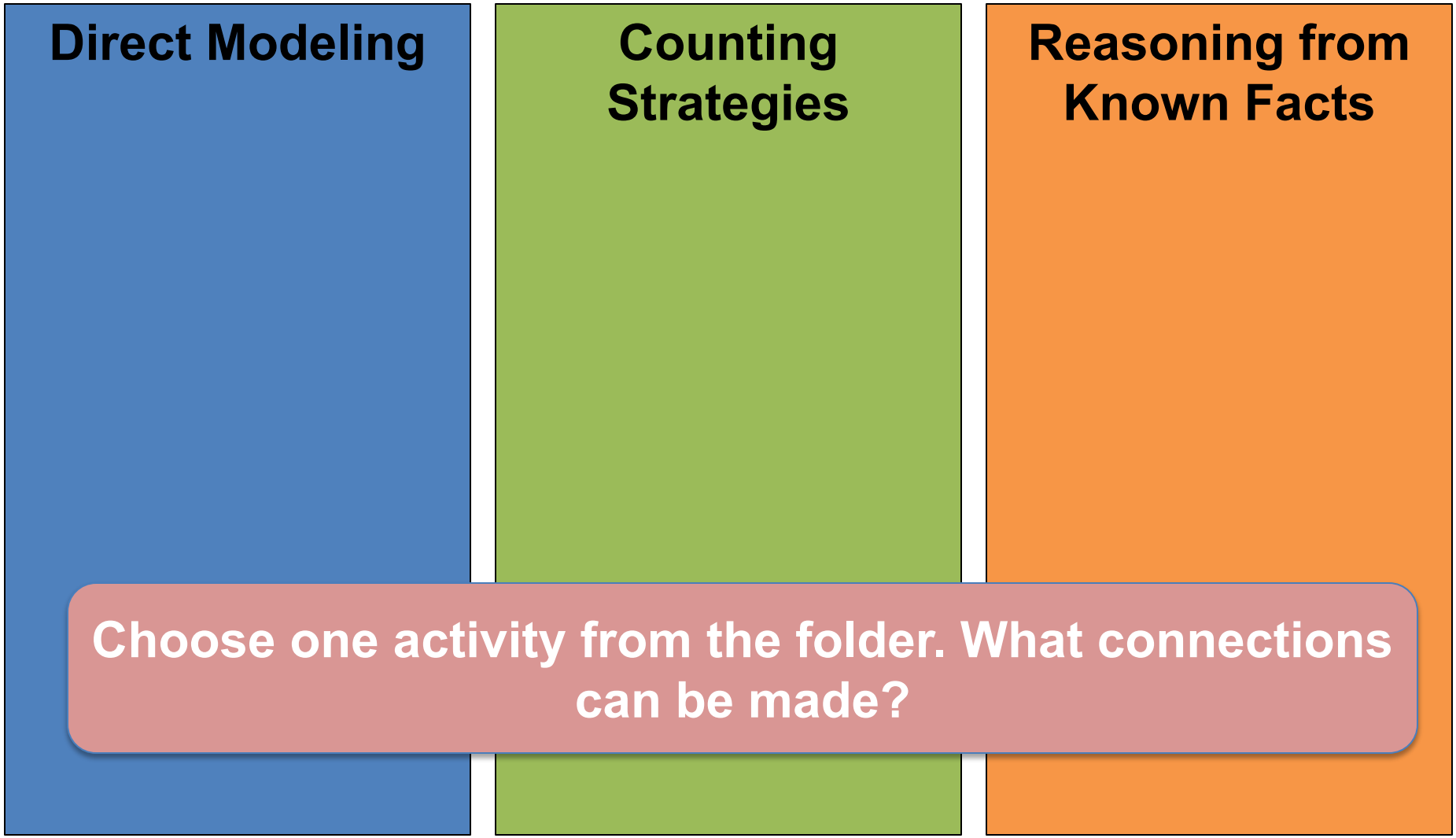
Show Tanya's method using a number line instead.

Which strategy do you think is best?

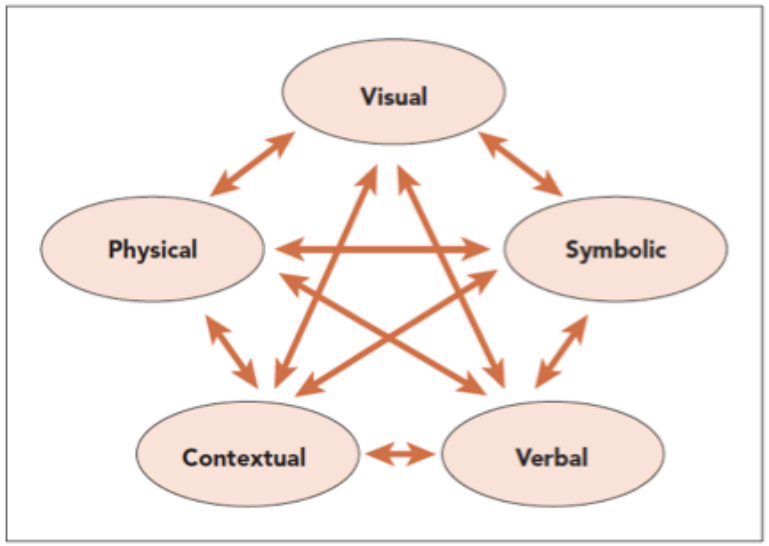
Carols got stuck. . .what strategy do you think will help him?

Is your strategy similar to . . ..

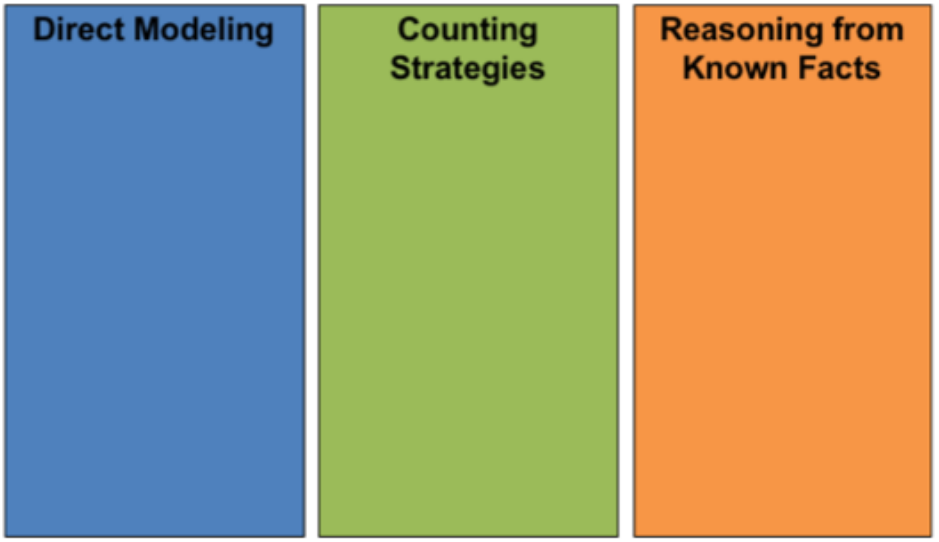
Stages of Conceptual Development



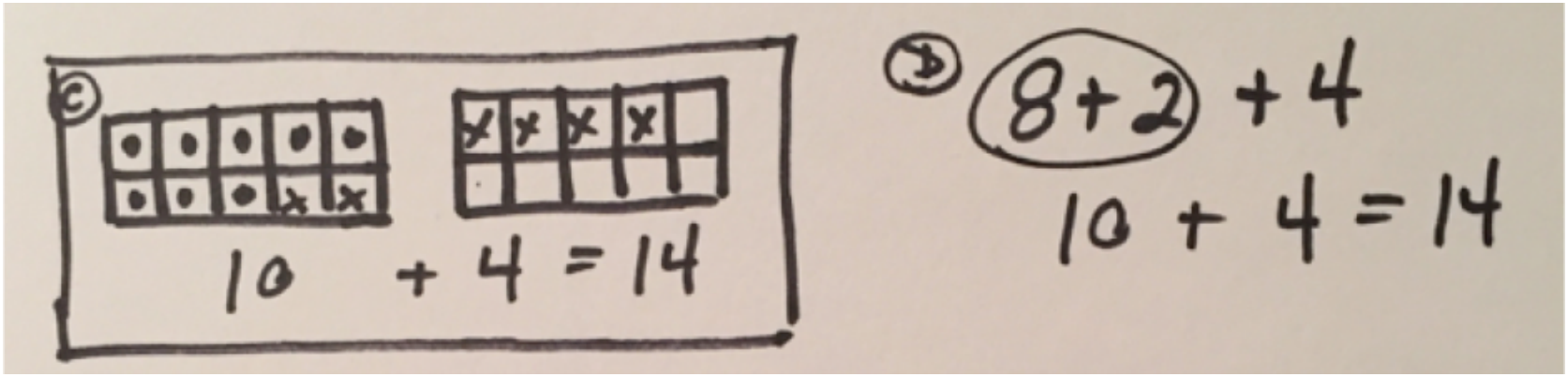
Carpenter, 1999; National Research Council, 2001



Connect Representations



Connect to Reasoning Development



Connect Strategies

Call to Action

- Jot down a take away from today on a post-it note.
- Share with someone in the room.
- Post on your way out.

Final Thoughts

Students are the ones that need to do the connecting.

Keep in touch.

Jennifer Leimberer

@Jleimberer1

@MathTrailblazers

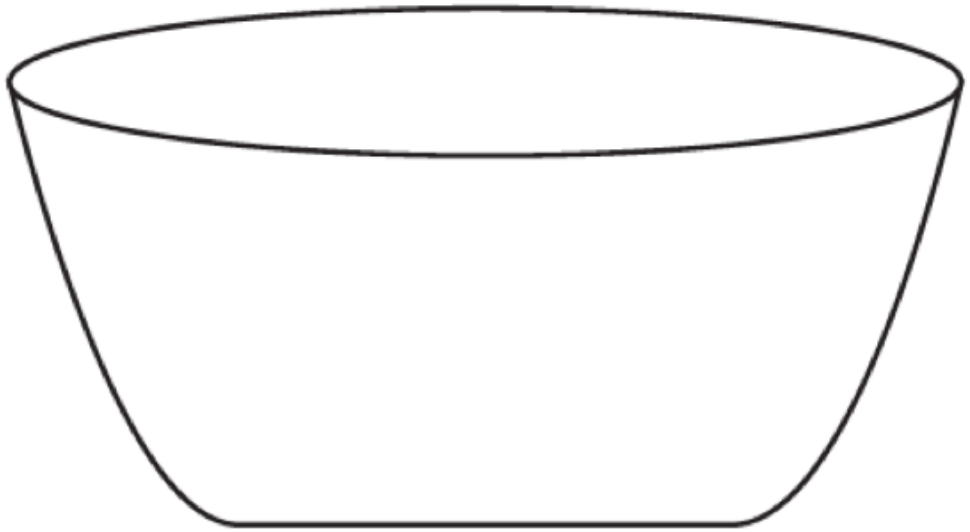
jeimb1@uic.edu

Tell a Story

There are 10 pieces of fruit. Some are in the bowl and some are out of the bowl.

Show and tell a story with the story mat and 10 cubes.

Write a number sentence for your story.



Direct Modeling

Counting Strategies

Flip All 10

- Place 1 chip above each number on the number path. All the chips should have the same color showing.
- Student 1 rolls the die and places the cube on the number on the number path as a marker. He flips over the chip above the number. For example, the number is 6 and the chip above the 6 is turned over. Once a chip is turned over during a game, it cannot be flipped over again.
- Student 2 rolls the die. He can go right or left on the number path. Student 2 starts at the number where the marker is and counts up or counts back. For example, he rolls a 2. He starts at 6 where the marker is and counts up 2 to 8. He flips that chip over.
- Student 1 rolls the die and follows the same procedure. If he or she cannot make a move, he skips a turn.
- The game ends when all the counters are flipped to the other color or there are no more moves.

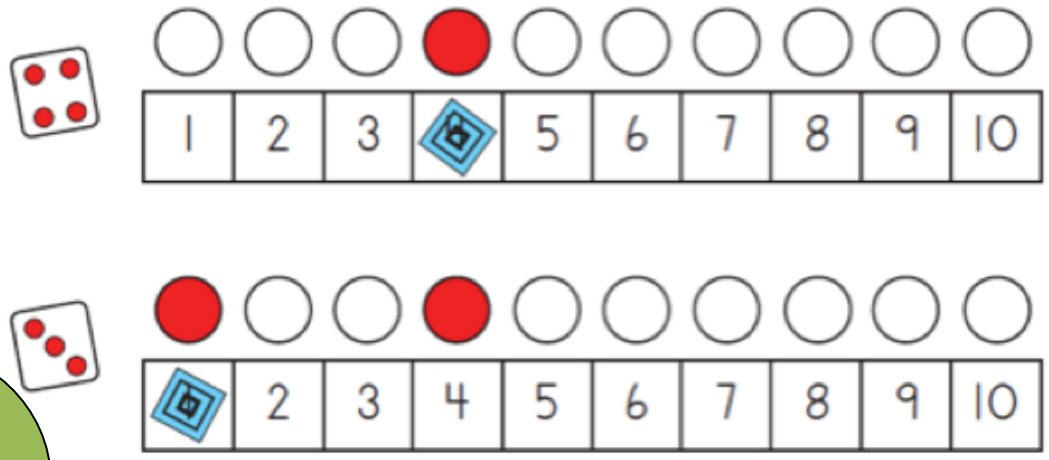


Figure 12: *Game progression of Flip All 10*

Materials:

- 1-10 Number Path
- 1 unifix cube
- 10 two-color chips
- 1 0-5 die (6 is a zero)

Direct Modeling

Counting Strategies

Make 5 (or 10)

Roll 10 (0-5) dice. (6 is a zero.)

Look for ways to make of 5 (or 10). Pull those to the side.

Roll the remaining dice and look again.

Repeat until all dice are paired to show partners to 5.

Counting Strategies

Reasoning from known facts

References

Brown, S., K. Pitvorec, K., Ditto, C., and Kelso, C. R. (2009). Reconceiving fidelity of implementation: An investigation of elementary whole-number lessons. *Journal for Research in Mathematics Education, 40* (4), 363–395.

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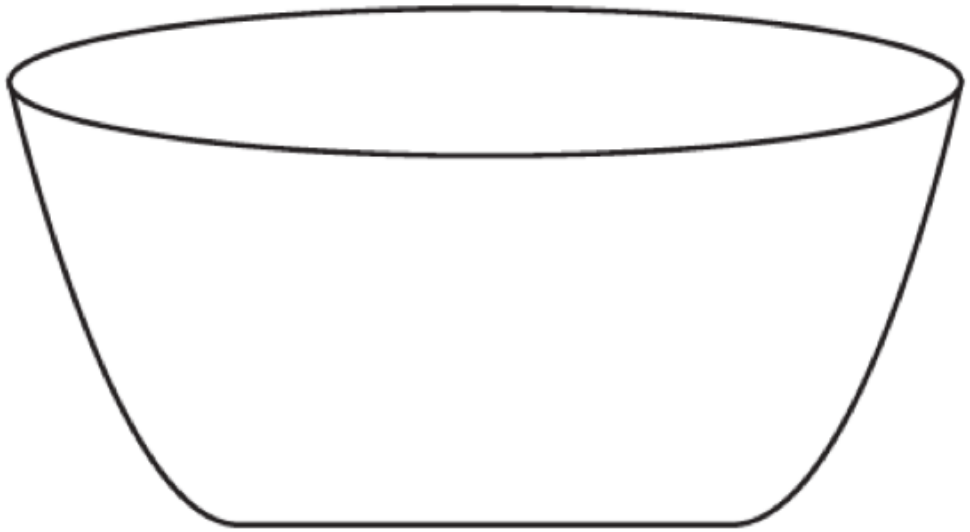
Webb, D. C., Boswinkel, N., & Dekker, T. (2008). Beneath the Tip of the Iceberg: Using Representations to Support Student Understanding. *Mathematics Teaching in the Middle School, 14*(2), 110-113.

Tell a Story

There are 10 pieces of fruit. Some are in the bowl and some are out of the bowl.

Show and tell a story with the story mat and 10 cubes.

Write a number sentence for your story.



Towers of Ten Game

Materials:

- **1 0-5 die**
- **10 unifix cubes in two colors, one tower per student**

Students take turns rolling a die and building a tower with that quantity. The other student then adds blocks to the tower to make ten.

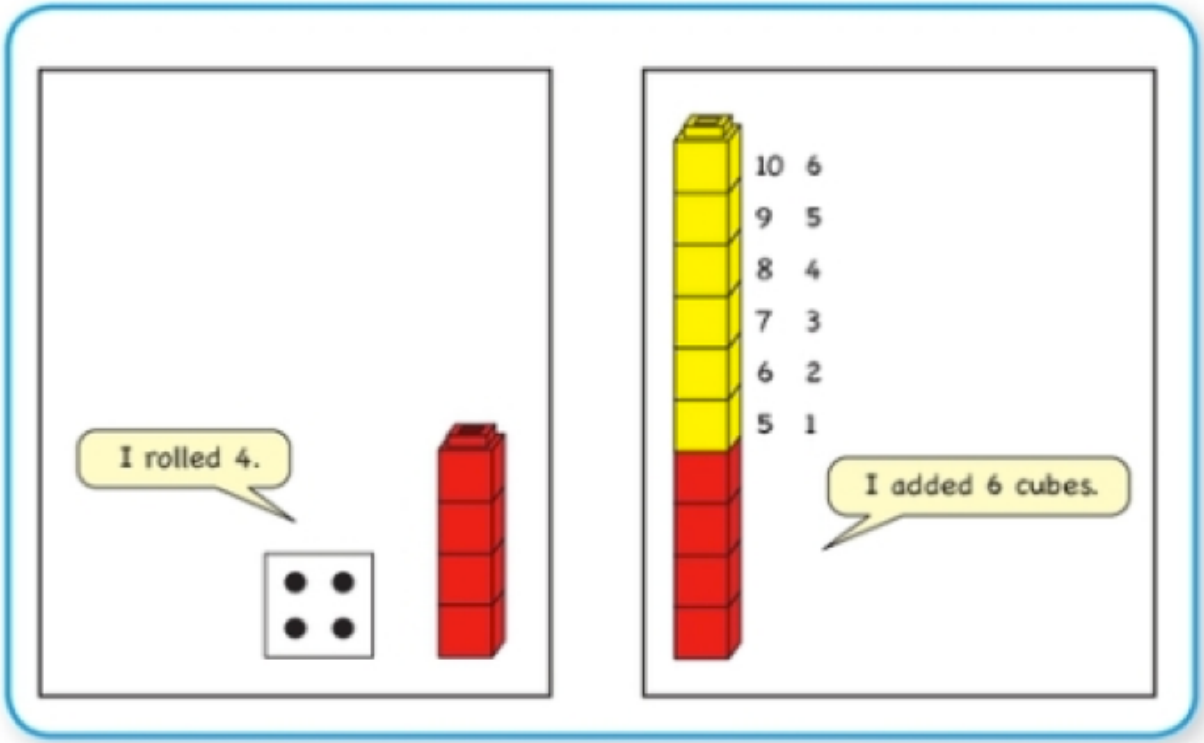


Figure 1: A sample round of Towers of Ten

Flip All 10

Materials:

- 1-10 Number Path
- 1 unifix cube
- 10 two-color chips
- 1 0-5 die (6 is a zero)

- Place 1 chip above each number on the number path. All the chips should have the same color showing.
- Student 1 rolls the die and places the cube on the number on the number path as a marker. He flips over the chip above the number. For example, the number is 6 and the chip above the 6 is turned over. Once a chip is turned over during a game, it cannot be flipped over again.
- Student 2 rolls the die. He can go right or left on the number path. Student 2 starts at the number where the marker is and counts up or counts back. For example, he rolls a 2. He starts at 6 where the marker is and counts up 2 to 8. He flips that chip over.
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- The game ends when all the counters are flipped to the other color or there are no more moves.

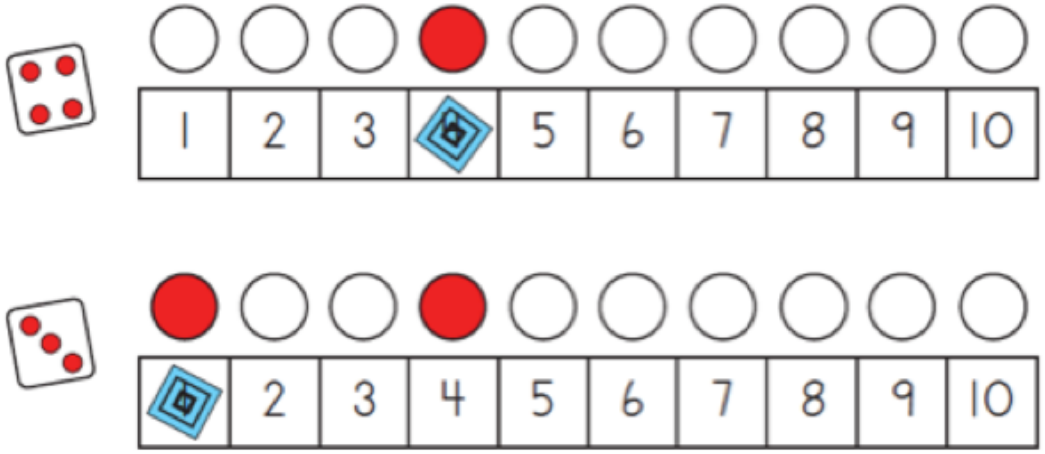


Figure 12: *Game progression of Flip All 10*

Find Ten Matches

Put two sets of 0-10 ten frame cards on the table, face down or face up.

Players take turns finding the partners to ten.

Goal, match all cards.

Make 5 (or 10)

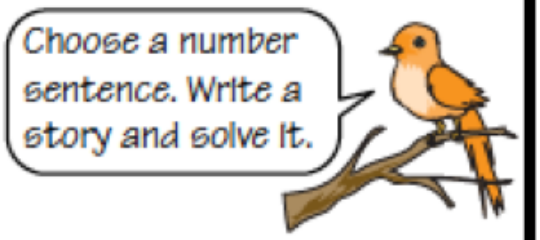
Roll 10 (0-5) dice. (6 is a zero.)

Look for ways to make of 5 (or 10). Pull those to the side.

Roll the remaining dice and look again.

Repeat until all dice are paired to show partners to 5.

Addition Stories



0 + 2

5 + 1

3 + 2

2 + 2

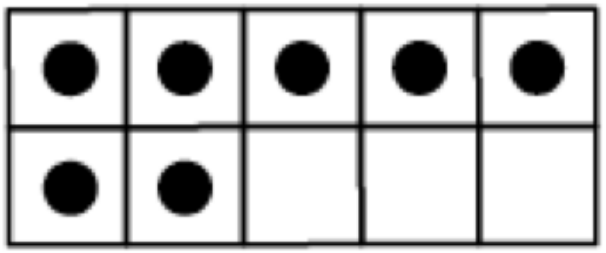
4 + 3

7 + 3

4 + 5

3 + 3

Play Ten Frame Flash: Subtract from Ten

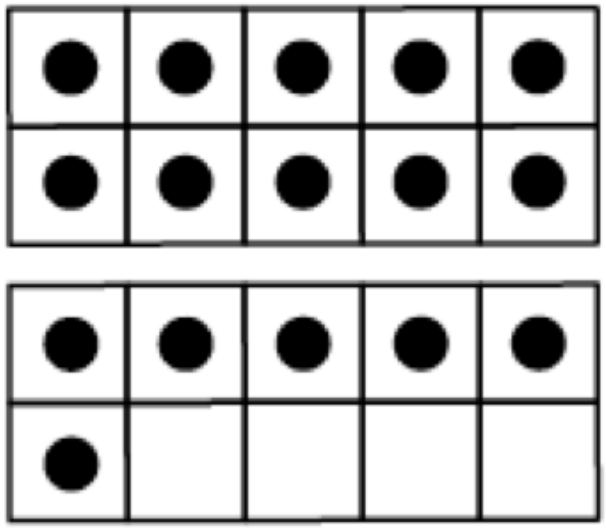


$10 - 3 = 7$

or

$10 - 7 = 3$

Play Minus Five



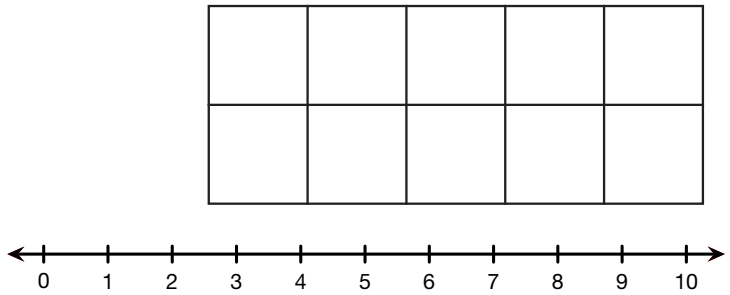
$16 - 5 = 10 + 1$ or 11

Name _____ Date _____

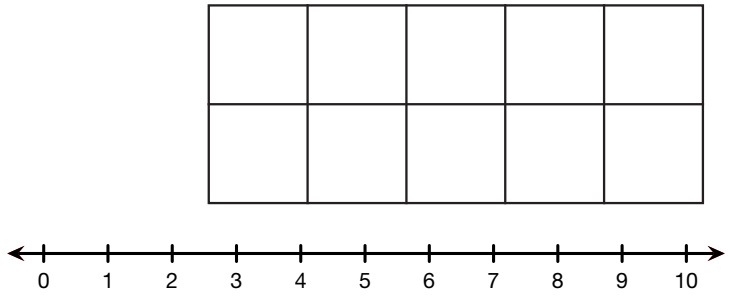
Birthday Party

Show how you solve each problem. Use connecting cubes, ten frames, or number lines.

1. John has 5 red balloons and 3 blue balloons. How many balloons does he have?

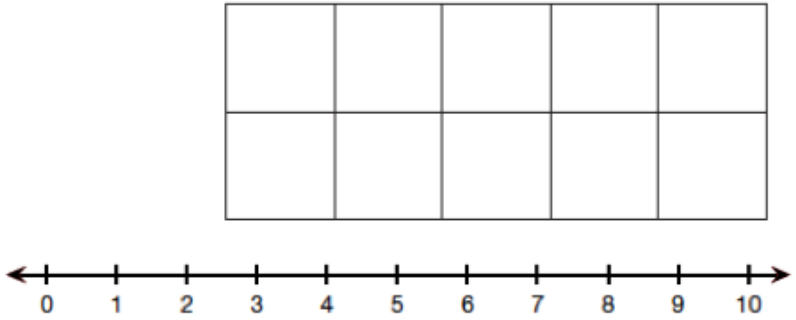


2. There are 9 children at the party. Four of the children are girls. How many are boys?

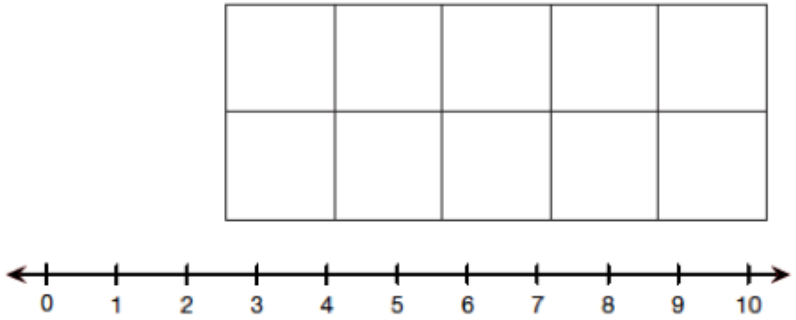


Name _____ Date _____

3. There are some candles on the cake. John put 4 more candles on the cake and now there are 7 candles. How many candles were on the cake?



4. The children played 5 games at the party. They played 2 games before lunch and some games after lunch. How many games did they play after lunch?

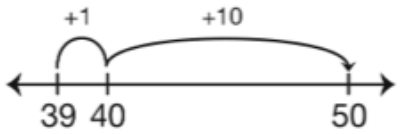
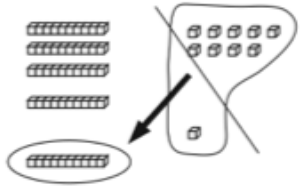


Start By

Solve the addition problems. Start each solution a different way. Circle the strategy you like best.

One Strategy	Another Strategy
1. A. 8 + 9 Start by adding 8 + 8.	**B.** 8 + 9 Start by splitting 8 into 7 + 1.
2. A. 15 + 6 Start by adding 15 + 5.	**B.** 15 + 6 Start by adding 5 + 6.

Possible Strategies for 39 + 11

Start by adding 39 + 10	39 + 10 = 49 49 + 1 = 50
Start by adding 39 + 1	 39 + 1 + 10 = 50
Start by adding 9 + 1	 5 skinnies or 50

0–10 Small Ten Frame Cards