MARSHMALLOW MEASUREMENT: EXPLORING LENGTH WITH NON-STANDARD AND STANDARD UNITS

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BUILDING STUDENT UNDERSTANDING

"It is only when you build from within that you really understand something. If children don't build from within and you just try to explain it to a child, then it's not really learned. It is only rote, and that's not real understanding."

---Ann Badeau, teacher (p. 1)

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (2015, 2nd edition; 1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

MATHEMATICS TEACHING PRACTICE: POSE PURPOSEFUL QUESTIONS

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Smith, M. S., Huinker, D., and Bill, V. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5*. Reston, Virginia: NCTM. (p. 97)

NCTM (2014). *Principles to Action: Ensuring Mathematical Success for All*. Reston, Virginia: Author. *(p. 35)*

A FRAMEWORK FOR TYPES OF QUESTIONS TO ENGAGE IN PRODUCTIVE MATHEMATICS DISCUSSIONS

Assessing Questions:

1. Gathering Information: Students recall facts, definitions, or procedures.

2. Probing Thinking: Students explain, elaborate, or clarify their thinking; including articulating the steps in solution methods or the completion of a task.

Advancing Questions:

3. Making the Mathematics Visible: Students discuss mathematical structures and make connections among mathematical ideas and relationships.

4. Encouraging Reflection and Justification: Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work. ---Smith et al. (2017) and NCTM (2014).

MEASUREMENT

Measurement of space is the intersection of spatial reasoning and number sense. Children must first identify and understand what aspect of an object they are going to measure.

Schifter, D., Bastable, V., & Russell, S. (2002). *Measuring Space in One, Two, and Three Dimensions. Pearson Learning Group. (p. 185)*

MEASUREMENT INSTRUCTION SHOULD…

- Focus on identifying the attribute to be measured.
- Estimation of the results of measurement both before and after measuring.
- Make comparisons between objects by matching.
- Measuring in both non-standard units and standard.
- Able to select the appropriate system to be used standard or metric.
- Pairing the attribute to be measured with the proper units for that attribute. Length must be measured with units of length (cm, in, m, etc.).
- Select an appropriate tool to measure and determine how accurate the measurement needs to be using the tool.

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms.* Sausalito: Math Solutions Publications. (p.xiv)

COMMON CORE STANDARDS K

Describe and compare measurable attributes.

CCSS.MATH.CONTENT.K.MD.A.1
Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

CCSS.MATH.CONTENT.K.MD.A.2
Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

COMMON CORE STANDARDS 1ST

Measure lengths indirectly and by iterating length units.

CCSS.MATH.CONTENT.1.MD.A.1
Order three objects by length; compare the lengths of two objects indirectly by using a third object.

CCSS.MATH.CONTENT.1.MD.A.2
Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

COMMON CORE STANDARDS 2ND

Measure and estimate lengths in standard units.

CCSS.MATH.CONTENT.2.MD.A.1
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

CCSS.MATH.CONTENT.2.MD.A.2
Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

CCSS.MATH.CONTENT.2.MD.A.3
Estimate lengths using units of inches, feet, centimeters, and meters.

CCSS.MATH.CONTENT.2.MD.A.4
Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

UNITS

1. Decide on the *attribute* to be measured.

2. Select a unit that has that *attribute*.

3. Compare the units, by filling, covering, matching, or some other method, with the *attribute* of the object being measured.

Van de Walle, J. (2004). *Elementary and Middle School Mathematics: TeachingDevelopmentally*. Boston: Pearson Education. *(p. 317)*

COMPARING BY MATCHING CENTER

Create a station with a variety of objects for students to order by length from shortest to tallest.

Or, give students a benchmark length (the green rod) and they have to find objects that are longer, shorter, or about the same length as that object in a bucket of objects.

NAME TRAINS

Partner 1					
Partner 2					

The name _____ is shorter.
The name _____ is longer.
Both _____ and _____
are the same length.

Who has the shortest name?
Longest name?

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms*. Sausalito: Math Solutions Publications. (pp.10-11)

Make a cube train that is the same length as your name.

Compare your name train with your partner's. Record the information on a recording sheet.

Other ideas:

Compare your name to a name in a book.

Use the name trains to line the class up in order by name length for recess.

Compare last name to first name.

UNIT ITERATION

"When a student lays down toothpicks to measure length and leaves gaps or overlaps, the student is struggling with *unit iteration*. He doesn't understand that the distance of the units altogether should be equal to the distance measured."

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms*. Sausalito: Math Solutions Publications. (p.xiv)

CROOKED PATHS

1. Draw some crooked paths on the sidewalk with sidewalk chalk or on the floor with tape. Let students free explore with measurement to decide which of the paths is longer (shorter).

2. Provide various manipulatives including string or rope they can use for measuring with non-standard units.

3. Resist the temptation to tell students how to measure and instead let them develop their approaches and watch for proper unit iteration and other measurement techniques like using a common unit (straw, yarn, cube, etc.).

Class share: How did you decide which path was longer?

RULER AS AN ITERATION OF UNITS

We are accustomed to distinguish counting from measuring, and all measuring is counting…when we count the children in a class, we measure the class as a whole – it is a large or small class, etc….

On the other hand, in measuring a continuous quantity… counting is equally necessary. We may apply a unit of measure to such a quantity and mark off the parts with perfect accuracy, but there is no measurement till we have *counted* the parts.

McLellan & Dewey, 1908, pp. 47-48

UNITS OF ITERATION ACTIVITY

Snake Imprints Using Non-standard units

- In whole group invite a student to come up and measure the length of an object using a non-standard unit like a paper clip or pencil.
- Discuss why it is important to count the number of units placed end-to-end to determine the length of the object without any gaps or overlaps.
- Use playdough to make a snake the length of the object. Starting at one end use the measurement tool (paper clip) and make repeated impressions in the clay repeatedly to measure the length of the object.
- Give students playdough and several objects to measure the length.

Dacey, L., Cavanagh, M., Findell C.R., Greenes, C.E., Sheffield, L.J., & Small, M. *(2003). Navigating through Measurement in Prekindergarten-Grade 2.* Reston, VA: NCTM, pp. 37-40.

UNITS OF ITERATION ACTIVITY

Snake Imprints Using Non-standard units

- Also give students non-standard measuring tools they can use to determine the objects length. (plastic links, paper clips, toothpicks, Cuisenaire rods).
- Students should make clay snakes to represent the length of each object to be measured.
- Ask students to estimate the number of non-standard units they will use before measuring.
- Then use the non-standard measuring tool to determine the length of the snake using clay impressions.
- Students could also measure using a second non-standard measuring tool and compare their results.

Dacey, L., Cavanagh, M., Findell C.R., Greenes, C.E., Sheffield, L.J., & Small, M. *(2003). Navigating through Measurement in Prekindergarten-Grade 2.* Reston, VA: NCTM, pp. 37-40.

GIANT STEPS, BABY STEPS

- Use tape to mark a start and finish line on the floor. (about 10ft)
- Have students take turns measuring the distance using giant steps, and baby steps.
- Students can work in pairs so one person steps and the other person counts and records the number of steps in a table or chart.
- Then in whole group compare the number of giant steps. Why do we have different measurements for the number of steps?
- Why do you think it took fewer giant steps than baby steps?
- Predict: If you were to measure using normal walking steps do you think it would take more walking steps than giant steps? Why?
- Measure again with normal walking steps and test your prediction.

Dacey, L., Cavanagh, M., Findell C.R., Greenes, C.E., Sheffield, L.J., & Small, M. *(2003). Navigating through Measurement in Prekindergarten-Grade 2*. Reston, VA: NCTM, pp. 32-33.

STUDENTS NEED TO UNDERSTAND THE INVERSE RELATIONSHIP

"When a student thinks that, when measuring with small units, a small total should result, the student does not yet know the *inverse relationship* between the size of the unit and the number of units-small units create a larger total and large units create a smaller total."

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2* Classrooms. Sausalito: Math Solutions Publications. (p.xiv)

How can exploring with non-standard units help students to develop this understanding?

MARSHMALLOW MEASUREMENT

Students use large and small marshmallows for measuring to "see" the inverse relationship between the size of the unit and the number of units it takes to measure.

https://www.teacherspayteachers.com/Product/Marshmallow-Measurement-FREE-131212

Does it take more large or small marshmallows to measure something? Why?

CONSERVATION OF LENGTH

Conservation of Length Video

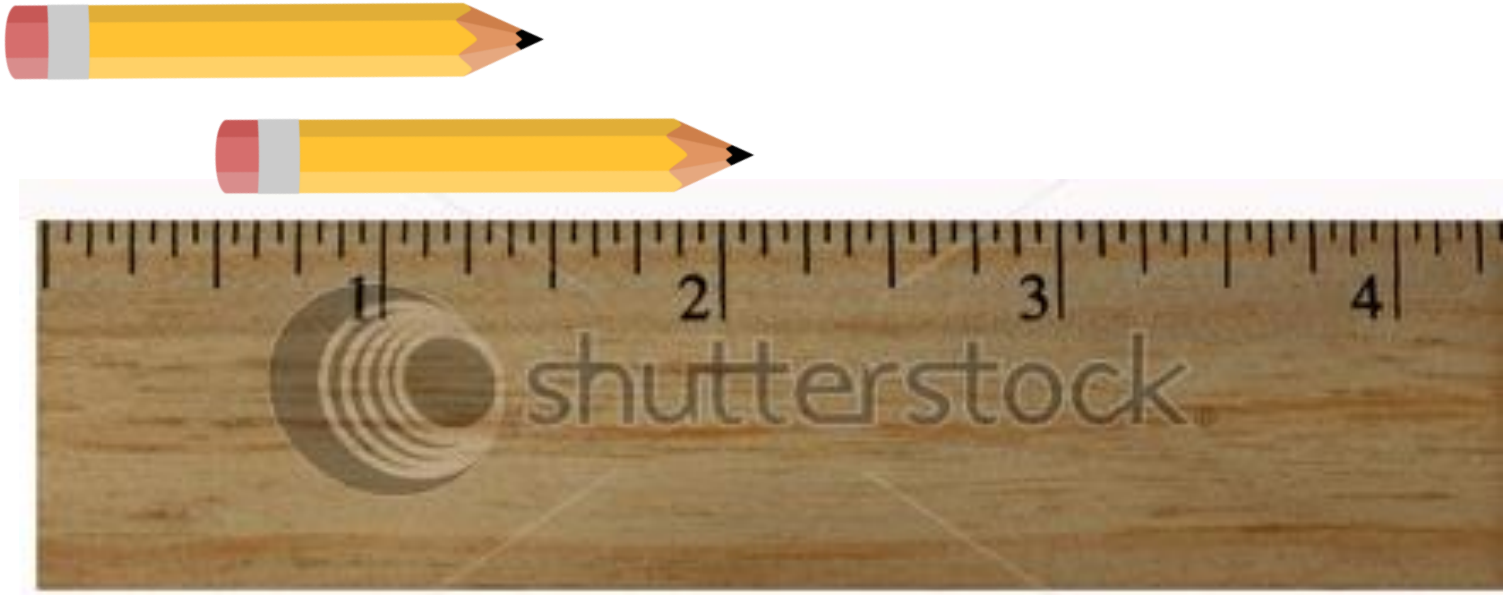
https://youtu.be/oc-dZVQ_aCM

Conservation of length includes understanding that lengths span fixed distances and the understanding that as an object is moved, its length does not change. For example, if children are shown two equal length rods aligned, they usually agree that they are the same length. If one is moved to project beyond the other, children 4½ to 6 years often state that the projecting rod is longer (at either end; some maintain, "both are longer"; the literature is replete with different interpretations of these data, but certainly children's notion of "length" is not mathematically accurate). At 5 to 7 years, many children hesitate or vacillate; beyond that, they quickly answer correctly. Conservation of length develops as the child learns to measure (Inhelder, Sinclair, and Bovet, 1974).

CONSERVATION OF LENGTH

"When a student compares the length of pencils that are not evenly lined up and thinks that the pencil that sticks out is longer, the student has not yet developed conservation of length – the idea that a different position does not change the length."

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms*. Sausalito: Math Solutions Publications. (p.xiv)



TRANSITIVE REASONING

"When a student knows that the marker is shorter than the pencil, and the pencil is shorter than the stick, but doesn't realize the marker therefore must be shorter than the stick, the student has yet to develop transitive reasoning. This is necessary in order for children to understand how rulers help us compare objects that are not side by side."

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms*. Sausalito: Math Solutions Publications. (p.xv)

TRANSITIVE REASONING: YARN HUNT

➢Challenge students to look around the room and list three objects they think are shorter than their yarn and three objects they think are longer than their yarn. Students can then measure the objects with their yarn to check their predictions.

Compared with My Yarn	
Shorter	Longer

➢Share results and discuss how the eraser was longer than one student's piece of yarn but still longer than another student's piece of yarn.

TRANSITIVE REASONING: YARN LINE-UP

- Hold up a piece of cut yarn and ask the students to describe the length. (focus on comparison language…longer than a pencil..shorter than me)
- Give each student in a group a piece of yarn of a different length. (Students could also be given objects that are different lengths.) Try to distribute them randomly. Then challenge the students to line themselves up from the one with the shortest piece of yarn to the longest piece of yarn. Before they line-up ask them to predict what will happened to the yarn as we look at our line?
- When you say "go" give them a time limit to line up in order.
- Once they are in-line have them hold up pieces of yarn all at the same time to see the progression.
- Then switch and have them line up longest to shortest.

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms*. Sausalito: Math Solutions Publications. (pp. 3-6)

UNDERSTANDING THE RULER

"When you put a pencil against the ruler between 2 and 8, and a student thinks the pencil is 8 inches long, the student doesn't understand that the number on the ruler represents the entire distance from the 'zero end' to that number."

Bachman, V. (2007). *Sizing up Measurement: Activities for Grades K-2 Classrooms*. Sausalito: Math Solutions Publications. (p.xv)

MAKING AND USING RULERS

Van de Walle, J., & Lovin, L. (2006). *Teaching student-centered mathematics, grades 3-5.* Boston, MA: Pearson, Inc.

"We often assume that students in the intermediate grades understand how rulers are used to measure. Evidence suggests otherwise, even for students in the fifth grade. Rather than trying to explain how to use a standard ruler, your time may well be better spent with at least one lesson in which students actually make and use their own rulers an compare them to standard rulers." (p. 259)

1. Use a long strip of tagboard about 5 cm wide.
2. Cut paper units (1 inch or 5 cm) in two different colors from narrow strips of construction paper.
3. Alternate colors and paste to a tagboard strip and don't start at the end of the tagboard.
4. Measure with the new 'rulers' before putting numbers on the rulers.

MORE THAN ONE WAY

Challenge students to find different ways to measure the same length with one ruler.

For example, start from either end; start at a point not at the end; measure different parts of the object and add the results. (p. 259)

Van de Walle, J., & Lovin, L. (2006). *Teaching student-centered mathematics, grades 3-5*. Boston. MA: Pearson, Inc.

UNITS OF ITERATION ACTIVITY
INCH BY INCH

- Read *Inch by Inch* by Leo Lionni (1995)
- Ask the children: Have you ever seen a worm outside? (Discuss their lengths)
- Show the children the pictures of animals they need to measure.
- https://www.teacherspayteachers.com/Product/Inchworm-Measuring-Fun-516918
- Discuss all the things we could measure on the animals and then decide what attribute they will measure.
- *As the children are working and you are circulating, what will you notice?*
- *This could also be used as a center activity.*

TEACHERS PAY TEACHERS FREEBIES

Sea Life Measurement: Animal cards that can be used for the Inch by Inch activity. Also includes water drops that can be used for non-standard measurement.

https://www.teacherspayteachers.com/Product/Sea-Life-Math-Measurement-Graphing-1803259

Jellybean Measurement: Measure pictures using jelly bean pictures as the unit. You could use real jelly beans.

https://www.teacherspayteachers.com/Product/Jellybean-Measuring-Common-Core-Aligned-FREEBIE-701761

MORE FREEBIES

Body Measurements: Students estimate how many non-standard units long their different body parts are and then measure and record. This activity has a nice recording sheet. Student can use the same non-standard unit for each body part or they can vary the non-standard unit used for more estimation.

https://www.teacherspayteachers.com/Product/Measure-ME-An-Activity-with-nonstandard-or-standard-units-Freebie-251559

Measurement Games: This activity has a tower building game and a straw length game. There is also a weighing game too.

https://www.teacherspayteachers.com/Product/Measurement-Games-Freebie-733573

Pencil images that can be used for ordering by length or as objects to measure using non-standard units (could be used as a center).

https://www.teacherspayteachers.com/Product/Pencil-Length-Measurement-Comparison-Activity-953526